01 Jan 2009

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Thomas Vojta
Missouri University of Science and Technology, vojtat@mst.edu

Chetan Kotabage

Jose A. Hoyos

Recommended Citation

T. Vojta et al., "Infinite-Randomness Quantum Critical Points Induced by Dissipation," *Physical Review B*, American Physical Society (APS), Jan 2009.
The definitive version is available at https://doi.org/10.1103/PhysRevB.79.024401

Selected for a Viewpoint in *Physics*

PHYSICAL REVIEW B **79**, 024401 (2009)

Infinite-randomness quantum critical points induced by dissipation

Thomas Vojta,[1] Chetan Kotabage,[1] and José A. Hoyos[1,2]

[1]*Department of Physics, Missouri University of Science and Technology, Rolla, Missouri 65409, USA*

[2]*Department of Physics, Duke University, Durham, North Carolina 27708, USA*

(Received 19 September 2008; revised manuscript received 13 November 2008; published 5 January 2009)

We develop a strong-disorder renormalization group to study quantum phase transitions with continuous $O(N)$ symmetry order parameters under the influence of both quenched disorder and dissipation. For Ohmic dissipation, as realized in Hertz's theory of the itinerant antiferromagnetic transition or in the superconductor-metal transition in nanowires, we find the transition to be governed by an exotic infinite-randomness fixed point in the same universality class as the (dissipationless) random transverse-field Ising model. We determine the critical behavior and calculate key observables at the transition and in the associated quantum Griffiths phase. We also briefly discuss the cases of super-Ohmic and sub-Ohmic dissipations.

DOI: 10.1103/PhysRevB.79.024401 PACS number(s): 05.70.Jk, 75.10.Lp, 75.10.Nr, 75.40.−s

I. INTRODUCTION

In recent years, it has become clear that quenched disorder, i.e., impurities, defects, or other types of imperfections can significantly modify the low-temperature behavior of quantum many-particle systems. At zero-temperature quantum phase transitions, the interplay between large-scale quantum fluctuations and random fluctuations leads to much more dramatic disorder effects than at classical thermal phase transitions, resulting in various exotic phenomena such as quantum Griffiths effects,[1–3] non-power-law dynamical scaling,[4,5] or even smeared phase transitions.[6,7] A recent review of some of these phenomena can be found in Ref. 8.

The quantum phase transitions in random transverse-field Ising magnets are among the most striking examples of such behavior. Utilizing a real-space renormalization-group (RG) technique due to Ma *et al.*,[9,10] now known as the strong-disorder RG (see Ref. 11 for a review), Fisher[4,5] showed that the one-dimensional random transverse-field Ising chain features an unconventional infinite-randomness critical point with ultraslow activated rather than power-law dynamical scaling. It is accompanied by strong power-law quantum Griffiths effects in the vicinity of the transition. While it was initially suspected that this scenario is special to one space dimension, Motrunich *et al.*[12] showed that the random transverse-field Ising models in two and three dimensions also display infinite-randomness critical points.

A dissipative environment further hampers the dynamics. In the experimentally relevant case of Ohmic damping, the large locally ordered droplets that are normally responsible for quantum Griffiths effects completely cease to tunnel.[13–15] Instead, they develop static order independently from the bulk system which destroys the sharp quantum phase transition by smearing.[6,7] A similar freezing of locally ordered droplets also occurs close to a quantum-percolation transition.[16,17]

The above behavior of Ising order parameters must be contrasted with that of *continuous* $O(N)$ symmetry order parameters. While the bulk ground-state phases of one-dimensional Heisenberg random quantum spin chains are governed by infinite-randomness fixed points,[9,10,18] higher-dimensional random quantum Heisenberg systems have more conventional ground states,[19,20] and their quantum phase transitions are governed by conventional critical points.[21–24] As in the Ising case, adding Ohmic dissipation hampers the dynamics of $O(N)$ symmetric order parameters. Vojta and Schmalian[25] showed that the "energy gap" of large locally ordered droplets is exponentially small in their volume leading to power-law quantum Griffiths effects analogous to those in the *dissipationless* random transverse-field Ising model. This analogy suggests the important question of whether Ohmic dissipation can induce an unconventional infinite-randomness quantum critical point for a continuous $O(N)$ symmetry order parameter.

In addition to its significance for the theory of quantum phase transitions, this question also has important experimental applications. Consider the antiferromagnetic quantum phase transition of itinerant electrons (as observed, e.g., in heavy fermion compounds[26,27]). Within the standard Hertz-Millis spin-fluctuation theory,[28,29] it is described by an $O(3)$ Landau-Ginzburg-Wilson (LGW) order-parameter field theory with Ohmic dissipation. The properties of this transition have been a long-standing unsolved problem; and given the fact that most experimental systems are rather dirty, studying the effects of disorder on the Hertz-Millis theory is of prime interest.

A second potential application is provided by the pair breaking superconductor-metal quantum phase transitions in nanowires.[30] It can be described by a one-dimensional $O(2)$ LGW theory with Ohmic dissipation.[31–33] There is an experimental evidence that the pair breaking in this systems is caused by surface magnetic impurities which necessarily also introduce quenched disorder.

In this paper, we investigate the quantum phase transition of a continuous-symmetry $O(N)$ order parameter under the combined influences of both quenched disorder and Ohmic dissipation. To this end we develop a strong-disorder RG suitable for this problem. The paper is organized as follows. In Sec. II we introduce our model: a dissipative LGW order-parameter field theory. In Sec. III we implement the strong-disorder RG in the large-N limit and relate it to that of the random transverse-field Ising model. We also summarize the solution and the resulting critical behavior. In Sec. IV, we calculate key observables close to the transition while Sec. V

deals with the case of non-Ohmic damping. We also show that our results do not rely on the large-N limit. We conclude in Sec. VI. A short account of part of this work has already been published in Ref. 34.

II. ORDER-PARAMETER FIELD THEORY

We start from a quantum LGW free-energy functional for an N-component ($N>1$) vector order parameter $\varphi=(\varphi_1,\ldots,\varphi_N)$ in d space dimensions. For the above-mentioned itinerant antiferromagnetic quantum phase transition, generically $d=3$ and $N=3$ while for the superconductor-metal transition in nanowires, $d=1$ and $N=2$. The LGW free energy can be derived from an appropriate microscopic Hamiltonian of disordered electrons using standard methods[28,29,35] (for a critical discussion of this approach, see Ref. 36). In the absence of quenched disorder, the action of our LGW theory reads

$$S=\int dydx\ \varphi(x)\Gamma(x,y)\varphi(y)+\frac{u}{2N}\int dx\ \varphi^4(x), \tag{1}$$

where $x\equiv(\mathbf{x},\tau)$ comprises imaginary time τ and position $\mathbf{x}$, $\int dx\equiv\int d\mathbf{x}\int_0^{1/T}d\tau$, and u is the standard quartic coefficient. $\Gamma(x,y)$ denotes the bare inverse propagator (two-point vertex) whose Fourier transform reads

$$\Gamma(\mathbf{q},\omega_n)=r+\xi_0^2\mathbf{q}^2+\gamma_0|\omega_n|^{2/z_0}. \tag{2}$$

Here, r is the bare distance from criticality (the bare gap), ξ_0 is a microscopic length scale, and ω_n is a Matsubara frequency. The nonanalytical frequency dependence of $\Gamma(\mathbf{q},\omega_n)$ is caused by the coupling of the order parameter to a dissipative bath. We are mostly interested in the case of overdamped (Ohmic) dynamics corresponding to a value of $z_0=2$. However, to demonstrate the special role of $z_0=2$, we also consider different values of z_0. The damping coefficient γ_0 depends on the coupling of the order parameter to the dissipative bath and the spectral density of the bath modes.

In the presence of quenched disorder, the functional form of the order-parameter field theory (1) does not change qualitatively, but the distance from criticality r becomes a random function of spatial position. Analogously, disorder appears in ξ_0, γ_0, and u.

Let us briefly comment on possible modifications of the two-point vertex (2) by mode-coupling effects. For the itinerant ferromagnetic quantum phase transition[37–39] and the superconductor-metal transition without magnetic impurities,[40,41] the coupling between the order-parameter fluctuations and the soft particle-hole excitations of the metal leads to a long-range interaction in space represented by a nonanalytical q dependence instead of the simple $\mathbf{q}^2$ term. In contrast, in our examples, the $\mathbf{q}^2$ term remains the leading term because the relevant modes are either gapped (for the superconductor-metal transition due to magnetic impurities) or couple too weakly to the order parameter (in the case of the itinerant antiferromagnetic transition).[42]

Our goal is the application of the real-space-based strong-disorder RG. We therefore need to discretize the continuum action (1) in space (but not in imaginary time) by defining discrete coordinates $\mathbf{x}_j$ and rotor variables $\varphi_j(\tau)$. These rotors are coarse-grained rather than microscopic variables. They represent the average order parameter in a volume ΔV large compared to the microscopic scale ξ_0 but small compared to the true correlation length ξ, i.e., $\varphi_j(\tau)=\int_{\Delta V}d\mathbf{y}\ \varphi(\mathbf{x}_j+\mathbf{y},\tau)$.

For simplicity, we first consider the large-N limit of our LGW theory. This will allow us to perform all of the following calculations explicitly. We will later show that the RG fixed point is the same for all $N>1$. The resulting discrete large-N action reads

$$S=T\sum_i\sum_{\omega_n}(r_i+\lambda_i+\gamma_i|\omega_n|^{2/z_0})|\phi_i(\omega_n)|^2 - T\sum_{\langle i,j\rangle}\sum_{\omega_n}\phi_i(-\omega_n)J_{ij}\phi_j(\omega_n), \tag{3}$$

where r_i, $\gamma_i>0$, and the nearest-neighbor interactions $J_{ij}>0$ are random quantities and $\phi_j(\omega_n)=\int_0^{1/T}\varphi_j(\tau)e^{i\omega_n\tau}d\tau$ is the Fourier transform of the rotor variable. The Lagrange multipliers λ_i enforce the large-N constraints $\langle[\varphi_i^{(k)}(\tau)]^2\rangle=1$ for each order-parameter component $\varphi_i^{(k)}$ at each site i; they have to be determined self-consistently. The renormalized local distance from criticality at site i is given by $\epsilon_i=r_i+\lambda_i$. In the disordered phase, *all* $\epsilon_i>0$.

III. STRONG-DISORDER RENORMALIZATION GROUP

The idea of the strong-disorder (Ma-Dasgupta-Hu) RG (Refs. 4, 5, 9, and 10) consists in the successive decimation of local high-energy degrees of freedom. It relies on the disorder distributions being broad and becomes exact in the limit of infinitely broad distributions. For now, we assume that our distributions are sufficiently broad. We will later show that their widths diverge at the critical RG fixed point, justifying the method.

A. Single-cluster solution

Let us start by considering a single rotor variable φ (i.e., a single cluster) with the action

$$S_{\rm cl}=T\sum_{\omega_n}(r+\lambda+\gamma|\omega_n|^{2/z_0})|\phi(\omega_n)|^2. \tag{4}$$

The value of the Lagrange multiplier λ is determined by the length constraint

$$1=\langle\phi^2\rangle=T\sum_{\omega_n}\frac{1}{r+\lambda+\gamma|\omega_n|^{2/z_0}}. \tag{5}$$

At zero temperature, the Matsubara sum can be turned into an integral, resulting in

$$1=\frac{1}{2\pi}\int_{-\infty}^{\infty}d\omega\frac{1}{\epsilon+\gamma|\omega_n|^{2/z_0}}, \tag{6}$$

where $\epsilon=r+\lambda$ is the renormalized distance from criticality.

To proceed, we now need to distinguish super-Ohmic, Ohmic, and sub-Ohmic dissipations. In the super-Ohmic case, $z_0<2$, the integral can be carried out straightforwardly giving $\langle\phi^2\rangle=c\epsilon^{(z_0-2)/2}\gamma^{-z_0/2}$ with c being a constant. Solving

for ϵ yields the relation between the gap and the damping constant (i.e., the cluster size)

$$\epsilon \sim (1/\gamma)^{z_0/(2-z_0)}. \tag{7}$$

In contrast, we need to introduce a high-frequency cutoff Λ to carry out the constraint integral in the Ohmic case $z_0=2$, giving $\langle\phi^2\rangle=\ln(1+\gamma\Lambda/\epsilon)/\pi\gamma$. The resulting dependence of the gap on the damping constant is exponential,

$$\epsilon = \gamma\Lambda/(e^{\pi\gamma}-1) \approx \gamma\Lambda e^{-\pi\gamma}, \tag{8}$$

signifying that a single Ohmic cluster is marginal, i.e., right at the lower critical "dimension" of the problem. In the sub-Ohmic case, $z_0>2$, the single-cluster physics changes dramatically. The constraint integral in Eq. (6) converges in the limit $\epsilon\to 0$. Thus, once $\gamma>\gamma_c=\Lambda^{(z_0-2)/z_0}z_0/[\pi(z_0-2)]$, Eq. (6) does not have a solution $\epsilon>0$ anymore, implying that the rotor has undergone a localization phase transition caused by the sub-Ohmic dissipation.

B. Recursion relations

In our large-N action (3), the competing independent local energies are the gaps ϵ_i and the interactions J_{ij} (the damping coefficient γ_i and the gap ϵ_i are not independent; they are coupled via the large-N constraint at site i). In the bare theory, J_{ij} and ϵ_i are independent random variables with probability distributions $P(J)$ and $R(\epsilon)$, respectively. Each step of the strong-disorder RG eliminates one rotor variable by first identifying the largest local energy $\Omega=\max(\epsilon_i,J_{ij})$ and then decimating the associated high-energy degree of freedom.

1. Decimating a site

Specifically, if the largest local energy is a gap, say ϵ_2, the corresponding rotor ϕ_2 is far away from criticality and does not contribute to the macroscopic order parameter. However, integrating out its fluctuations generates effective interactions between all pairs of sites that couple to ϕ_2. If the disorder distributions are broad, ϵ_2 is much larger than all local energies associated with the neighboring sites. Thus, ϕ_2 can be integrated out in perturbation theory with the unperturbed part of the action being

$$S_0 = T\sum_{\omega_n} (\epsilon_2 + \gamma_2|\omega_n|^{2/z_0})|\phi_2(\omega_n)|^2, \tag{9}$$

while $S_1=S-S_0$ is the perturbation. Up to second order in perturbation theory, we only need to consider the interaction of ϕ_2 with the neighboring sites j; thus,

$$S_1 = -T\sum_{j\neq 2,\omega_n} J_{2j}\phi_2(-\omega_n)\phi_j(\omega_n). \tag{10}$$

The partition function can now be written as

$$Z = \int D[\phi_2]\prod_{j\neq 2} D[\phi_j]e^{-S} = Z_0\int \prod_{j\neq 2} D[\phi_j]\langle e^{-S_1}\rangle_0$$
$$= Z_0\int \prod_{j\neq 2} D[\phi_j]e^{-\tilde{S}}, \tag{11}$$

where $\int D[\phi_j]$ comprises integration over all frequency components of ϕ_j, Z_0 is the partition function associated with the action S_0, and $\langle\cdot\rangle_0$ denotes the average with respect to S_0. The renormalized action $\tilde{S}$ can be calculated in cumulant expansion

$$\tilde{S} = -\ln\langle e^{-S_1}\rangle_0 = \langle S_1\rangle_0 - \frac{1}{2}[\langle S_1^2\rangle_0 - \langle S_1\rangle_0^2] \pm \cdots. \tag{12}$$

Evaluating the averages, we obtain $\langle S_1\rangle_0=0$ due to symmetry and

$$\langle S_1^2\rangle_0 = T\sum_{\omega_n}\left[\sum_j \frac{J_{2j}^2|\phi_j(\omega_n)|^2}{\epsilon_2+\gamma_2|\omega_n|^{2/z_0}} + 2\sum_{i\neq j}\frac{J_{i2}J_{2j}\phi_i^*(\omega_n)\phi_j(\omega_n)}{\epsilon_2+\gamma_2|\omega_n|^{2/z_0}}\right]. \tag{13}$$

The first term in the square brackets just gives subleading renormalizations of the gaps ϵ_j of the neighboring sites and can thus be dropped. The second term provides the renormalized interactions $\tilde{J}_{ij}$ between all sites that used to couple to ϕ_2. Their leading low-frequency behavior is $\tilde{J}_{ij}=J_{i2}J_{2j}/\epsilon_2$ independent of the exponent z_0. This term has to be added to the interaction J_{ij} already coupling sites i and j if any. Consequently, the final recursion relation for the RG step reads

$$\tilde{J}_{ij} = J_{ij} + \frac{J_{i2}J_{2j}}{\epsilon_2}. \tag{14}$$

At the end of the RG step, ϕ_2 is dropped from the action. Note that the multiplicative structure of the effective interaction in Eq. (14) is a direct consequence of second-order perturbation theory. It does not depend on details of the model; in particular, it is valid for *any* z_0.

2. Decimating a bond

Let us now consider the RG step in the case of the largest local energy being an interaction, say J_{23}, coupling sites 2 and 3. For broad disorder distributions, $J_{23}\gg\epsilon_2,\epsilon_3$. Thus, the two rotors ϕ_2 and ϕ_3 are essentially parallel and can be replaced by a single rotor $\tilde{\phi}_2$ which represents the entire cluster comprising ϕ_2 and ϕ_3. The moment $\tilde{\mu}_2$ of the effective rotor, i.e., the number of original sites in the cluster is the sum of the moments μ_2 and μ_3 of the original rotors,

$$\tilde{\mu}_2 = \mu_2 + \mu_3. \tag{15}$$

To find the renormalized gap $\tilde{\epsilon}_2$ of the effective rotor, we solve exactly the two-site problem involving ϕ_2 and ϕ_3 while treating the couplings to all other sites as perturbations. The two-site action is given by

$$S_0 = T\sum_{\omega_n}\sum_{i=2,3}(r_i+\lambda_i+\gamma_i|\omega_n|^{2/z_0})|\phi_i(\omega_n)|^2$$
$$-T\sum_{\omega_n} J_{23}\phi_2(-\omega_n)\phi_3(\omega_n). \tag{16}$$

It is subjected to the large-N length constraints

$$1 = \langle\phi_2^2\rangle = T\sum_{\omega_n}\frac{d_3}{d_2d_3-J^2/4},$$

$$1 = \langle \phi_3^2 \rangle = T \sum_{\omega_n} \frac{d_2}{d_2 d_3 - J^2/4}, \tag{17}$$

with $d_j = r_j + \lambda_j + \gamma_j |\omega_n|^{2/z_0}$. They determine the Lagrange multipliers λ_i. (It is important to note that the value of $r_i + \lambda_i$ in the two-site cluster is different from the single-site ϵ_i.)

To integrate out the high-energy mode, we diagonalize the quadratic form in Eq. (16) separately for each Matsubara frequency. The two eigenvalues read

$$\kappa_{a,b} = \frac{1}{2}[d_2 + d_3 \pm \sqrt{(d_2 - d_3)^2 + J_{23}^2}]$$
$$= \frac{1}{2}(d_2 + d_3 \pm J_{23}) + O\left(\frac{\epsilon_j}{J_{23}}, \frac{\omega_n}{J_{23}}\right). \tag{18}$$

The corresponding eigenmodes are given by $\psi_a = \alpha \phi_2 + \beta \phi_3$ and $\psi_b = -\beta \phi_2 + \alpha \phi_3$ with

$$\alpha = \frac{d_3 - d_2 + \sqrt{(d_2 - d_3)^2 + J^2}}{\sqrt{[d_3 - d_2 + \sqrt{(d_2 - d_3)^2 + J^2}]^2 + J^2}},$$

$$\beta = \frac{J}{\sqrt{[d_3 - d_2 + \sqrt{(d_2 - d_3)^2 + J^2}]^2 + J^2}}. \tag{19}$$

The higher eigenvalue κ_b is at least J_{23} above the lower eigenvalue κ_a; we thus integrate out the corresponding mode leaving us with the effective action $\tilde{S} = T \Sigma_{\omega_n} \lambda_a |\psi_a(\omega_n)|^2$ and a length constraint $\langle \psi_a^2 \rangle = \langle (\alpha \phi_2 + \beta \phi_3)^2 \rangle \neq 1$.

We define the renormalized rotor variable by rescaling $\tilde{\phi}_2 = \psi_a / \langle \psi_a^2 \rangle^{1/2}$ because we wish it to fulfill the same length constraint $\langle \tilde{\phi}_2^2 \rangle = 1$ as all other rotor variables. Inserting this definition into the (diagonalized) two-site action (16) allows us to identify the renormalized gap, damping constant, and interactions with the neighbors;

$$\tilde{\epsilon}_2 = \frac{1}{2} \langle \psi_a^2 \rangle (r_2 + \lambda_2 + r_3 + \lambda_3 - J),$$

$$\tilde{\gamma}_2 = \frac{1}{2} \langle \psi_a^2 \rangle (\gamma_2 + \gamma_3),$$

$$\tilde{J}_{2j} = \langle \psi_a^2 \rangle^{1/2} (\alpha J_{2j} + \beta J_{3j})|_{\omega_n \to 0}. \tag{20}$$

To proceed further, we need explicit results for the Lagrange multipliers λ_2 and λ_3 as well as $\langle \psi_a^2 \rangle$. This requires the solution of the two coupled integral equations (17). In the case of Ohmic dissipation, the integrals are rational and can be done exactly. In the limit of strong disorder, we obtain $r_2 + \lambda_2 + r_3 + \lambda_3 - J = 2\epsilon_2 \epsilon_3 / J$ and $\alpha|_{\omega_n \to 0} = \beta|_{\omega_n \to 0} = \sqrt{2}/2$. Moreover, $\langle \psi_a^2 \rangle$ is bounded between 1 and 2 and approaches 2 in the asymptotic limit $\Omega \to 0$. This leads to the recursion relations

$$\tilde{J}_{2j} = J_{2j} + J_{3j}, \tag{21}$$

$$\tilde{\epsilon}_2 = 2 \frac{\epsilon_2 \epsilon_3}{J_{23}}, \tag{22}$$

implying an additive relation for the renormalized damping constant,

$$\tilde{\gamma}_2 = \gamma_2 + \gamma_3. \tag{23}$$

We emphasize that the multiplicative form of Eq. (22) is *not* independent of the functional form of the action (3). In contrast to the recursion relation (14) for the interactions, the recursion relation (22) for the gaps is special to the case of Ohmic dissipation. It is related to the fact that the gap ϵ of single cluster depends exponentially on the damping constant (and thus on the cluster size), $\epsilon = \gamma \Lambda e^{-\pi \gamma}$, as derived in Eq. (8). We will come back to this point in Sec. V where we discuss the case of non-Ohmic dissipation.

Although the prefactor in Eq. (22) is larger than 1, this does not mean that the renormalized gap can become larger than the decimated ones in the weak-disorder limit. Using the methods of Ref. 43 we showed that the exact value (20) of $\tilde{\epsilon}_2$ [calculated within the two-site action (16)] is always less than the decimated gaps (ϵ_2 and ϵ_3) for all $\epsilon_2, \epsilon_3 \leq J_2$. Therefore, the system flows toward the infinite-randomness fixed point for all bare disorder strengths, ensuring the internal consistency of the RG. The net result of a single RG step is the elimination of one rotor and the reduction of the maximum local energy Ω as well as renormalizations of the remaining energies and reconnections of the lattice.

The RG recursion relations (14), (15), (21), and (22) completely define the RG procedure. They are identical to the corresponding relations for the dissipationless random transverse-field Ising model.[4,5,12] We thus conclude that our system belongs to the same universality class. Note, however, that there are some subtle differences in the behavior of some observables due to the continuous symmetry of the order parameter and the Ohmic damping, as will be discussed in Sec. IV.

C. RG flow equations and fixed points

In this section, we briefly summarize Fisher's solution[4,5] of the strong-disorder RG defined by the recursions (14), (15), (21), and (22) to the extent necessary for the purposes of this paper. In one space dimension, the RG step does not change the lattice topology because the interactions remain between nearest-neighbor sites only and ϵ and J remain statistically independent. Therefore, the theory can be formulated in terms of the probability distributions $P(J)$ and $R(\epsilon)$. Fisher derived RG flow equations for these distributions and solved them analytically. There are three types of nontrivial fixed points corresponding to the ordered and disordered quantum Griffiths phases and the quantum critical point that separates them.

The most remarkable feature of the critical fixed point is that the probability distributions P and R broaden without limit under renormalization, even on a logarithmic scale. Using logarithmic variables $\Gamma = \ln(\Omega_I / \Omega)$ [where Ω_I is of the order of the initial (bare) value of Ω], $\zeta = \ln(\Omega / J)$, and $\beta = \ln(\Omega / \epsilon)$ the probability distributions $\mathcal{P}(\zeta)$ and $\mathcal{R}(\beta)$ at the critical fixed point read

$$\mathcal{P}(\zeta) = \frac{1}{\Gamma} e^{-\zeta/\Gamma}, \quad \mathcal{R}(\beta) = \frac{1}{\Gamma} e^{-\beta/\Gamma}. \tag{24}$$

The diverging widths of the probability distributions give the critical point its name, viz. infinite-randomness critical point. They also *a posteriori* justify the method because the perturbative recursion relations (14)–(22) become exact in the limit of infinitely broad distributions (i.e., approaching the critical point).

The complete critical behavior can be found by including the moments and lengths of the clusters in the RG procedure. It is characterized by three exponents $\nu=2$, $\psi=1/2$, and $\phi=(1+\sqrt{5})/2$. The correlation length exponent ν determines how the average correlation length ξ diverges if one approaches the critical point via

$$\xi \sim |r|^{-\nu}. \tag{25}$$

Here r denotes the fully renormalized dimensionless distance from criticality which is given by $r \sim [\ln(\epsilon)] - [\ln(J)]$ in terms of the bare variables ($[\cdot]$ denotes the disorder average).[44]

The tunneling exponent ψ controls the dynamical scaling, i.e., the relation between length scale L and energy scale Ω. It is of activated rather than power-law type,

$$\ln(\Omega_I/\Omega) \sim L^{\psi}, \tag{26}$$

which is a direct consequence of the multiplicative structure of the recursions (14) and (22). ψ also controls the density n_Ω of clusters surviving at an energy scale Ω in the RG procedure. Its scaling form is given by

$$n_\Omega(r) = [\ln(\Omega_I/\Omega)]^{-d/\psi} X_n[r^{\nu\psi} \ln(\Omega_I/\Omega)] \tag{27}$$

with the scaling function behaving as $X_n(0)=\text{const}$ and $X_n(y\to\infty) \sim y^{d/\psi} e^{-cdy}$ where c is a constant. As a result, the cluster density decreases as $n_\Omega \sim [\ln(\Omega_I/\Omega)]^{-d/\psi}$ at criticality while it behaves as $n_\Omega \sim r^{d\nu}\Omega^{d/z}$ in the disordered quantum Griffiths phase ($r>0$). The dynamical exponent z varies with $z \sim r^{-\nu\psi}$ in the Griffiths phase.

Finally, the exponent ϕ describes how the typical moment μ_Ω of a surviving cluster depends on the energy scale Ω. The scaling form of μ_Ω reads

$$\mu_\Omega(r) = [\ln(\Omega_I/\Omega)]^{\phi} X_\mu[r^{\nu\psi} \ln(\Omega_I/\Omega)]. \tag{28}$$

The scaling function behaves as $X_\mu(0)=\text{const}$ and $X_\mu(y\to\infty) \sim y^{1-\phi}$. Thus, at criticality the typical moment increases as $\mu_\Omega \sim [\ln(\Omega_I/\Omega)]^{\phi}$ while it behaves as $\mu_\Omega \sim r^{\nu\psi(1-\phi)} \ln(\Omega_I/\Omega)$ in the disordered quantum Griffiths phase.

The strong-disorder RG steps discussed in Sec. III B generate effective interactions between sites that were previously uncoupled. In dimensions $d>1$, this changes the lattice connectivity and it introduces statistical correlations between J and ϵ. Therefore, the theory cannot be formulated in terms of individual probability distributions of these variables, and a closed-form analytical solution appears to be impossible. However, Motrunich *et al.*[12] numerically implemented the recursion relations (14)–(22) in two dimensions, keeping track of all reconnections of the lattice under the RG. They found an infinite-randomness critical point very similar to that in one dimension. In fact, the critical behavior described in Eqs. (25)–(28) is also valid in two dimensions but with different exponent values. Various numerical implementations[12,45–47] of the strong-disorder RG yielded $\psi=0.42-0.6$, $\phi=1.7-2.5$, and $\nu=1.07-1.25$. In three dimensions, the RG flow toward an infinite-randomness fixed point has been confirmed,[12] but reliable estimates of the exponent values are still missing.

The strong-disorder RG allows one to identify the infinite-randomness fixed point and confirm its stability but, strictly, it cannot answer the question of whether or not a weakly or moderately disordered system will flow toward this fixed point, because if the disorder is weak the strong-disorder RG step is not very accurate. [An internal consistency check[43] of the RG in the weak-disorder limit can be achieved by computing exactly rather than perturbatively the renormalized couplings (gaps and interactions) within the relevant two-site or three-site clusters (see Sec. III B).] For our system, additional insight can be gained from the results of a conventional perturbative (replica-based) renormalization group. Building on earlier work,[48] Kirkpatrick and Belitz[35] showed that the perturbative RG always takes the system to large disorder strength even if the bare disorder is very small. Moreover, by taking rare region effects into account in an approximate way, Narayanan *et al.*[49,50] showed that there is no stable weak-disorder fixed point; instead, the perturbative RG shows runaway flow toward large disorder. This strongly suggests that our infinite-randomness critical point is universal and governs the quantum phase transition for all nonzero disorder strength.

IV. OBSERVABLES

The strategy for calculating, within the strong-disorder RG, thermodynamic observables such as the susceptibility as a function of temperature consists in running the RG from the initial energy scale Ω_I down to $\Omega=T$. The high-energy degrees of freedom eliminated in this procedure generally do not make significant contributions to the low-energy behavior of observables. At best, they change nonuniversal constants. All clusters surviving at energy scale $\Omega=T$ can be considered to be independent because they are coupled by interactions J much smaller than T. The desired observable is thus simply the sum of independent contributions from the individual surviving clusters. Frequency-dependent observables can be determined analogously.

A. Single-cluster results

In order to proceed, we therefore need to calculate the relevant observables for single clusters. To do so, we add a source term to the single-cluster action (4). It reads

$$S_H = -T\sum_{\omega_n} H(\omega_n)\phi(-\omega_n) \tag{29}$$

with $H(\omega_n)=\int_0^{1/T} H(\tau) e^{i\omega_n\tau} d\tau$ being the Fourier transform of the source field conjugate to the order parameter. Because the theory defined by $S_{\rm cl}+S_H$ is still Gaussian, the partition function Z_H in the presence of the field can be easily evalu-

ated. The dynamic (Matsubara) susceptibility is then given by

$$\chi(i\omega_n) = \frac{1}{T}\frac{\partial^2 \ln Z_H}{\partial H(\omega_n)\,\partial H(-\omega_n)} = \frac{1}{r+\lambda+\gamma|\omega_n|^{2/z_0}}. \quad (30)$$

For the temperature-dependent static susceptibility, we set $\omega_n=0$ and find the distance from criticality, $\epsilon(T)=r+\lambda(T)$ as a function of temperature. To this end we solve the finite-temperature constraint (5) yielding

$$\epsilon(T) = \begin{cases} \epsilon(0) + aT & [\gamma T^{2/z0} \ll \epsilon(0)] \\ T & [\gamma T^{2/z0} \gg \epsilon(0)], \end{cases} \quad (31)$$

where $\epsilon(0)$ is the zero-temperature value determined by the constraint integral (6). In the super-Ohmic and Ohmic cases, $\epsilon(0)$ is given by Eqs. (7) and (8), respectively. The constant a is given by $a=\pi\gamma$ in the Ohmic case and $a=2/(2-z_0)$ in the super-Ohmic case.

If the rotor variable ϕ represents a cluster of moment (number of sites) μ, its contribution to the uniform susceptibility is proportional to μ^2 while the contribution to the local susceptibility is proportional to μ. By combining this with Eqs. (30) and (31), we obtain the uniform static order-parameter susceptibility as a function of temperature of a cluster of moment μ and distance ϵ from criticality,

$$\chi_{\rm cl}(T) = \begin{cases} \mu^2/\epsilon & (\epsilon \gg T) \\ \mu^2/T & (\epsilon \ll T). \end{cases} \quad (32)$$

The corresponding results for the average *local* susceptibility read

$$\chi^{\rm loc}_{\rm cl}(T) = \begin{cases} \mu/\epsilon & (\epsilon \gg T) \\ \mu/T & (\epsilon \ll T). \end{cases} \quad (33)$$

To calculate the specific heat we also need the total-energy contribution of a single cluster which behaves as

$$\Delta E_{\rm cl} \sim T \quad (\epsilon \ll T). \quad (34)$$

This is an important difference from the random transverse-field Ising-model case[5] and stems from the fact that our rotor variables have an unbounded spectrum.

We now turn to the dynamical order-parameter susceptibility at zero temperature (focusing on Ohmic dissipation). From Eq. (30), we obtain in imaginary time formalism

$$\chi_{\rm cl}(i\omega_n) = \frac{\mu^2}{\epsilon+\gamma|\omega_n|}. \quad (35)$$

After Wick rotation $i\omega_n \to \omega+i0$ to real frequencies, this leads to $\chi_{\rm cl}(\omega+i0)=\mu^2/(\epsilon-i\gamma\omega)$ implying

$$\mathrm{Im}\,\chi_{\rm cl}(\omega+i0) = \frac{\mu^2\gamma\omega}{\epsilon^2+\gamma^2\omega^2}. \quad (36)$$

Analogously, the dynamical local susceptibility reads

$$\mathrm{Im}\,\chi^{\rm loc}_{\rm cl}(\omega+i0) = \frac{\mu\gamma\omega}{\epsilon^2+\gamma^2\omega^2}. \quad (37)$$

B. Summing over all clusters

We now combine the single-cluster observables summarized in Sec. IV A with the strong-disorder RG results for density and moment of the surviving clusters given in Eqs. (27) and (28). We focus on the critical point and the disordered Griffiths phase. On the ordered side of the transitions, the scenario is dimensionality dependent because in $d>1$ an infinite percolating RG cluster forms already at a finite-energy scale.[12]

To obtain the uniform static order-parameter susceptibility $\chi(r,T)$ and the corresponding local susceptibility $\chi^{\rm loc}(r,T)$, we run the RG to the energy scale $\Omega=T$ and sum over all surviving clusters. Using Eqs. (27), (28), and (32), we obtain the scaling form

$$\chi(r,T) = \frac{1}{T}n_T(r)\mu_T^2(r) = \frac{1}{T}[\ln(\Omega_I/T)]^{2\phi-d/\psi}\Theta_\chi[r^{\nu\psi}\ln(\Omega_I/T)] \quad (38)$$

with the scaling function Θ_χ given by $\Theta_\chi(y)=X_n(y)X_\mu^2(y)$. At criticality, $r=0$, this leads to $\chi\sim[\ln(\Omega_I/T)]^{2\phi-d/\psi}/T$. In the Griffiths phase we need to use the large-argument limit of the scaling function giving $\chi\sim T^{d/z-1}r^{d\nu+2\nu\psi(1-\phi)}\ln^2(\Omega_I/T)$. Thus, χ shows the nonuniversal power-law temperature dependence characteristic of a quantum Griffiths phase. For $z>d$, the susceptibility actually diverges with $T\to 0$. Along the same lines, the scaling form of the local susceptibility is found to be

$$\chi^{\rm loc}(r,T) = \frac{1}{T}[\ln(\Omega_I/T)]^{\phi-d/\psi}\Theta^{\rm loc}_\chi[r^{\nu\psi}\ln(\Omega_I/T)], \quad (39)$$

with $\Theta^{\rm loc}_\chi(y)=X_n(y)X_\mu(y)$. This reduces to $\chi^{\rm loc}\sim[\ln(\Omega_I/T)]^{\phi-d/\psi}/T$ at criticality and to $\chi^{\rm loc}\sim T^{d/z-1}r^{d\nu+\nu\psi(1-\phi)}\ln(\Omega_I/T)$ in the disordered Griffiths phase.

The scaling form (38) of the susceptibility can also be used to infer the shape of the phase boundary close to the quantum phase transition. The finite-temperature transition corresponds to a singularity in $\Theta_\chi(y)$ at some nonzero argument y_c. This yields the unusual form $T_c\sim\exp(-\mathrm{const}|r|^{-\nu\psi})$ shown in Fig. 1. The crossover line between the quantum critical and quantum paramagnetic regions displays analogous behavior.

The specific heat C can be found by first adding the total-energy contributions of all surviving clusters,

$$\Delta E(r,T) = Tn_T(r) = T[\ln(\Omega_I/T)]^{-d/\psi}\Theta_E[r^{\nu\psi}\ln(\Omega_I/T)] \quad (40)$$

with $\Theta_E(y)=X_n(y)$. After taking the temperature derivative this gives $C\sim[\ln(\Omega_I/T)]^{-d/\psi}$ at criticality and $C\sim r^{d\nu}T^{d/z}$ in the Griffiths phase.

To calculate the dependence of the low-temperature order parameter m on an external (conjugate) field H, we run the RG to the energy scale $\Omega_H=\mu_\Omega H\gg T$. All decimated clusters have $\epsilon\gg\mu H$ and do not contribute significantly to the order parameter. All surviving clusters have $\epsilon\ll\mu H$ and are fully polarized. Summing over all surviving clusters therefore gives

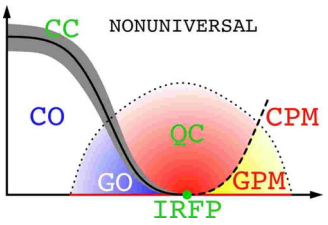

FIG. 1. (Color online) Temperature-coupling phase diagram for Ohmic dissipation. IRFP denotes the infinite-randomness critical point. The ordered phase is divided into a conventional (CO) region and a quantum Griffiths (GO) region. On the disordered side of the transition, there is a quantum Griffiths paramagnet (GPM) followed by a conventional quantum paramagnet (CPM). The phase boundary (solid curve) and the crossover line (dashed curve) between the quantum critical (QC) region and the quantum paramagnetic regions take unusual exponential forms leading to a wide quantum critical region. In the classical critical region (CC) close to the phase boundary classical thermal fluctuations dominate. At sufficiently high temperatures (above the dotted dome), the behavior is nonuniversal.

$$m(r,H) = n_{\Omega_H}(r)\mu_{\Omega_H}(r) = [\ln(\Omega_I/\Omega_H)]^{\phi-d/\psi}\Theta_m[r^{\nu\psi}\ln(\Omega_I/\Omega_H)], \quad (41)$$

where $\Theta_m(y)=X_n(y)X_\mu(y)$. After resolving the implicit field dependence caused by the moment in the definition of the energy scale $\Omega_H=\mu_\Omega H$, we find $m\sim[\ln(\Omega_I/H)]^{\phi-d/\psi}$ (with double-logarithmic corrections) at criticality, $r=0$. For $r>0$, we obtain a nonuniversal power-law field dependence, $m\sim H^{d/z}r^{d\nu+\nu\psi(1-\phi)(1+d/z)}[\ln(\Omega/H)]^{1+d/z}$, characteristic of a quantum Griffiths phase.

Finally, to find the zero-temperature dynamic susceptibility χ at external frequency ω, we run the RG to the energy scale $\Omega_\omega=\gamma\omega=\gamma_0\mu_\Omega\omega$ ($\gamma=\mu\gamma_0$ is the effective damping constant of a cluster of moment μ). All decimated clusters (having $\epsilon\gg\gamma\omega$) only make negligible contributions to χ. The surviving clusters have $\epsilon\ll\gamma\omega$ which simplifies Eq. (36) to $\mathrm{Im}\,\chi_{\mathrm{cl}}(\omega)=\mu/\gamma_0\omega$. In the same limit, the local dynamic susceptibility reads $\mathrm{Im}\,\chi^{\mathrm{loc}}_{\mathrm{cl}}(\omega)=1/\gamma_0\omega$. Using Eqs. (27) and (28) we now sum over all surviving clusters to obtain the scaling form

$$\mathrm{Im}\,\chi(r,\omega) = \frac{1}{\gamma_0\omega}[\ln(\Omega_I/\Omega_\omega)]^{\phi-d/\psi}\Theta_{\mathrm{dyn}}[r^{\nu\psi}\ln(\Omega_I/\Omega_\omega)], \quad (42)$$

with $\Theta_{\mathrm{dyn}}(y)=X_n(y)X_\mu(y)$. After resolving the implicit frequency dependence brought about by the moment in the definition of Ω_ω, the leading low-frequency behavior of the dynamic susceptibility at criticality is $\mathrm{Im}\,\chi\sim[\ln(\Omega_I/\gamma_0\omega)]^{\phi-d/\psi}/(\gamma_0\omega)$. In the disordered Griffiths phase, we obtain

$$\mathrm{Im}\,\chi\sim(\gamma_0\omega)^{d/z-1}r^{d\nu+\nu\psi(1-\phi)(1+d/z)}[\ln(\Omega_I/\gamma_0\omega)]^{1+d/z}.$$

The differences from the random transverse-field Ising-model results[51,52] have two reasons: (i) the additional frequency dependence hidden in the effective damping constant γ and (ii) the difference between a Lorentzian spectrum in our case and the δ-function spectrum in the Ising case. The local dynamic susceptibility can be found along the same lines, yielding the scaling form

$$\mathrm{Im}\,\chi^{\mathrm{loc}}(r,\omega) = \frac{1}{\gamma_0\omega}[\ln(\Omega_I/\Omega_\omega)]^{-d/\psi}\Theta^{\mathrm{loc}}_{\mathrm{dyn}}[r^{\nu\psi}\ln(\Omega_I/\Omega_\omega)] \quad (43)$$

with $\Theta^{\mathrm{loc}}_{\mathrm{dyn}}(y)=X_n(y)$. At criticality, this leads to $\mathrm{Im}\,\chi^{\mathrm{loc}}\sim[\ln(\Omega_I/\gamma_0\omega)]^{-d/\psi}/\gamma_0\omega$, and in the disordered Griffiths phase we get

$$\mathrm{Im}\,\chi^{\mathrm{loc}}\sim(\gamma_0\omega)^{d/z-1}r^{d\nu+\nu\psi(1-\phi)d/z}[\ln(\Omega_I/\gamma_0\omega)]^{d/z}.$$

V. GENERALIZATIONS

A. Non-Ohmic Dissipation

In this section we briefly discuss how our results change if we replace the Ohmic damping term ($z_0=2$) in the starting actions (1) and (2) with a non-Ohmic term ($z_0\neq 2$). We are interested in the range $z_0=1-\infty$; $z_0=1$ corresponds to undamped (dissipationless) dynamics, $1<z_0<2$ is the so-called super-Ohmic case (damping qualitatively weaker than Ohmic damping), and for $z_0>2$ the damping is sub-Ohmic (qualitatively stronger than Ohmic).

Let us first consider sub-Ohmic damping, $z_0>2$. In this case, the crucial observation is that a single cluster with sufficiently large damping constant can undergo a freezing or localization transition independent of the bulk system. In Sec. III A we showed that this transition occurs when the damping constant γ becomes larger than $\gamma_c=\Lambda^{(z_0-2)/z_0}z_0/[\pi(z_0-2)]$. Within the strong-disorder RG, the damping constant γ renormalizes additively. Thus, even for very small bare dissipation, sufficiently large and strongly damped clusters will be formed under the RG (as long as $\mu\to\infty$ with $\Omega\to 0$). Once they are formed, their quantum dynamics freezes. Consequently, for $z_0>2$ the global quantum phase transition is destroyed by smearing.[6]

In the super-Ohmic case, $1\leq z_0<2$, the behavior is less exotic. To study this case, we repeat the derivation of the strong-disorder RG recursion relations described in Sec. III B for $1\leq z_0<2$. As was already pointed out, the multiplicative form of the recursion (14) for the interactions J follows directly from the structure of second-order perturbation theory and does not depend on z_0. In contrast, the recursion for the gaps ϵ does depend on the value of z_0. Repeating the exact solution of the two-site cluster for the super-Ohmic case, we find

$$\tilde{\epsilon}_2^{-x} = \alpha[\epsilon_2^{-x}+\epsilon_3^{-x}] \quad (44)$$

instead of the multiplicative form (22). Here $x=(2-z_0)/z_0$ and α is a constant. This form also follows from the fact that the damping constants add, $\tilde{\gamma}_2=\gamma_2+\gamma_3$, together with the power-law dependence $\epsilon\sim\gamma^{z_0/(z_0-2)}=\gamma^{-1/x}$ of the single-cluster gap on the damping constant derived in Eq. (7). For undamped dynamics, $z_0=1$, Eq. (44) reduces to the dirty boson result $1/\tilde{\epsilon}_2=1/\epsilon_2+1/\epsilon_3$ derived by Altman *et al.*[53] These authors also solved the resulting flow equations for $z_0=1$ and found Kosterlitz-Thouless-type flows.

While a full solution of the RG flow equations in the generic case $1<z_0<2$ remains a task for the future, the qualitative critical behavior can be inferred from the recursion relation (44). As a result of the additive form of Eq. (44), the local gaps ϵ are much more weakly renormalized than the interactions which are governed by the multiplicative recursion (22). Near criticality, the distribution of the interactions J thus becomes highly singular while that of the gaps ϵ remains narrower. We therefore expect the critical point not to be of infinite-randomness type but conventional with power-law scaling $\tau \sim \xi^z$, although the dynamical exponent z can become arbitrarily large as $z_0 \to 2^-$. Similar behavior was found at a percolation quantum phase transition.[54]

B. Generic $N>1$

So far, all of our explicit calculations have been for the large-N limit of the $O(N)$ order-parameter field theory. In this section we show that the results do not change qualitatively for all $N>1$, i.e., all continuous-symmetry cases. In order to do so, we reanalyze the recursion relations (14) and (22) for generic N [the relations (15) and (21) trivially carry over for all N]. For definiteness, we focus on the case of Ohmic dissipation.

As discussed above, the multiplicative form of the recursion (14) for the interactions relies on the structure of second-order perturbation theory only. It is thus valid for all N including the discrete Ising case. In contrast, the form of the recursion (22), which describes how the local gap ϵ (i.e., distance from criticality) changes if two clusters are combined, potentially does depend on N. To understand this dependence, we first look at the related problem of the dependence of ϵ on the size (moment) of the cluster.

By invoking the quantum-to-classical mapping it was recently shown[25] that the gap depends exponentially on the size, $\epsilon \sim e^{-c\mu}$ (with c being a constant), for all continuous-symmetry cases $N>1$. This follows from the fact that classical one-dimensional continuous-symmetry $O(N)$ models with $1/r^2$ interaction are known to be exactly *at* their lower critical dimension[55–57] implying an exponential dependence of the correlation length on the coupling strength. Alternatively, one can explicitly estimate the strength of the *transverse* fluctuations in a putative ordered phase and notice the logarithmic divergence of $\int_0^\Lambda d\omega/\gamma\omega$. The exponential size (moment) dependence of the gap ϵ requires a multiplicative structure of the recursion relation (22) for the merging of two clusters because their moments simply add [see Eq. (15)]. We thus conclude that this multiplicative structure is valid for all continuous-symmetry cases, $N>1$.

Consequently, for sufficiently broad disorder distributions, the complete set of recursion relations (14), (15), (21), and (22) is valid for all $N>1$, and with it the resulting infinite-randomness scenario of Sec. III C. Possible N-dependent prefactors modify nonuniversal quantities only. An analogous conclusion was drawn in the undamped case, $z_0=1$, in Refs. 53 and 58.

The universal behavior of all continuous-symmetry cases has to be contrasted with the case of Ising symmetry, $N=1$. In the Ising case, the gap does not depend exponentially on the cluster size. Instead, for sufficiently large Ohmic dissipation, the cluster dynamics freezes; i.e., it undergoes the localization transition of the dissipative two-state system.[59] The resulting behavior of an Ising system with Ohmic dissipation is thus very similar to that of a continuous-symmetry system with sub-Ohmic dissipation (as discussed in Sec. V A). Sufficiently large clusters freeze independently from the rest of the system which leads to a destruction of the global quantum phase transition by smearing. This behavior was predicted in Ref. 6 and recently confirmed by an analytical strong-disorder RG (Ref. 7) as well as numerical simulations.[60,61]

VI. CONCLUSIONS

In summary, we have studied quantum phase transitions in systems with continuous-symmetry $O(N)$ order parameters under the influence of both quenched disorder and dissipative dynamics. To this end, we have applied a strong-disorder RG to the LGW order-parameter field theory of the transition. For Ohmic dissipation, we have found an exotic infinite-randomness critical point in the same universality class as the random transverse-field Ising chain. In the sub-Ohmic case, the quantum phase transition is destroyed by smearing, while super-Ohmic damping (including the undamped case) leads to conventional behavior. These results must be contrasted with the case of Ising symmetry for which an infinite-randomness critical point occurs in the absence of damping[4,5] while Ohmic dissipation causes a smeared quantum phase transition.[6,7]

All these different behaviors and their relations can be understood with the help of a general classification[8,25] of phase transitions in the presence of weak disorder. This classification is based on the effective dimensionality of the defects or, equivalently, the rare regions. If finite-size regions are exactly at the lower critical dimension of the problem, the critical point is of infinite-randomness type (accompanied by power-law quantum Griffiths singularities). Here this applies to continuous-symmetry order parameters with Ohmic dissipation as well as dissipationless Ising order parameters. If the rare regions are below the lower critical dimension, the behavior is conventional (continuous-symmetry order parameters with super-Ohmic dissipation); and if they are above the lower critical dimension, individual regions order (freeze) independently, leading to a smeared transition (continuous-symmetry order parameters with sub-Ohmic damping or Ising systems with at least Ohmic damping).

It is worth noting that Del Maestro *et al.*[33] very recently studied the large-N action (3) in one dimension by numerically solving the saddle-point equations. All their results are in beautiful agreement with our predictions; i.e., they confirmed that the quantum critical point is of infinite-randomness type and in the universality class of the random transverse-field Ising model.

We now turn to potential experimental realizations of our theory. One application is the Hertz-Millis theory[28,29] of the (incommensurate) itinerant antiferromagnetic quantum phase transition. In this theory, the LGW free energy (1) is derived from a microscopic Hamiltonian of interacting electrons by

integrating out the fermionic degrees of freedom in favor of the order-parameter field φ. While this procedure involves integrating out soft (gapless) particle-hole excitations and is thus potentially dangerous,[36] the resulting order-parameter field theory of the antiferromagnetic transition appears to be internally consistent and free of additional singularities at least in three dimensions. However, the applicability of the theory to realistic systems is still a controversial question, in particular, for the much-studied heavy fermion compounds where several experimental results are in pronounced disagreement with the theoretical predictions.[26,27] Different scenarios to explain the discrepancies are discussed in the literature (see Ref. 62 for a recent review), and one much-discussed reason are disorder effects.[63]

Our theory provides explicit results on how the interplay of dissipation and disorder in the vicinity of the itinerant antiferromagnetic quantum phase transition can yield activated dynamics, quantum Griffiths phenomena, and non-Fermi-liquid behavior. We expect this to make an experimental verification or falsification of the disorder scenario much easier. Note that a generic metallic system will have extra complications not contained in the LGW free energy (1). Specifically, attention must be paid to the long-range Ruderman-Kittel-Kasuya-Yosida (RKKY) part of the interaction between the magnetic fluctuations. It can produce an extra sub-Ohmic dissipation of locally ordered clusters[64] which leads to freezing into a "cluster glass" phase at a low nonuniversal temperature T_{CG} determined by the strength of the RKKY interactions. This phase replaces part of the quantum Griffiths regions. Its properties and the zero- and finite-temperature transitions to the surrounding phases are not fully explored yet (the transitions may be of fluctuation-driven first order at low temperatures[65]). The behavior of observables in the broad quantum critical region above the cluster glass phase will be controlled by our infinite-randomness critical point. Possible phase diagram scenarios are sketched in Fig. 2.

Another potential application that has attracted considerable attention recently is the superconductor-metal quantum phase transition occurring as a function of wire thickness in ultrathin nanowires.[30] The clean version of this transition was studied by means of a one-dimensional LGW theory (1) with a complex order parameter (equivalent to $N=2$) and Ohmic dissipation.[31,32] However, there is an experimental evidence for the pair breaking in this system being caused by magnetic impurities at the surface of the nanowire. This inevitably introduces quenched disorder due to the random positions of the magnetic impurities. Our theory thus describes the thermodynamics of this quantum phase transition. With proper modifications, it should also apply to arrays of resistively shunted Josephson junctions.

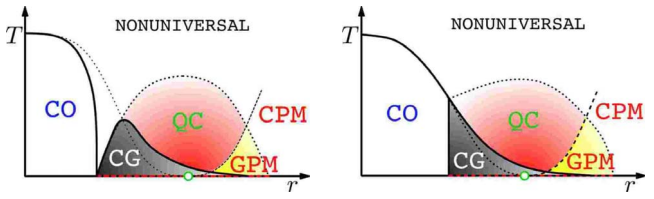

FIG. 2. (Color online) Schematic phase diagrams of a disordered itinerant quantum antiferromagnet close to the quantum phase transition showing two possible scenarios for the appearance of a cluster glass phase, which is denoted by CG. The other labels are as in Fig. 1.

So far, we have focused on the thermodynamics close to the quantum phase transition. Transport properties can also be calculated within the strong-disorder RG by following the approach of Refs. 51 and 52. Calculations along these lines are underway; their results will be reported elsewhere.

ACKNOWLEDGMENTS

This work was supported in part by the NSF under Grants No. DMR-0339147 and No. DMR-0506953, by the Research Corporation, and by the University of Missouri Research Board. We gratefully acknowledge discussions with A. Del Maestro and S. Sachdev as well as the hospitality of the Aspen Center for Physics during part of this research.

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