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Effects of localization and amplification on intensity distribution of light transmitted through random media

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We numerically study the statistical distribution of intensity of light transmitted through quasi-one-dimensional random media by varying the dimensionless conductance g and the amount of absorption or gain. A markedly non-Rayleigh distribution is found to be well fitted by the analytical formula of Nieuwenhuizen *et al.* [Phys. Rev. Lett. **74**, 2674 (1995)] with a single parameter g'. We show that in the passive random system g' is uniquely related to g, while in amplifying or absorbing random media g' also depends on the gain or absorption coefficient.

Light transport in a quasi-one-dimensional system is described by three directly measurable quantities: T_{ab}, the transmission coefficient from an incoming channel a to an outgoing channel b; $T_a = \Sigma_b T_{ab}$, the total transmission coefficient from channel a to all outgoing channels; $\widetilde{g} = \Sigma_{a,b} T_{ab}$, the transmittance [1,2]. A wave transmitted through a random medium is a coherent sum of a large number of contributions due to different propagation paths. In the diffusive regime, $\langle \widetilde{g} \rangle \equiv g \gg 1$, the contributions T_{ab} are largely uncorrelated. A Gaussian distribution of the field components leads to the renowned Rayleigh distribution $P(s_{ab}) = \exp(-s_{ab})$ of the normalized intensity [3], $s_{ab} = T_{ab}/\langle T_{ab} \rangle$, with a simple relation between the moments of the distribution: $\langle s_{ab}^n \rangle = n! \langle s_{ab} \rangle^n$. This factorization cannot be exact [6], since it implies the complete absence of nonlocal correlations [7]. Kogan *et al.* [8] related the distribution of the normalized total transmission $s_a = T_a/\langle T_a \rangle$ to that of s_{ab}:

$$P(s_{ab}) = \int_0^\infty \frac{ds_a}{s_a} P(s_a) \exp\left[-\frac{s_{ab}}{s_a}\right]. \qquad (1)$$

The Gaussian distribution of $P(s_a)$ for $g \gg 1$ leads to a deviation from a Rayleigh distribution at large intensity $s_{ab} \gg g$. The deviation has the same origin as the universal conductance fluctuations and nonlocal intensity correlations in transport [1,2]. The distribution of the intensity was obtained in a closed form in Refs. [9,10]:

$$P(s_a) = \int_{-i\infty}^{i\infty} \frac{dx}{2\pi i} \exp[x s_a - \Phi_0(g',x)], \qquad (2)$$

where $\Phi_0(g',x) = g' \ln^2(\sqrt{1+x/g'} + \sqrt{x/g'})$. Equation (2) was derived under the assumption of $g \gg 1$; thus $g' \equiv g$. The expressions for $P(s_a)$ and $P(s_{ab})$ have been verified in experiments [5,11–13]. Unexpectedly, the experiments demonstrated that Eq. (2) worked well even for moderate values of $g \sim 10$, and in the presence of significant absorption. More-

over, based on the statistics of the transmitted intensity, the localization criterion

$$g' \equiv \tfrac{2}{3}\mathrm{var}(s_a) \equiv \tfrac{4}{3}[\mathrm{var}(s_{ab}) - 1] \qquad (3)$$

equals unity was surmised [11]. Such a definition of g' can be used, irrespective of whether Eq. (2) holds. However, if Eq. (2) is applicable, g' obtained from Eq. (3) should match the one obtained from the fit of the entire distribution of s_{ab} with Eqs. (1) and (2) [10]. One question to be addressed in this paper is whether Eqs. (1) and (2) still hold in the regime of incipient photon localization $g \sim 1$, and, if so, how the intensity distribution fitting parameter g' is related to the system properties.

We will also investigate the effects of optical amplification and absorption on the transmitted intensity distribution. The effect of the amplification on purely one-dimensional (1D) transport has been studied by a number of groups [14]. In a quasi-1D system, the probability of reflectance by a random amplifying medium was also obtained by Beenakker *et al.* [15]. Zyuzin showed that the fluctuation of T_a grows faster than its average value in an amplifying random medium near the lasing threshold [16]. However, it is not clear whether the statistical distribution of the transmission coefficient of an active system differs qualitatively from that of a passive system. Brouwer demonstrated that Eq. (2) is inapplicable in the limit of strong absorption [17,18], whereas we are interested in weakly absorbing systems with $\sqrt{D\tau} > L$, where D is the diffusion coefficient, and τ is the absorption time.

In this paper, we numerically calculate the statistical distribution of transmitted intensity $P(s_{ab})$ in a quasi-1D system. In particular, we check whether $P(s_{ab})$ still satisfies Eqs. (1) and (2). In a real experiment or a numerical study, the local intensity distribution $P(s_{ab})$ is obtained by collecting the data of transmitted intensity from many random configurations. Among them, there exist rare configurations that could lase in the presence of gain. The light intensity would diverge if gain saturation were neglected. In the diffusive regime $g \gg 1$, this problem is limited only to the immediate vicinity of the diffusive lasing threshold [16]. For the systems we considered, $g < 10$, strong fluctuation of the lasing

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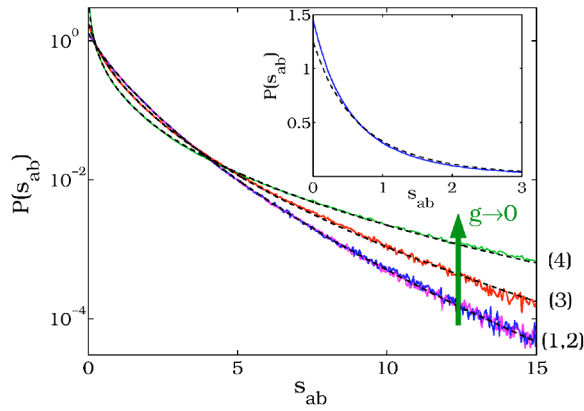*Electronic address: a-yamilov@northwestern.edu

FIG. 1. Numerically calculated $P(s_{ab})$ for four samples. Dashed lines represent the fit with Eqs. (1) and (2). The inset shows $P(s_{ab})$ calculated from Eqs. (1) and (2) with the kernel $\Phi_0(g',x)$ (solid line), or $\Phi_1(g',x)$ (dashed line). $g'=3$.
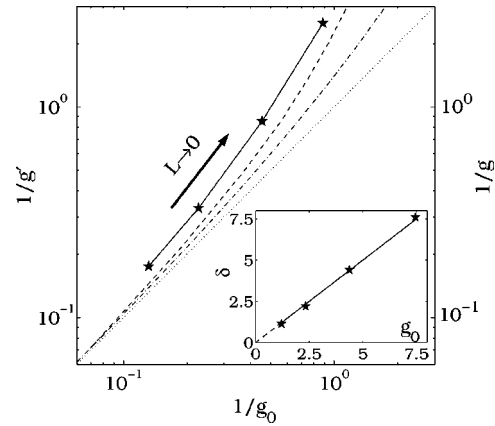

FIG. 2. Symbols depict $1/g'$ versus $1/g_0$ for samples 1, 3–5. In the zeroth order of $1/g_0$, $g'(g_0)=g_0$ (dotted line). The dash-dotted line is $g'(g_0)$ with a first-order correction. Dashed line plots the exact solution $g(g_0)$ from Ref. [22]. The inset shows the linear dependence of δ on g_0. The solid line has a slope of 1.

threshold results in a non-negligible percentage of realizations whose lasing threshold is met even at moderate gain (i.e., not very close to the diffusive lasing threshold). Although gain saturation could prevent the divergence of laser intensity, the actual value of the saturated intensity depends on the properties of the gain material. In order to eliminate any material-dependent effect on $P(s_{ab})$, we disregard the contributions of the lasing configurations to the intensity distribution [19]. Our numerical studies show that the resultant $P(s_{ab})$ can be well described by Eqs. (1) and (2) with a reduced g'.

We numerically obtain $P(s_{ab})$ as the histogram of the intensity transmitted through a quasi-1D system: a 2D metallic waveguide filled with circular dielectric scatterers. We employ the method developed earlier in Ref. [20] for passive and Ref. [19] for amplifying and absorbing systems. In quasi-1D geometry the transition from diffusion $g \gg 1$ to localization $g \lesssim 1$ can be achieved even in the weak scattering regime by increasing the system length L beyond the localization length ξ. First, we study the intensity distribution in passive systems as g approaches 1. The random system is characterized by the Thouless number $\delta = \delta\nu/\Delta\nu$. The average mode linewidth $\delta\nu$ is obtained from the width of $|C_E(\Delta\nu)|^2$ divided by a numerical factor 1.46 [$C_E(\Delta\nu)$ is the spectral field correlation function]. The average mode spacing is $\Delta\nu = c/(\pi L' N n_{eff}^2)$, where $L'=L+2z_b$ is the effective length of the random medium; z_b accounts for the boundary effects. N is the number of waveguide modes; c is the speed of light in vacuum; and n_{eff} is the effective refractive index. All these parameters can be determined independently [19,20]. For a passive diffusive system δ coincides with the dimensionless conductance g. We consider four samples labeled from 1 to 4. The lengths of the first two samples are related as $L_2=2L_1$, and the scatterer density is chosen in such a way that an increase of L would be offset by a change of the transport parameters, mostly by the transport mean free path l, to give the identical value of $\delta=4.4$. Samples 3 and 4 had the same scatterer densities as sample 1, but L was increased, yielding $\delta=2.2$ and $\delta=1.13$, respectively. Figure 1 shows that samples 1 and 2 are fitted by Eqs. (1) and (2) with

the same $g'=2.9$. This means that the distribution of local intensity is determined by a single parameter g', which depends only on δ (or g; see below). Samples 3 and 4 are fitted well with $g'=1.25$ and $g'=0.4$. This agreement confirms the applicability of Eqs. (1) and (2) down to $g'=0.4$, even smaller than the value reported in the microwave experiments of Refs. [5,11,13]. In what follows we give an argument that may shed some light on this agreement

In Ref. [21] Van Langen *et al.* found that the proper perturbation series should be constructed for $\Phi(g',x)$, the kernel in Eq. (2). The first order (in $1/g$) correction to $\Phi_0(g',x)$ of Refs. [9,10] was calculated [21]. In the inset of Fig. 1 we plotted $P(s_{ab})$ calculated within both approximations. We see that Eq. (2) with Φ_0 starts to fail for the $s_{ab} < 1$ region, while the asymptotic behavior for $s_{ab} \gtrsim 1$ is preserved. In the transition from diffusion to the localization threshold ($g \sim 1$), $P(s_{ab} \gtrsim 1)$ should be well described by Eqs. (1) and (2) with decreasing g', a quantitative change. But for $g \ll 1$ $P(s_{ab})$ changes to the lognormal distribution, a qualitative change, not captured by the Φ_0 approximation. The numerically obtained intensity distribution, indeed, shows some deviation from Eqs. (1) and (2) at small values of s_{ab}. The deviation, however, is noticeable only when $P(s_{ab})$ is plotted on a linear scale. On a logarithmic scale, the distribution has a pronounced tail at large s_{ab}, where the correct asymptotic prevails.

Next, we would like to analyze the value of g'. The following four quantities should coincide in a diffusive system: g', g, δ, and $g_0=(\pi/2)n_{eff}Nl/L'$. Figure 2 shows the relations of these four quantities in the regime of incipient localization. The numerical data include sample 5 with $\delta=7.6$. The inset of Fig. 2 shows that for the studied systems $\delta \simeq g_0$, and thus our data for $\delta(g_0)$ should lie along the dotted line of slope 1 in the main plot. The rigorous solution $g(g_0)$ obtained by Mirlin in Ref. [22] is shown in Fig. 2 with the dashed line. Our data for $1/g'$ lie above the dotted and dashed lines, revealing the tendency of g' to decrease below g_0, δ and g in the regime $g_0 \sim 1$. The first order (in $1/g_0$) correction to $g'(g_0) \simeq g_0$ can be obtained from the variance
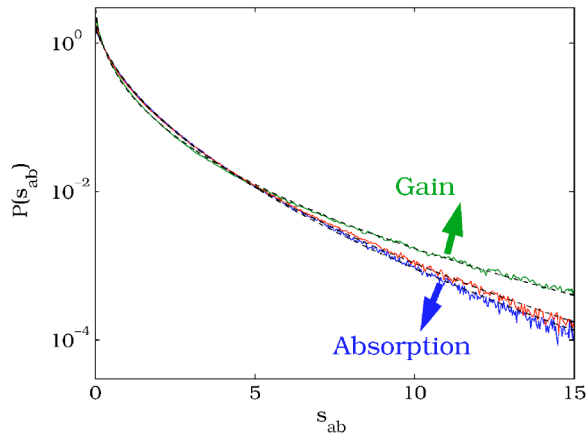
FIG. 3. Numerically calculated $P(s_{ab})$ for sample 3 with and without gain/absorption. The arrows show the direction of increasing gain/absorption. Dashed lines represent the fit with Eqs. (1) and (2) with g' equal to 1.5 (absorption), 1.25 (passive), and 0.65 (gain).
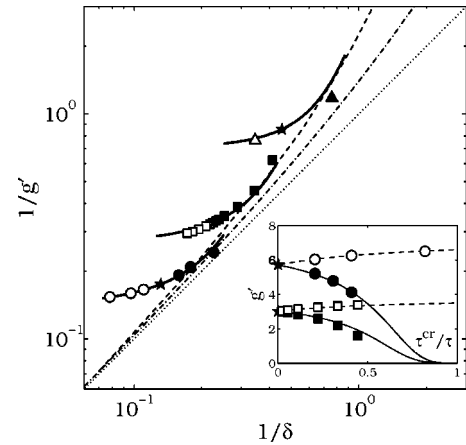

FIG. 4. The numerical data for $1/g'$ versus $1/\delta$ for samples 1 (squares), 3 (triangles), and 5 (circles). Solid (open) symbols represent the systems with gain (absorption). The pentagons correspond to the passive system. The dotted, dashed, and dash-dotted lines are the same as in Fig. 2. Solid lines represent $g'(\delta)$. The inset shows the explicit dependence of g' on the amplification or absorption time τ normalized to τ_{cr}. Dashed (solid) line plots Eq. (4) with positive (negative) τ.

of the intensity distribution, $\text{var}(s_{ab}) \simeq 1 + 4/3g_0 + 8/15g_0^2$ upto the second order of $1/g_0$ [2]. The approximation of $g'(g_0)$ obtained by substitution of the above expression into Eq. (3) is plotted in Fig. 2 with the dash-dotted line. The fact that the data points of $1/g'$ are above the dash-dotted line indicates that the higher-order terms are not negligible and their contributions lead to a further reduction of g'.

Next, we study the effect of coherent amplification on the intensity distribution. Figure 3 compares the local intensity distributions found in sample 3 with and without gain or absorption. The value of the absorption time is the same as that of the amplification time. The latter was equal to $\tau = 5\tau^{cr}$, where τ^{cr} is the critical amplification time corresponding to the diffusive lasing threshold [23]. Even at such a low level of gain, some of the random realizations lased, due to strong fluctuation of the lasing threshold. The numerical results presented in Fig. 3 contain only the contributions from nonlasing realizations.

First of all, we would like to point out that Eqs. (1) and (2) give a good fit to the numerically obtained $P(s_{ab})$ with gain or absorption. In Refs. [5,13] it was also found that Eqs. (1) and (2) describe well the intensity distribution even for absorbing systems. Here, we present a systematic study of the effect of absorption on $P(S_{ab})$ and also consider the effect of amplification. Figure 3 shows that the presence of gain leads to an increase of $P(S_{ab})$ in the regions $S_{ab} \ll 1$ and $S_{ab} \gg 1$ and, therefore, an enhancement of intensity fluctuations. The effect of absorption is exactly the opposite. From this qualitative analysis and Eq. (3), we see that amplification reduces the value of g', while absorption increases it.

Figure 4 shows the effect of amplification or absorption on g' for samples 1, 3, and 5. We can still define δ for amplifying or absorbing systems through the correlation functions [19]. In contrast to g_0, δ depends on the amount of gain or absorption. The change of δ is directly related to the change of the correlation linewidth $\delta\nu$, while $\Delta\nu$ remains the same as in a passive system. The superlinear increase of $1/g'$

with $1/\delta$ in Fig. 4 demonstrates that g' decreases significantly faster than δ with increasing gain. This indicates that intensity fluctuations are more sensitive to amplification than the average mode linewidth $\delta\nu$. The long propagation paths extract more gain, leading to an enhancement of nonlocal correlations [19]. The enhanced correlation results in a further deviation from Rayleigh statistics. The absorption, on the other hand, causes a reduction of fluctuations. The sublinear decrease of $1/g'$ with $1/\delta$ in Fig. 4 reveals that g' increases more slowly than δ with increasing absorption. The dependence of g' on the absorption time τ can be determined perturbatively from the known result for $\text{var}(s_{ab})$ in an absorbing system of $g_0 \gg 1$:

$$\frac{4}{3g'(g_0, \tau)} \equiv \text{var}(s_{ab}) - 1 = \frac{4}{3g_0}A_2(\tau) + \frac{8}{15g_0^2}A_3(\tau), \quad (4)$$

where $A_2(\tau)$ and $A_3(\tau)$ are given in Refs. [18,24]. The expressions for A_i were derived for absorbing systems, where τ is positive. In the negative absorption model, τ becomes negative for systems with gain. In Ref. [19] the applicability of the negative-τ expressions was discussed. Obviously, the contribution of the lasing realizations should be omitted to avoid divergence of $\text{var}(s_{ab})$. For samples 1 and 5, which have larger g_0, we compared our data to Eq. (4) in the inset of Fig. 4. Also, $\delta(g_0, \tau)$ can be found from the width of $|C_E(\Delta\nu)|^2$ [19]. By eliminating τ from $g'(g_0, \tau)$ and $\delta(g_0, \tau)$ we obtained $g'(\delta)$, shown as the solid lines in Fig. 4. The deviations of our data points from the solid line increase with decrease of g_0, because the contributions of the higher-order terms in $1/g_0$ cannot be neglected.

In Ref. [11] g' was proposed as a localization criterion parameter. Table I shows that in passive systems the criteria

TABLE I. Comparison of localization threshold criteria in a passive system.

g'	g_0	δ	g
1.0	2.0	2.0	1.3
0.3	1.0	1.0	0.43
0.7	1.6	1.6	1.0

based on $g'=1$, $g_0=1$, $g=1$, and $\delta=1$ differ by only a numerical factor of order 1. In the presence of gain or absorption the situation is essentially different. By definition, g_0 contains only information about the passive system, whereas g' and δ account for the effect of amplification or absorption. In this case the difference between g', δ, and g is not merely numerical. Amplification (absorption) results in a decrease (increase) of g' and δ, but an increase (decrease) of g. With the increase of absorption, δ increases without a bound, while $g'(\tau\to0)$ approaches $4g'(\tau\to\infty)/3$ in the limit $g\gg1$. In sharp contrast, we see that in amplifying systems g' diminishes superlinearly with τ^{cr}/τ (inset of Fig. 4), while δ decreases almost linearly with τ^{cr}/τ as reported in Ref. [19]. Our numerical results in the inset of Fig. 4 also suggests that g' falls below unity prior to δ in an amplifying system.

Therefore, g' is more sensitive to amplification but less sensitive to absorption than δ.

In conclusion, we numerically calculated the statistical distribution of the intensity of light transmitted through a quasi-1D random medium. In a passive system $P(s_{ab})$ is well described by Eqs. (1) and (2) down to $g'=0.4$, far beyond the $g'\gg1$ regime where it was originally derived. Surprisingly, Eqs. (1) and (2) also hold for amplifying and absorbing systems with g' different from the value of the passive system. Our data show that g' decreases superlinearly with increasing gain constant, but increases sublinearly with absorption constant.

The results we obtained for passive random media are also applicable to the mesoscopic transport of other scalar waves, such as noninteracting electron wave functions and acoustic waves, which is governed by essentially the same wave equation. Yet the phenomenon of coherent amplification does not have its counterpart in any fermionic system, e.g., the electronic system.

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