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CONTEXTUAL BIPOLARITY AND ITS QUALITY CRITERIA IN BIPOLAR LINGUISTIC SUMMARIES

BIPOLARNOŚĆ KONTEKSTOWA I KRYTERIA JEJ JAKOŚCI W BIPOLARNYCH PODSUMOWANIACH LINGWISTYCZNYCH

Abstract

Bipolar linguistic summaries of data are assumed to be an extension of the 'classical' linguistic summarization, a data mining technique revealing complex patterns present in data in a human-consistent form. The extension proposal is based on the possibilistic interpretation of the 'and possibly' operator and introduced notion of context, which results in the introduction of the new 'contextual and possibly' operator. As the end user is expecting the most relevant summaries, ways of determining the quality of summary propositions (quality measures) needs to be developed. Here we focus on specific insights into the quality measures of proposed bipolar linguistic summaries of data and present some basic examples of their correctness and necessity of introduction.

Keywords: *bipolarity, context, linguistic summaries, quality criteria*

Streszczenie

Bipolarne podsumowania lingwistyczne są rozwinięciem „klasycznego" podejścia do lingwistycznego podsumowania danych, techniki ich eksploracji, której celem jest odszukanie obecnych w nich wzorców oraz zaprezentowanie ich w przystępnej dla człowieka formie. Propozycja rozwinięcia oparta jest na posybilistycznej interpretacji operatora „and possibly" oraz wprowadzonym pojęciu kontekstu, w wyniku czego zaproponowano nowy operator „contextual and possibly". Ponieważ użytkownik oczekuje prezentacji najbardziej trafnych podsumowań, konieczne jest zaproponowanie sposobów określania ich poprawności, zwanych dalej wskaźnikami jakości podsumowań. W niniejszym artykule skupiono się na szczegółowym spojrzeniu na wskaźniki jakości bipolarnych podsumowań danych i przedstawiono proste przykłady świadczące o ich poprawności oraz konieczności ich wprowadzenia.

Słowa kluczowe: *bipolarność, kontekst, kryteria jakości, podsumowania lingwistyczne*

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1. Introduction

The aim of data mining is to discover patterns in data in a form interesting and clear to the end user. A promising way to achieve this is to use (quasi) natural language. This has been a motivation for the linguistic data summaries introduced by Yager [11] and further developed by him [12] and other contributors, notably Kacprzyk and Zadrożny [8, 9].

Recently, an important role of bipolarity of user preferences, in particular in fuzzy linguistic querying [17], has been noticed. Its essence is in considering both positive and negative evaluations of objects in question which are not necessarily complements of each other. This entails the need to introduce logical connectives other than simple conjunction and disjunction.

An important and most interesting line of research focuses on the treatment of negative evaluations as obligatory while the positive evaluations as somehow secondary. This results in the introduction and study of the 'and possibly' logical connective [1]. Moreover, the concept of bipolar queries involving such a connective has been proposed [2] to better model user preferences as exemplified by the query 'Find an apartment, cheap and possibly located close to a station'.

In our previous papers [4, 5] we began to study if the relationship between fuzzy linguistic queries and linguistic data summaries may be adopted for bipolar queries. The results were positive and led us to the concept of bipolar linguistic summaries of data. In this paper, we focus on two quality criteria of such new type of linguistic summaries, introduced in [5] and referring to the notion of the context of a summary.

The structure of the paper is as follows. In Section 2, we briefly review the basics of fuzzy linguistic queries and 'classical' linguistic summaries, and introduce the notation to be used in the rest of the paper. In Section 3, we discuss the concepts of bipolar queries and bipolar linguistic summaries. Section 4 reports on the computational experiments focused on comparing different summary contexts and discusses the results obtained.

The preliminary version of this paper was presented at the FedCSIS'2013 conference [3].

2. Fuzzy linguistic queries and linguistic data summaries

2.1. Fuzzy linguistic queries

In classical query languages, such as SQL, preferences of users must be expressed precisely. However, due to the fact that their original form is a natural language expression, they are very often imprecise. For example, one may be concerned primarily with the cost while looking for an apartment to rent and express his or her preference as:

Find cheap apartments in Kraków (1)

In an approach, referred to here as fuzzy linguistic queries, such imprecise terms (e.g. cheap) are represented by fuzzy sets defined in the domains of respective attributes.

Usually, a dictionary of linguistic terms is assumed as a part of an implementation which contains predefined linguistic terms and corresponding fuzzy sets as well as terms defined by the users. Linguistic terms collected in a dictionary are a starting point to derive meaningful linguistic summaries of a database.

2.2. Linguistic summaries of data

As linguistic summaries we understand a (quasi) natural language sentences that grasp some characteristic features of data collected in a database. We use Zadeh's calculus of linguistically quantified propositions as the underlying formalism. The statement representing a linguistic summary points out some properties shared by a number of data items and the proportion of these data items is expressed using a linguistic quantifier. Yager [11, 12] first proposed the use of linguistically quantified propositions to summarise data in a user consistent way. That idea has been further developed, cf., e.g., Kacprzyk and Yager [6], and Kacprzyk, Yager and Zadrożny [7, 8].

Assuming $R = \{t_1, ..., t_n\}$ is a set of tuples (a relation) in a database, representing, for example, a set of employees; $A = \{A_1, ..., A_m\}$ is a set of attributes defining schema of the relation R, for example, Comfort, Price, No. of rooms etc., in a real estate database $A_j(t_i)$ denotes a value of attribute A_j for a tuple t_i), the linguistic summary of a set R is a linguistically quantified proposition which is an instantiation of one of the following abstract protoforms [16] of type I and type II, respectively:

$$Q_{t \in R} S(t) \tag{2}$$

$$Q_{t \in R}(U(t), S(t)) \tag{3}$$

(also denoted 'Q of R are S' and 'Q of U are S', respectively) then a linguistic summary is composed of the following elements:

- Summariser S which is a fuzzy predicate representing, for example, an expression an apartment is comfortable, formed using attributes of the set A;
- Qualifier U (optional) which is another fuzzy predicate representing, for example, a set of cheap apartments;
- Linguistic quantifier Q, for example, most expressing the proportion of tuples satisfying the summariser (optionally, among those satisfying a qualifier);
- Truth (validity) T of the summary, i.e. a number from [0, 1] expressing the truth of a respective linguistically quantified proposition in the scope of summarised data.

In Yager's original approach [11] the linguistic quantifiers are represented using Zadeh's definition [15]. A proportional, non-decreasing linguistic quantifier Q is represented by a fuzzy set in [0, 1] and $\mu_{Q(x)}$ states the degree to which the proportion of $100 \cdot x\%$ of elements of the universe match the proportion expressed by the quantifier Q. Thus, the formulas for the truth degree of type I and type II linguistic summaries, are respectively:

$$T\left(Q_{\{t \in R\}} S(t)\right) = \mu_Q \left[\frac{1}{n} \sum_{i=1}^{n} \mu_S(t_i)\right] \tag{4}$$

$$T\left(Q_{\{t\in R\}}\left(U(t),S(t)\right)\right)=\mu_Q\left(\frac{\sum_{i=1}^{n}\left(\mu_U(t_i)\wedge\mu_S(t_i)\right)}{\sum_{i=1}^{n}\mu_U(t_i)}\right) \tag{5}$$

3. Bipolar queries and bipolar linguistic summaries of data

3.1. Bipolar queries

In classical approaches to preferences modelling, notably in database querying, it is usually assumed that an alternative (tuple) is either accepted or rejected. However, the results of many studies, cf. [2], seem to suggest that the decision maker often comes up with somehow independent evaluations of the positive and negative features of the alternatives in question. This leads to a general concept of *bipolar query* against the database, which evaluation results in two degrees corresponding to the satisfaction of the positive and negative condition.

Most of the research on bipolar queries is focused on a special case where the positive and negative conditions are interpreted in an asymmetric way, cf. [2]. Namely, the latter is treated as a *constraint*, denoted *C*, which has to be satisfied, while the former plays the role of a mere *preference*, denoted ~*P*.

We follow the approach of Lacroix and Lavency [10], Yager [13, 14] and Bordogna and Pasi [1], adapted for database querying by Zadrożny and Kacprzyk [18], which combine both conditions using the 'and possibly' operator which aggregates their satisfaction degrees depending on the possibility of a simultaneous matching of both conditions.

Thus, the bipolar query's condition may be formally written as:

$$C \text{ and possibly } P \tag{6}$$

Such a bipolar query would be denoted (*C*, *P*) and interpreted as follows.

If there is a tuple which satisfies both conditions, then and only then is it actually *possible* to satisfy both of them and each tuple of data has to do so, which turns (*C*, *P*) into the conjunction of both conditions, $C \wedge P$. On the other hand, if there is no such a tuple, then condition *P* is ignored.

As an example, consider the query:

$$\textit{Find apartments that are } \text{comfortable } \textit{and possibly } \text{cheap} \tag{7}$$

to a databases shown in Tab. 1–2. Let us assume that apartments priced below 250k PLN (250 000 of Polish zlotys) are in general considered as *cheap* (to a high degree, whenever we refer to satisfying a fuzzy condition or matching a summary, we mean to a high degree) and Comfort higher than 7.5 means a comfortable apartment. Then, it is possible to find in the *Sample real estate database 1* (Tab. 1) an apartment that is both *comfortable* and *cheap*, e.g. apartments No. 1 or 2, which, as stated earlier, turns the example query (7) into:

Find apartments that are *comfortable* and *cheap*

and returns tuples No. 1 and 2.

Let us contrast this to the result against the *Sample real estate database 2* (Tab. 2), where it is not possible to satisfy both conditions as there is no *cheap* apartment, which, as a result, ignores condition *P* and returns all four tuples.

The matching degree of the (*C*, *P*) query against a tuple *t* may be formalised as [10]:

$$T\left(C\left(t\right)\textit{and possibly}\,P\left(t\right)\right)=C\left(t\right)\wedge\left(\exists s\left(C\left(s\right)\wedge P\left(s\right)\right)\Rightarrow P\left(t\right)\right) \tag{8}$$

Table 1

Sample real estate database 1

Apt. No.	Comfort [1–10]	Price [k PLN]	Rooms
1	9.0	250	4
2	8.6	229	3
3	9.3	895	8
4	9.1	830	9

Table 2

Sample real estate database 2

Apt. No.	Comfort [1–10]	Price [k PLN]	Rooms
1	9.3	455	6
2	8.9	429	5
3	9.3	895	8
4	9.1	830	9

3.2. Bipolar linguistic summaries

Let us start with a brief remainder of the point of departure of our work. In [4] we proposed the concept of a bipolar linguistic summary using, as a starting point the concept of a bipolar query and a link between fuzzy linguistic queries and 'classical' linguistic summaries pointed out earlier in our works. We to follow the same concept with bipolar queries and bipolar linguistic summaries.

The earlier proposed interpretation of '*C* and possibly *P*' expressed by (8) makes this proposition true for a tuple *t* only if either of two conditions hold:

1) *t* satisfies both conditions *C* and *P*, or
2) *t* satisfies *C* and there is no tuple in the whole database which satisfies both conditions.

Thus, the the straightforward use of the formula (6) to instantiate the summariser in the linguistic summary protoforms (2)–(3) and its interpretation via (8) does not make much sense. Namely, the expression (6) may be appropriate to represent preferences of the user (as it is exemplified by query (7)) who does not know if there is an interference between conditions *C* and *P* with respect to the content of the queried database. However, e.g., the following proposition:

Most apartments (in the database) are *comfortable* and possibly *cheap*

is rather meaningless in the role of a summary as 'the system' knows if there is or isn't such an interference and should incorporate this information into results.

The main idea behind the interpretation of the bipolar linguistic summaries proposed by us is to relate the '*C* and possibly *P*' to *a part* the database of instead of the whole database. Let us consider the following example summary:

Most apartments (in the database) are *confortable* and possibly,
with respect to aparments of *similar size* (assuming ± 1 room), *cheap*

An apartment matches such a summary if:

1) it is comfortable and cheap, or
2) it is comfortable and there is no other apartment of similar size that is both comfortable and cheap.

Taking this into consideration and assuming that 430–460 k PLN priced apartments this time could be considered *cheap* (to much lover degree than ≤ 250 k PLN obviously), the above summary is true (still to a high degree) for both *Sample real estate databases 1* and 2 (Tab. 1–2).

A characteristic feature of such a summary is the use of a summariser employing an extended version of the 'and possibly' operator, which we will refer to as the 'contextual and possibly' operator. This operator may be expressed as:

$$C \text{ and possibly } P \text{ with respect to } W \tag{9}$$

For the purposes of contextual bipolar queries (and, thus, bipolar linguistic summaries) the predicates C and P should be interpreted as the required and desired conditions, respectively, while the predicate W denotes the *context* in which the possibility of satisfying both C and P will be assessed, separately for each tuple. Then, the formula (9) is interpreted as:

$$\begin{aligned} &T\left(C(t) \textit{ and possibly } P(t) \textit{ with respect to } W(t)\right) \\ &\quad = C(t) \wedge \left(\exists s \left(W(t,s) \wedge C(s) \wedge P(s)\right) \Rightarrow P(t)\right) \end{aligned} \tag{10}$$

Our preliminary computational experiments show that usage of standard De Morgan triples ($\wedge_{\min}$, $\vee_{\max}$, $\neg$), ($\wedge_{\Pi}$, $\vee_{\Pi}$, $\neg$) and ($\wedge_{L}$, $\vee_{L}$, $\neg$) with *t*- and *s*-norms: Minimum and Maximum; Product and Probabilistic sum; and Łukasiewicz's *t*- and *s*-norm, respectively), both with the *S*- and *R*-implication, in (10) may lead to somehow counter-intuitive results in terms of bipolar queries evaluation. Thus we use the ($\wedge_{\min}$, $\vee_{\max}$, $\neg$) De Morgan triple and Goguen *R*-implication which turns (10) into:

$$\begin{aligned} &T\left(C(t) \textit{ and possibly } P(t) \textit{ with respect to } W(t)\right) = \\ &\begin{cases} \min\left(C(t), 1\right) & \textit{for } \exists WCP(t) = 1 \\ \min\left(C(t), \min\left(1, \dfrac{P(t)}{\exists WCP(t)}\right)\right) & \textit{otherwise} \end{cases} \end{aligned} \tag{11}$$

where $\exists WCP(t)$ denotes $\max_{s \in R} \min(W(t, s), C(s), P(s))$ in this context.

3.3. Summary context quality criteria

In [5] we stated that the quality of the summary context W itself and the whole implication premise in (10) (i.e. $\exists WCP(t)$) have to be considered when measuring the quality of the bipolar linguistic summaries.

If P and/or W are such that the $\exists WCP(t)$ is true to a very low or a very high degree for *most* of tuples, then the summarizer (9) does not make much sense even if the truth value of the whole summary is high. This is due to the behaviour of the bipolar query 'C and possibly P' which turns into C and $C \wedge P$, respectively, when the truth degree of $\exists WCP(t)$ (i.e. $\exists_{s \in R} C(s) \wedge P(s)$) is close to 0 and close to 1.

The introduction of the context W partially alleviates this problem, but W has to be chosen carefully. If for most t's there does not exist $s \in R/\{t\}$ such that $W(t, s)$, then the premise of the implication is most often false and the summary is true for any P.

We propose a solution to those problems in a form of quality measures incorporating following linguistically quantified propositions:

$$Q_{t \in R} \exists_{s \in R \setminus \{t\}} W(t, s) \tag{12}$$

$$Q_{t \in R} \exists_{s \in R \setminus \{t\}} C(s) \wedge P(s) \wedge W(t, s) \tag{13}$$

Namely, if the truth of (12) for a summary is too small (lower than some threshold value), then such a summary should be discarded. Also, if the truth of (13) is too small (too close to 0; lower than the third threshold value) or too high (too close to 1; larger than the second threshold value) then the summary also shouldn't be taken into account. Obviously, if the first threshold is violated, then the third one also is. On the other hand, even if the first threshold is satisfied, the summary may still fail to satisfy thresholds two or three and should be discarded.

Tuple t is excluded from the range of the existential quantifiers in (12)–(13) as if the only tuple related via W with t is only t itself, then, naturally, the resulting summary is of no interest.

4. Computational examples and discussion

As a confirmation of the need to introduce quality criteria (12) and (13), let us consider sample databases similar to those used in section 3.2, shown in Tab. 3–6 (Tab. 3–5 presents data to which bipolar linguistic queries should not be used and need to be filtered out, whereas Tab. 6 shows an example in a favour of them – the proposed quality criteria support this).

For simplicity we limited components of a summary to:

- One quantifier *most* defined as unitary quantifier: $most(x) = x$;
- Two fuzzy predicates based on Comfort and Price attributes, instantiated for clarity by only one linguistic value each: *comfortable* and *cheap*, respectively, shown on Fig. 1;
- (Crisp) context W = *similar size* true iff $|Rooms(t) - Rooms(s)| \leq 1$.

Table 3

Sample real estate database 3

Apt. No.	Comfort [1]	Price [k PLN]	Rooms
1	7.8	213	3
2	8.1	349	5
3	7.6	629	7
4	8.0	712	9

Table 4

Sample real estate database 4

Apt. No.	Comfort [1–10]	Price [k PLN]	Rooms
1	8.2	475	5
2	7.8	489	5
3	8.9	629	7
4	7.5	655	8

Table 5

Sample real estate database 5

Apt. No.	Comfort [1–10]	Price [k PLN]	Rooms
1	8.0	205	3
2	7.5	213	3
3	7.8	245	5
4	8.3	249	6

Table 6

Sample real estate database 6

Apt. No.	Comfort [1–10]	Price [k PLN]	Rooms
1	8.0	205	3
2	7.5	213	3
3	7.8	345	5
4	8.3	359	6

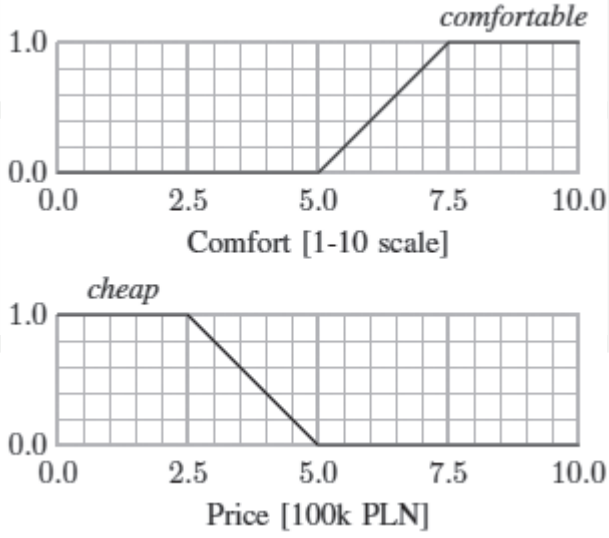

Fig. 1. Membership functions of *comfortable* (condition *C* – upper plot) and *cheap* (condition *P* – lower plot) predicates based on Comfort and Price attributes, respectively

All of which taken together result in an example summary considered:

Most apartments (in the sample database) are comfortable and possibly, with respect to apartments of similar size, cheap (14)

4.1. Discussion of results

A brief analysis of results obtained for individual tables shows that:

- Tab. 3 – truth value of the summary (14) equals 1.00 because of the wrongly chosen context *W*, where the majority of tuples (in this example all of them) are in its own 'neighborhood'. This context should not be taken into consideration during summarisation.
  Situation identified by a low value of criterion (12): 0.00.
- Tab. 4 – truth value of the summary (14) equals 0.86 making it a feasible component of the returned set of summaries despite that apartments No. 3 and 4 are not *cheap* and the remaining two are considered *cheap* in a very low degree (0.10 and 0.04). This summary should be replaced by: '*Most* apartments (in the database) are *comfortable*', '*Most* apartments (in the database) are *comfortable* and *NOT cheap*' or even '*Almost none of* apartments (in the database) are *cheap*'.
  Identified by a low value of criterion (13): 0.0.
- Tab. 5 – truth value of the summary (14) equals 1.00 regardless to the context *W* (all apartments are *cheap* to the same, or at least similar, degree), therefore the summary itself should be replaced by a 'classical' one such as '*Most* apartments (in the sample database) are *comfortable* and *cheap*'.
  Identified by a high value of criterion (13): 1.00.
- Tab. 6 – truth value of the summary (14) equals 0.98, which suggest that this summary could be considered as a component of the resulting set of summaries. Value 1.00 of criterion (12) indicates the properly chosen context and value 0.80 of criterion (13) (rather high) suggests caution in selecting this summary, however competitive 'classical' summary receives lover truth value: 0.80.

We focused here on showing the benefits of using 'contextual and possibly' operator in the scope of linguistic data summarization, presenting both a theoretical and semantic justification of this concept, and intuitively appealing examples of correctness of the proposed criteria.

Contextual bipolarity employed in the summaries manifests itself by determining dynamically for each tuple, a context *W*, in which the possibility of matching the conditions *C* and *P* simultaneously is checked. This property of bipolar summaries offers the possibility to discover more interesting patterns in data utilising a very human-specific bipolar approach to preferences.

Examples clearly argue in favour of introduced additional quality criteria (measures) and confirm that their help to distinguish interesting summaries from among all with high truth values. Additional studies are needed in order to clearly determine the best summaries, yet already the results are promising.

5. Concluding remarks

Preliminary computational results of bipolar linguistic summaries proposed in [4], demonstrated the need for new quality criteria to determine the true quality of the summary. In [5] we introduced two of them which have been studied deeper here. The results presented in section 4 show that proposed criteria fulfil their role and help select bipolar linguistic

summaries that are valuable and interesting for an end user. Due to a conceptual character of the paper and for simplicity, we have considered here only qualifier-free bipolar linguistic summaries, however, they may be extended to include qualifiers in an obvious way.

Future works in this subject will mainly cover combining introduced criteria with other known quality measures, in order to determine a single value of the quality of the linguistic summary on one hand, and for evaluating and selecting linguistic summaries by means of heuristic methods on the other hand.

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