

**TECHNICAL TRANSACTIONS** | **CZASOPISMO TECHNICZNE**  
MECHANICS | MECHANIKA  
1-M/2016

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**MODELLING OF HEAT TRANSFER IN GROUND HEAT  
EXCHANGERS**

**MODELOWANIE PRZENOSZENIA CIEPŁA  
W WYMIENNIKACH GRUNTOWYCH**

**Abstract**

The results of the numerical simulation of heat conduction in ground with a horizontal, tubular (parallel pipes) heat exchanger installed are presented in this paper. On the basis of analysis of courses of temperature isolines in the ground, a simplified mathematical heat transfer model in a horizontal ground exchanger was developed. The ground thermal diffusivity was assumed to be a variable of location. On the basis of the model, the temperature profiles in the ground with a heat exchanger installed were determined.

*Keywords: ground heat exchangers, non-homogeneous ground thermal properties*

**Streszczenie**

Przedstawiono wyniki symulacji numerycznej przewodzenia ciepła w gruncie, w którym znajduje się szereg równoległych, poziomych rur wymiennika ciepła. Na podstawie analizy przebiegów izolinii temperatur w gruncie opracowano uproszczony model matematyczny przenoszenia ciepła w poziomym gruntowym wymienniku. W modelu przyjęto, że dyfuzyjność cieplna gruntu jest zmienna z położeniem. Wykorzystując opracowany model wyznaczono profile temperatur w gruncie z zainstalowanym wymiennikiem.

*Słowa kluczowe: gruntowe wymienniki ciepła, niejednorodne właściwości termiczne gruntu*

**DOI:**

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## 1. Introduction

Ground heat exchangers are the essential components of ground-source heat pump installations. A basic criterion of ground heat exchangers is their orientation – this paper deals with horizontally heat exchangers. An overview of the application of ground heat exchangers and numerical models relating to them was presented by Florides and Kalogirou [1].

Ground temperature varies according to its depth and is the result of both a periodic variability of the ground surface temperature and the ground thermal inertia. During cold periods, the ground temperature exceeds the ground surface temperature by several degrees K – the reverse situation occurs during warm periods. An analysis of receiving heat from various ground depths in different time periods was presented by Gan [2]. If the heat is received from the ground during cold periods, it is better to place the exchanger pipes deep down. Conversely, if the heat is received during warm periods (e.g. for heating water in swimming pool) it is advantageous to place the pipes at a shallow depth, close to the surface.

The thermal properties of the ground are characterised by thermal diffusivity:

$$a = \frac{k}{c \cdot \rho} \quad (1)$$

where:  $k$  – heat conduction coefficient,  $c$  – heat capacity,  $\rho$  – density.

Ground typically consists of three phases: solid, liquid and gaseous. Fluid phases, i.e. water and air, fill the spaces between the grains. Therefore, the ground is a multi-phase system [3]. For modelling the heat transfer in the ground, it is convenient to use a substitute thermal diffusivity coefficient taking into account both the heat conduction in the solid and the heat transport in the fluid filling the void space. Because the ground humidity depends on the ground location, the variability in thermal diffusivity should be taken into account when modelling.

The aim of this work is to determine the temperature profiles in the ground in which an exchanger associated with a heat pump is installed. The calculations were carried out for ground physical parameters varying with the position with the application of the model based on one-dimensional heat conduction equation with an internal heat source. A possibility of application of 1D model has proved the simulations performed with the use of the ANSYS Transient Thermal application. The effect of the physical ground properties on the temperature distribution in the ground affecting the operation of the ground heat exchanger associated with a heat pump is analysed.

## 2. Simulation of heat conduction in the ground cooled with a system of horizontal pipes using the ANSYS application

In cases of slight changes between the inlet and the outlet temperature of the operating fluid in a horizontal ground heat exchanger, the ground heating and the ground cooling can be considered as a two-dimensional problem. In this paper, the flow through three parallel,

coupled, horizontal pipes located in the ground is analysed. The problem can be described by the two-dimensional transient heat conduction equation:

$$\frac{\partial T}{\partial t} = a \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (2)$$

where:  $T$  – temperature,  $t$  – time,  $x, y$  – position coordinates.

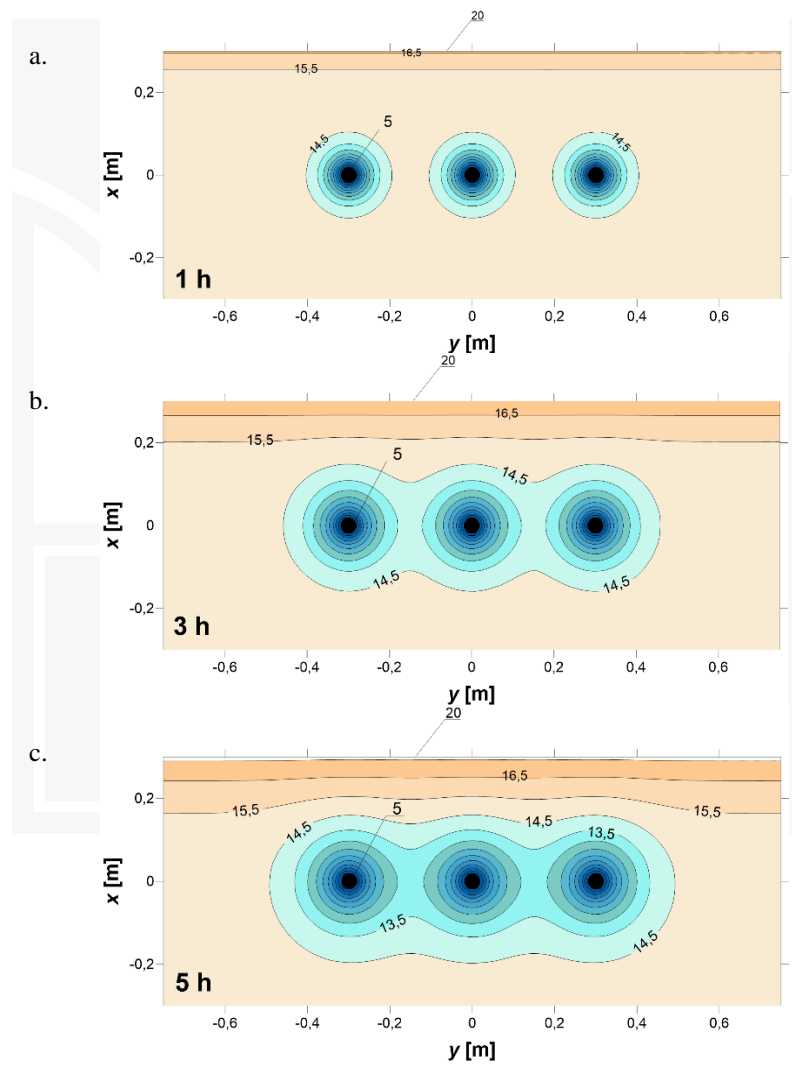


Fig.1. The cooling of ground with exchanger pipes  
a) 1hr, b) 3hrs, c) 5hrs

The initial condition results from the assumption of the uniform ground temperature at the beginning of the process. The boundary condition is related to the constant (in time and space) value of temperature of all the tubes surface. For calculations, the following data were used:  $a = 0.384 \cdot 10^{-6} \text{ m}^2/\text{s}$ , temperature of pipe surface  $5^\circ\text{C}$ , initial ground temperature  $T_i = 15^\circ\text{C}$ , and dimensions of the analysed ground block –  $1.5\text{m} \times 0.6\text{m} \times 0.3\text{m}$  (number of nodes: 381270). Furthermore, the following assumptions were made: the outer diameter of pipes was 38 mm, the distance between the axes of pipes was 300 mm, the ambient temperature was  $20^\circ\text{C}$ , the heat transfer coefficient between the environment and the ground  $h_0$  was  $10 \text{ W}/\text{m}^2\text{K}$ . The calculations relate to ground of homogenous thermal properties. The results of the calculations are presented in Figs. 1a, b, c [4]. The figures are related to the cross-section of the system of pipes. As can be seen, at the beginning, each pipe cools down the ground individually, independently of the other pipes. However, after some time, the temperature fronts come together and isolines become more rectilinear and parallel to the horizontal position. Thus, the system behaves similarly to cooling down the ground by infinite plate. The similarity is even larger when there are more pipes installed, and the more densely arranged they are side by side. Therefore, in the case of large number of pipes the heat transfer in the considered system is practically 1D.

### 3. Heat transfer in ground with non-homogenous thermal properties

#### 3.1. Heat conduction in porous materials

The ground thermal properties change considerably even if the parameters affecting the heat transfer only change slightly. The heat transfer process in the ground is determined by the heat conduction of the grains of minerals present in the ground, the heat conduction of the air and water in the ground void space, possible natural convection inside the void space and the thermal resistance at contact points between the grains. In humid ground below  $0^\circ\text{C}$ , the heat transfer process is more complicated because of the phase transition of the water portion into ice.

Russel derived the following relationship to calculate the effective heat conductivity of porous material as a function of the properties of solid and fluid matter [5]:

$$k = k_s \cdot \frac{\varepsilon^{2/3} + \kappa}{\varepsilon^{2/3} \cdot (1 - \varepsilon^{1/3}) + \kappa} \quad (3)$$

where  $\varepsilon$  is the porosity and  $\kappa$  is defined with the formula:

$$\kappa = k_s / (k_f - k_s) \quad (4)$$

where  $k$ ,  $k_s$  and  $k_f$  respectively denote the heat conductivity of porous material, solid and fluid matter.

If the pores are filled with water, usually  $k_f > k_s$ , then  $\kappa > 0$  and finally  $k > k_s$ . However, if  $k_f < k_s$ , which is always valid for the case of gas in pores, then  $\kappa < 0$  and  $k < k_s$ . In the first case, the greater the porosity (water content), the greater the heat conduction coefficient. In

the case of gas filling the pores, the greater the porosity, the lower the heat conduction coefficient of the system. The Russel model meets the following conditions: for  $\varepsilon = 1$ , there is equality  $k = k_f$  but for  $\varepsilon = 0$ , there is  $k = k_s$ . Moreover, the Russel model describes well an effect of a kind of phase filling the ground on the heat conduction coefficient.

### 3.2. Mathematical model of ground exchanger

On the basis of the heat transfer simulation it should be noted that the constant temperature lines do not differ significantly from those which occur when the exchanger is used as a flat slab. This provides the basis for utilising the one-dimensional equations of heat conduction for the modelling of horizontal ground heat exchangers. It is assumed that the only mechanism of heat transport in the ground was conduction.

The equation of heat conduction has the form [6-8]:

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left[ a(x) \frac{\partial T}{\partial x} \right] + \frac{q_v}{c\rho} \quad (5)$$

where:  $x$  – position coordinate (distance from the ground surface),  $q_v$  – rate of heat generation per volume unit.

The boundary condition at the surface of the ground has the form:

$$x = 0 \quad T = T_0 \quad (6)$$

The ground surface temperature  $T_0$  varies in time according to the formula:

$$T_0 = T_b + B \cdot \cos \left[ \frac{2 \cdot \pi}{t_c} \cdot (t - t_{\max}) \right] \quad (7)$$

where  $t_c = 365$  days. The following values, valid for climatic conditions in Cracow, were used in calculations:  $T_b = 8.5^\circ\text{C}$ ,  $B = 10.4$  K,  $t_{\max} = 198$  days.

The second boundary condition is related to the ground at a great depth, where the temperature is constant:

$$x \rightarrow h_{\text{inf}} \quad T = T_b \quad (8)$$

The rate of heat generation  $q_v$  in equation (5) is the thermal power produced in a volume  $\Delta V = A_g \cdot \Delta x$  [9]. This quantity is connected with the rate of heat transfer  $\dot{Q}$  between the working liquid flowing in a ground heat exchanger and the ground. The calculations were done for  $N$  nodes so  $\Delta x = h_{\text{inf}}/N$ . Since exchanger pipes are arranged at distance  $h$  from the ground surface, then:

$$q_v = \begin{cases} 0 & \text{for } i < n \\ -\dot{Q}_{\text{GHE}} / \Delta V & \text{for } i = n \\ 0 & \text{for } i > n \end{cases} \quad (9)$$

where  $i = 1, 2, \dots, n, \dots, N$ ;  $n = \text{Int}(h/\Delta x)$ .

The ground thermal diffusivity is described with a step function because the variation of ground humidity with the ground depth is step. Two cases are considered:

- a) The thermal diffusivity in the outer ground layer ( $< 2$  m deep) is higher than the diffusivity in the bottom layer. The function  $a(x)$  has the form:

$$a(x) = \begin{cases} 1.0 \cdot 10^{-6} \text{ m}^2/\text{s} & \text{for } x < 2 \text{ m} \\ 0.2 \cdot 10^{-6} \text{ m}^2/\text{s} & \text{for } x \geq 2 \text{ m} \end{cases} \quad (10)$$

In this case, the ground heat exchanger is placed at a depth of 1 m.

- b) The thermal diffusivity in the upper ground layer ( $< 1$  m) is lower than the diffusivity in the bottom layer. The function  $a(x)$  has the form:

$$a(x) = \begin{cases} 0.2 \cdot 10^{-6} \text{ m}^2/\text{s} & \text{for } x < 1 \text{ m} \\ 1.0 \cdot 10^{-6} \text{ m}^2/\text{s} & \text{for } x \geq 1 \text{ m} \end{cases} \quad (11)$$

In this case, the ground heat exchanger is placed at a depth of 2 m.

### 3.3. Computational temperature profiles in the ground

If the thermal diffusivity is high, the temperature gradients are low; therefore, the lines of temperature profiles are steep. Conversely, if the thermal diffusivity is low, the temperature gradients are high; thus the inclination angle of temperature profiles lines is small. In the case of high thermal diffusivity, the temperature in the outer layer approximates the ground temperature whereas in the case of low thermal diffusivity, the outer ground layer is a kind of buffer isolating the bottom layer from the impact of the ground surface.

The presence of a heat exchanger installed in the ground is connected with the heat collection from the ground which results in disturbance of natural temperature profiles. The calculations were carried out for the heat flux collected from the ground  $\dot{Q} = 5000$  W and for the ground surface area  $A_g = 500$  m<sup>2</sup>.

The results of calculations of temperature profiles are depicted in Figs. 2 and 3. The calculations relate to the following operating conditions: the average twenty-four-hour ambient temperature is lower than 7.5°C, the ground temperature at the depth of the arrangement of exchanger pipes is higher than 1°C [10].

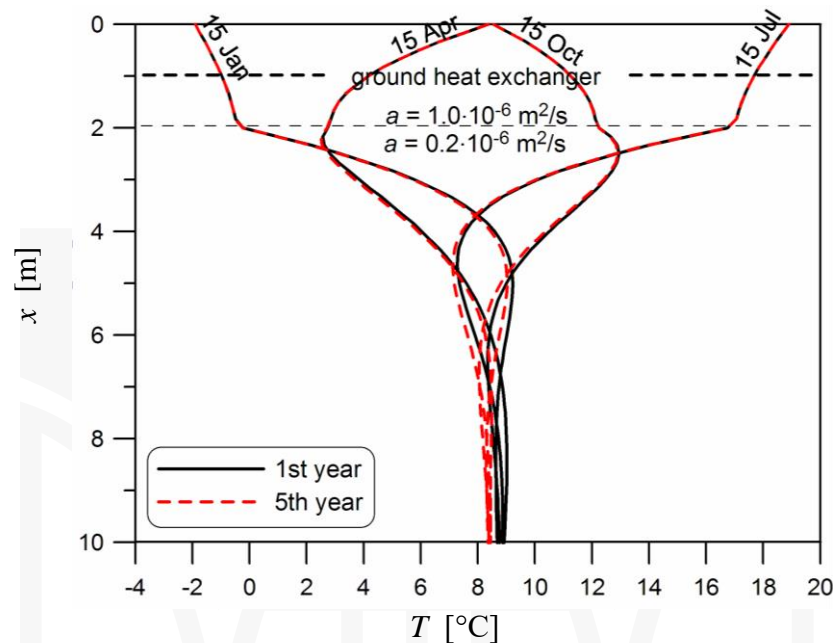


Fig. 2. Ground temperature profiles with heat exchanger installed in the case of higher thermal diffusivity in the outer ground layer

Fig. 2 presents the ground temperature profiles if the exchanger is located at a depth of 1 m, while in the depth range from 0 to 2 m, the ground thermal diffusivity is higher than in deeper layer. The shapes of temperature profiles result from the collection of heat mainly from the ground surface because of good thermal diffusivity and a shallow location of exchanger. The low thermal diffusivity ground layer below 2 m has a slight influence in the heat transfer process.

Fig. 3 presents the results of calculations relating to the location of a ground heat exchanger at a depth of 2 m while below a depth of 1 m, the ground thermal diffusivity is lower than in deeper layers. The low thermal diffusivity of the upper ground layer (0÷1 m) hampers the heat transfer between the ground surface and the cooled ground at the depth of the arrangement of the exchanger pipes. Therefore, the heat is transferred from the deeper ground layer which results in the deformation of their temperature profiles. At a depth of 10 m, the ground temperature is unstable over time – this is in contrast to the previous case.

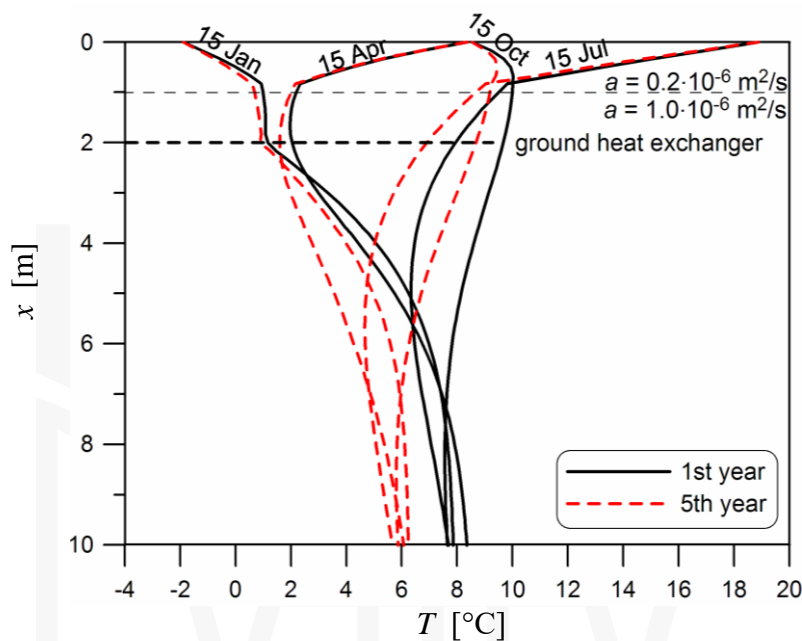


Fig. 3. Ground temperature profiles with heat exchanger installed in the case of lower thermal diffusivity in the outer ground layer

The significant differences in the courses of temperature profiles for the first and the fifth year of operation only occur in the case presented in Fig. 3. In numerical calculations, it was necessary to properly adjust the ground depth where the temperature is stabilised ( $h_{inf}$ ). For the ground thermal diffusivity described with formula (11), it was necessary to assume the value  $h_{inf} = 50$  m.

#### 4. Conclusions

- It was noticed that for the simulation of heat conduction for a system of parallel pipes of a ground heat exchanger that after some time, the temperature isolines are similar to straight lines. This gives a basis for using one-dimensional equations of heat conduction for the modelling of horizontal ground heat exchangers.
- If a higher diffusivity ground layer is located above a lower diffusivity layer, the upper layer does not affect the temperature distribution in the lower layer because the upper layer temperature approximates the ground surface temperature.
- If a higher diffusivity ground layer is located below a lower diffusivity layer, the upper ground layer is a buffer isolating the bottom layer from the impact of the ground surface.
- When a heat exchanger is installed in the ground and the thermal diffusivity



of the upper layer is low, the temperature profiles in the bottom layer undergo considerable and long-term changes over time. After several years of the operation of an exchanger in such conditions, ground could be cool down significantly. This is due to not enough compensation of the heat transfer from the surroundings.

### Nomenclature

- $a$  – thermal diffusivity of the ground,  $\text{m}^2/\text{s}$ ,  
 $A_g$  – area of the ground surface,  $\text{m}^2$   
 $B$  – annual amplitude of the ground surface temperature,  $^\circ\text{C}$ ,  
 $c$  – ground heat capacity,  $\text{J}/(\text{kgK})$   
 $h$  – distance between the heat exchanger and the surface of the ground,  $\text{m}$   
 $h_{inf}$  – distance from the surface of the ground where the ground temperature is undisturbed,  $\text{m}$   
 $k$  – ground thermal conductivity,  $\text{W}/(\text{mK})$   
 $q_v$  – rate of heat generation per unit volume,  $\text{W}/\text{m}^3$   
 $\dot{Q}$  – rate of heat transfer,  $\text{W}$ ,  
 $t$  – time,  $\text{s}$   
 $t_{max}$  – time lag from the beginning of the year to the occurrence of the highest temperature in a year,  $\text{s}$   
 $T$  – temperature of the ground,  $^\circ\text{C}$ ,  
 $T_0$  – average daily ground surface temperature,  $^\circ\text{C}$ ,  
 $T_b$  – average annual ground surface temperature,  $^\circ\text{C}$ ,  
 $x$  – position coordinate,  $\text{m}$   
 $\rho$  – ground density,  $\text{kg}/\text{m}^3$

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