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## MINIMIZING OF AIR CONSUMPTION FOR THE AIR CUSHION WITH MULTIPLE OUTLET NOZZLES

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## MINIMALIZACJA ZUŻYCIA POWIETRZA DLA PODUSZKI PNEUMATYCZNEJ Z WIELOMA DYSZAMI WYLOTOWYMI

### Abstract

The paper presents the results of research on minimizing the air consumption of the air cushions used to move the transport platform. This kind of transport becomes more popular in industry. It is applied especially to move heavy machinery and equipment in factories with hardened floors. The working medium is air. This system has many advantages but it is characterized by high air consumption. The paper takes the issues of searching solutions to minimizing the air consumption. To perform the computational analysis a mathematical model was defined. Simulations were performed by using Maple software.

*Keywords: air cushion, air consumption, minimizing, Maple*

### Streszczenie

W artykule przedstawiono analizę, której celem jest minimalizacja zużycia powietrza przez poduszki pneumatyczne stosowane w platformach transportowych. W tym transporcie wykorzystuje się jako czynnik roboczy powietrze, które przepływa przez poduszki powodując unoszenie ładunku. System ten ma wiele cech korzystnych, charakteryzuje go jednak znaczne zużycie powietrza. W referacie podjęto zadanie polegające na poszukiwaniu rozwiązań umożliwiających minimalizację zużycia powietrza. Do tego celu zbudowano model matematyczny oraz przeprowadzono obliczenia symulacyjne za pomocą programu Maple.

*Słowa kluczowe: poduszka pneumatyczna, zużycie powietrza, minimalizacja, Maple*

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### Denotations

$A$	– length of the carrying plate [m]
$B$	– height of the carrying plate [m]
$F_o$	– force from a load [kg]
$R_1, R_2$	– external and internal radius of the surface cooperating with the floor [m]
$p_o$	– ambient pressure [MPa]
$T$	– ambient temperature [K]
$p_1$	– inlet pressure [MPa]
$p_{zas}$	– supply pressure [MPa]
$p_2, p_3$	– pressure in air chambers [MPa]
$V_2, V_3$	– volume in air chambers [m <sup>3</sup> ]
$h_f$	– height of the air slit [m]
$h_p$	– height of lifting [m]
$A_i$	– cross sectional area of the nozzle [m]
$v_i$	– average air flow velocity [m/s]
$\mu_i$	– discharge coefficient [–]
$d_i$	– diameter of the nozzle [m]
$\rho_i$	– air density for given cross sections [kg/ m <sup>3</sup> ]
$\rho_p$	– ambient air density [kg/ m <sup>3</sup> ]
$\varphi$	– velocity coefficient [–]
$\kappa$	– adiabatic index [–]
$\eta$	– dynamic viscosity [Pa·s]
$R$	– universal gas constant [J/kgK]
$Q_i$	– volumetric flow rate [m <sup>3</sup> /s]
$i$	– parameter depends on the number of nozzles [–]
$a$	– number of nozzles [–]
$x_i$	– decision variable [–]
$\hat{a}_i$	– optimal point [–]
$\Phi$	– set of acceptable solutions [–]

## 1. Introduction

In the industrial plants, there is a need to transport heavy loads and also locate different types of machines. For this purpose are used expensive lifting devices such as cranes, winches, forklifts, etc. Current production halls being built using new technologies ensure the high quality surfaces, which are horizontal and smooth. These are excellent conditions for the transportation systems using the air cushions. The air cushion is a structurally simple mechanism that is characterized by a small height, so it is easily to slip it under the devices [1, 2]. The air flow under the air cushion causes the formation of air film, which reduces friction for the floor surface to a value close to zero, so people can move the load, which weight is more than few tons using strength of muscles (Fig. 1) [11].

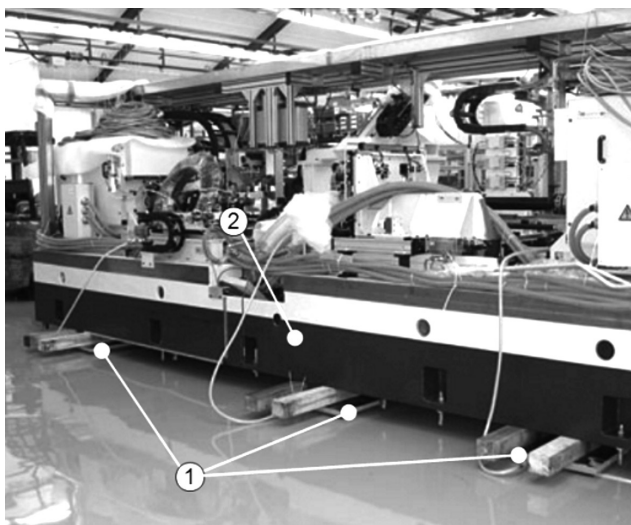


Fig. 1. An Installation weighing 30 tons moved easily on air cushions: 1 – air cushion, 2 – cargo  
 Rys. 1. Instalacja o masie 30 ton przesuwana na poduszkach pneumatycznych: 1 – poduszka pneumatyczna, 2 – ładunek

## 2. Determination of the volume flow rate

To determine the volumetric flow rate in the air cushion a mathematical model was created. The mathematical model was constructed using a simplified model of the air cushion as shown in Fig. 2. The air cushion is built with an internal 4 and an external 7 flexible torus. Torus is fixed to the carrying plate 1 where a supply collector 6 is made. The throttle nozzle 2 is installed to the plate 1. The air flows through the collector 6 to a volume  $V_2$  in the torus further by the throttle nozzle 2 to a volume  $V_3$  and next by the slit created between rigid surface 9 and the cushion torus 7. The air cushion is described by the physical and geometrical parameters as shown in the schema of air cushion (Fig. 2).

In the air cushion beyond the main outlet nozzle 2 and the lower chamber exist additional nozzles 3. These are arranged on the lower surface of the cushion 7 uniformly on a circle of a certain radius  $R_d$ .

With the assumption of the flow continuity and steady state, the volume flow rate in individual points of the air cushion can be expressed by formulas (1)–(5). The flow rate through the supply collector nozzle has been described by equation:

$$Q_1 = \mu_1 \cdot A_1 \cdot v_1 = \frac{\mu_1 \cdot \pi \cdot d_1^2}{4} \cdot \sqrt{\frac{2}{\rho_p} \cdot (p_{zas} - p_1)} \quad (1)$$

where:

- $\mu_1$  – discharge coefficient,
- $A_1$  – cross sectional area of the nozzle,

- $v_1$  – average air flow velocity,  
 $d_2$  – diameter of the nozzle,  
 $\rho_p$  – ambient air density,  
 $p_1$  – inlet pressure,  
 $p_3$  – pressure in air chamber.

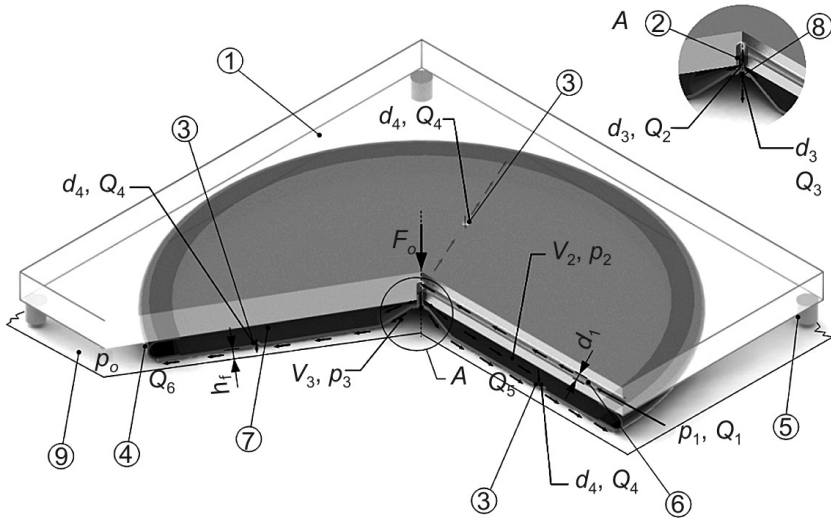


Fig. 2. Model of the air cushion with multi-nozzle air outflow: 1 – carrying plate, 2, 3 – nozzles, 4 – side surface of the air chamber, 5 – bracket, 6 – supply collector, 7 – cushion bearing surface, 8 – fixing plate, 9 – rigid surface

Rys. 2. Model poduszki pneumatycznej z wieloma dyszami: 1 – płyta nośna, 2, 3 – dysze, 4 – powierzchnia boczna komory powietrznej, 5 – podpora, 6 – kanał zasilający, 7 – powierzchnia nośna poduszki, 8 – płytka mocująca, 9 – podłoże

The volumetric flow rate through the nozzles respectively to the side chamber and the lower chamber were expressed by the following equations:

$$Q_2 = \mu_2 \cdot A_2 \cdot v_2 = \frac{\varphi \cdot \mu_2 \cdot \pi \cdot d_2^2}{4} \cdot \sqrt{v_1^2 + \frac{2 \cdot \kappa}{\kappa - 1} \cdot \left( \frac{p_1}{\rho_1} - \frac{p_2}{\rho_2} \right)} \quad (2)$$

$$Q_3 = \mu_3 \cdot A_3 \cdot v_3 = \frac{\varphi \cdot \mu_3 \cdot \pi \cdot d_3^2}{4} \cdot \sqrt{v_1^2 + \frac{2 \cdot \kappa}{\kappa - 1} \cdot \left( \frac{p_1}{\rho_1} - \frac{p_3}{\rho_3} \right)} = Q_5 \quad (3)$$

The volumetric flow rate each of the nozzles placed in the surface cooperating with the floor of the side chamber with assumption of the load uniformity and the symmetry of system:

$$Q_4 = \mu_4 \cdot A_4 \cdot v_4 = \frac{\mu_4 \cdot \pi \cdot d_4^2}{4} \cdot \sqrt{\frac{2}{\rho_2} \cdot (p_2 - p_3)} \quad (4)$$

The total volumetric flow rate through the slit between the cushion and the rigid surface:

$$Q_6 = Q_5 + a \cdot Q_4 = \frac{\pi \cdot p_3}{6 \cdot \eta \cdot \ln\left(\frac{R_1}{R_2}\right)} \cdot h_f^3 \quad (5)$$

where:

- $a$  – number of nozzles,
- $h_f$  – height of the air slit,
- $p_3$  – pressure in air chamber,
- $\eta$  – dynamic viscosity,
- $R_1, R_2$  – external and internal radius of the surface cooperating with the floor.

### 3. Minimizing of the air consumption

To determine the minimum value of volumetric flow rate correlation (6) [7] was defined:

$$\hat{a} \in \Phi = \underset{s \in \Phi}{\wedge} Q(x) \geq Q(\hat{a}) \quad (6)$$

where:

- $x_i$  – decision variable,
- $\hat{a}_i$  – optimal point,
- $Q$  – volumetric flow rate,
- $\Phi$  – set of acceptable solutions.

The decision variables are assigned to the geometrical and physical parameters of the air cushion with restrictions:

- $x_1 = R_1$  for  $x_{1A} \leq R_1 \leq x_{1B}$  [m] where  $R_1$  is the external radius of the torus cooperating with the floor.
- $x_2 = R_2$  for  $x_{2A} \leq R_2 \leq x_{2B}$  [m] where  $R_2$  is the internal radius of the torus cooperating with the floor.
- $x_3 = p_1$  for  $x_{3A} \leq p_1 \leq x_{3B}$  [Pa] where  $p_1$  is the inlet pressure.
- $x_4 = a$  for  $x_{4A} \leq a \leq x_{4B}$  [-] where  $a$  is the number of nozzles.
- $x_5 = d_4$  for  $x_{5A} \leq d_4 \leq x_{5B}$  [m] where  $d_4$  is the diameter of the nozzle.
- $x_6 = h_f$  for  $x_{6A} \leq h_f \leq x_{6B}$  [m] where  $h_f$  is slit height.

The minimization was performed by using Maple software. For minimizing problem Maple used linear programming and modified Newton method. Adopting minimum of objective function  $Q_6(x_1 \dots x_6)$  gives problem solution. Data adopted for the analysis:

$$T = 298.15 \text{ [K]}; \quad \eta = 1.79 \cdot 10^{-5} \text{ [Pa} \cdot \text{s]}; \quad R = 287.05 \left[ \frac{\text{J}}{\text{kg} \cdot \text{K}} \right]; \quad \mu_4 = 0.05 \text{ [-]}; \quad \mu_5 = 0.4 \text{ [-]};$$

$$p_2 = 0.71 \cdot 10^5 \text{ [Pa]}; \quad p_3 = 0.69 \cdot 10^5 \text{ [Pa]}; \quad x_{1A} = 0.22 \text{ [m]}; \quad x_{1B} = 0.3 \text{ [m]}; \quad x_{2A} = 0.1 \text{ [m]};$$

$$x_{2B} = 0.2 \text{ [m]}; \quad x_{3A} = 1 \cdot 10^5 \text{ [Pa]}; \quad x_{3B} = 3 \cdot 10^5 \text{ [Pa]}; \quad x_{4A} = 0 \text{ [-]}; \quad x_{4B} = 20 \text{ [-]};$$

$$x_{5A} = 0.005 \text{ [m]}; \quad x_{5B} = 0.01 \text{ [m]}; \quad x_{6A} = 0.5 \cdot 10^{-4} \text{ [m]}; \quad x_{6B} = 1 \cdot 10^{-4} \text{ [m]}$$

For minimization task by using Maple software NLPsolve function was applied. Definition of that function is shown in Fig. 3.

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Optimization[NLPsolve](Q6,
R1 = 0.22 ..0.3,
R2 = 0.1 ..0.2,
p1 = 1 · 105 ..3 · 105,
a = 0 ..20,
d4 = 0.005 ..0.01,
hf = 0.50 · 10-4 ..1 · 10-4,
initialpoint = [R1 = 0.27, R2 = 0.15, p1 = 200000, d4 = 0.08, hf = 0.75 · 10-4, a = 10],
assume = nonnegative, method = modifiednewton).

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Fig. 3. The definition of NLPsolve function in the optimization task by using the Maple software  
Rys. 3. Definicja zadania optymalizacji za pomocą funkcji NLPsolve w programie Maple

#### 4. Conclusions

Analysis of the air cushion parameters affecting to the air consumption was performed. It was found that parameters have a significant impact on air consumption. As a result of

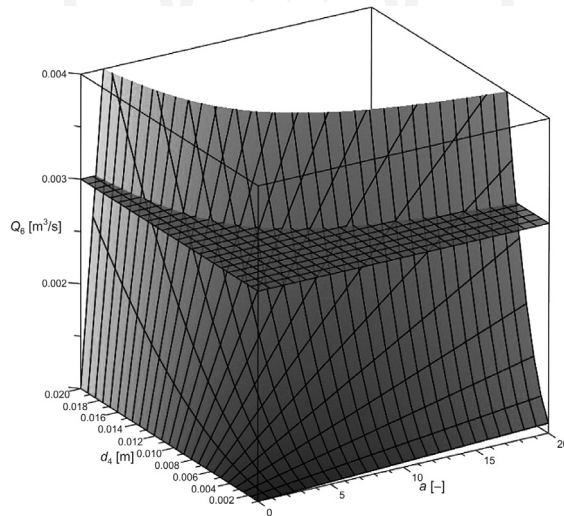


Fig. 4. The volumetric flow rate at the outlet  $Q_6$  [m<sup>3</sup>/s] dependence of nozzle's number  $a$  [-] and nozzle's diameter  $d_4$  [m]  
Rys. 4. Zależność objętościowego natężenia przepływu na wylocie  $Q_6$  [m<sup>3</sup>/s] od ilości dysz  $a$  [-] oraz średnicy dyszy  $d_4$  [m]

minimization  $Q_6 = 0.0004$  [m<sup>3</sup>/s] was obtained for the following values of the decision variables:  $R_1 = 0.3$  [m];  $R_2 = 0.1$  [m];  $h_f = 0.5 \cdot 10^{-4}$  [m];  $d_4 = 0.01$  [m];  $p_1 = 1 \cdot 10^5$  [Pa];  $a = 0$  [-]. The results of the decision variables are on the limits of sets. The minimization including six decision variables showed that in addition to the significant impact of  $R_1$  and  $R_2$  to the needs of air consumption important are the inlet pressure  $p_1$ , height of the air slit  $h_f$ , diameter of outlet nozzle  $d_4$  and number of nozzles  $a$ . After the adoption of  $x_1, x_2, x_3, x_6$  as constant values the influence of diameter of outlet nozzle  $d_4$  and number of nozzles  $a$  for air consumption can be determined. Adopted data:  $R_1 = 0.24$  [m];  $R_2 = 0.16$  [m];  $h_f = 0.5 \cdot 10^{-4}$  [m];  $p_1 = 26 \cdot 10^5$  [Pa]. In Fig. 4 the volumetric flow rate at the outlet  $Q_6$  in dependence of diameter of outlet nozzle  $d_4$  and number of nozzles  $a$  is shown. The vertical surface shows the case when mass of load is equal to  $m = 252$  [kg] with a supply pressure  $p_1 = 0.26$  [MPa]. The best solution is for the minimum of  $d_4$  and  $a$ , where  $Q_6 = 0.001$  [m<sup>3</sup>/s].

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