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**Microscopic examination of hot spots giving rise to nonlinearity in superconducting resonators**Cihan Kurter,<sup>1</sup> Alexander P. Zhuravel,<sup>2</sup> Alexey V. Ustinov,<sup>3</sup> and Steven M. Anlage<sup>1</sup><sup>1</sup>*Center for Nanophysics and Advanced Materials, Department of Physics, University of Maryland, College Park, Maryland 20742-4111, USA*<sup>2</sup>*B. Verkin Institute for Low Temperature Physics and Engineering, National Academy of Sciences of Ukraine, UA-61103 Kharkov, Ukraine*<sup>3</sup>*Physikalisches Institut and DFG-Center for Functional Nanostructures (CFN),**Karlsruhe Institute of Technology, DE-76128 Karlsruhe, Germany*

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We investigate the microscopic origins of nonlinear rf response in superconducting electromagnetic resonators. Strong nonlinearity appearing in the transmission spectra at high input powers manifests itself through the emergence of jumplike features near the resonant frequency that evolve toward lower quality factor with higher insertion loss as the rf input power is increased. We directly relate these characteristics to the dynamics of localized normal regions (hot spots) caused by microscopic features in the superconducting material making up the resonator. A clear observation of hot-spot formation inside a Nb thin film self-resonant structure is presented by employing the microwave laser scanning microscope, and a direct link between microscopic and macroscopic manifestations of nonlinearity is established.

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**I. INTRODUCTION**

Microwave resonators utilizing superconducting thin films have enabled the development of various nonlinear devices including bifurcation amplifiers,<sup>1</sup> hot electron mixers,<sup>2</sup> and kinetic inductance photon detectors,<sup>3,4</sup> among other examples. They have been crucial elements in circuits designed for quantum computation.<sup>5</sup> Recently, these on-chip resonators have started to appear in metamaterial designs promising new possibilities such as negative refractive index,<sup>6</sup> electromagnetically induced transparency,<sup>7</sup> tunable THz response,<sup>8</sup> and parametric amplification of negative-index photons.<sup>9</sup>

Nonlinearity is manifested in many forms, including a reduction in quality factor, an increase in insertion loss, shift of the resonant frequency,<sup>10</sup> or appearance of abrupt jumps in the transmission/reflection characteristics.<sup>11</sup> Understanding the microscopic origin of such a response is essential. Various mechanisms have been proposed to explain the nonlinear characteristics at elevated microwave power, including both intrinsic and extrinsic types. Among them, hot-spot formation is found to be one of the major mechanisms, which was both theoretically and experimentally studied in superconducting strip conductors and resonators.<sup>12</sup>

A clear and compelling connection between the microscopic models and the macroscopic picture of nonlinearity revealed by microwave measurements is still lacking. Here we establish this connection through a study of the rf response of a two-dimensional spiral resonator made up of a 200-nm Nb thin film on a quartz substrate via the laser scanning microscopy (LSM) technique.

**II. EXPERIMENT**

The resonant structures have been designed as engineered meta-atoms (planar spirals) of rf metamaterials intended to create a strong magnetic response in the sub-100 MHz range.<sup>13,14</sup> Prior results show that the spirals act as very compact self-resonant structures supporting up to 10 half-wave-length standing waves of rf current.<sup>14</sup> The free space stimulation of these spiral resonators is done by sandwiching them between

two rf antennas made up of loop-terminated coaxial cables whose other ends are connected to an rf network analyzer. Exciting the spirals results in Lorentzian-type resonant peaks in transmission for low levels of rf stimulus power.<sup>14</sup> The peaks do not change in shape or position over a broad range of input powers. When the spirals are strongly driven, the resonant characteristics begin to develop notches near the center of the transmission peaks.<sup>11</sup> Those nonlinear features evolve with rf power and are believed to be associated with the existence of non-superconducting regions in the Nb thin films causing resistive hot-spot formation.

The high-power properties of superconducting strip resonators are very sensitive to the geometrical structure of the current-carrying conductors.<sup>15</sup> Our continuous circular spirals have a unique geometry (see the left inset of Fig. 1) in which the currents flowing in neighboring turns are parallel and approximately equal in magnitude, at least for the first few resonant modes, in contrast with the current flow profile in meander-line resonators used for photon counting.<sup>4</sup> In the spiral, the magnetic fields generated by flowing currents largely cancel in the region between the strips, leading to a self-field pattern nearly parallel to the plane of the spiral. This geometrical effect greatly reduces the accumulation of rf currents at the edges of the strip. Moreover, the continuous arc of the spiral allows for a relatively homogenous and uniform flow along the windings compared to sharp-cornered structures that have a tendency to pile up the currents at the corners and edges.<sup>16,17</sup>

**III. RESULTS AND DISCUSSION****A. Global Microwave Transmission Measurements**

The main panel of Fig. 1 shows the global microwave transmission response of a Nb spiral resonator for a set of rf input powers at 4.4 K. The spiral has an outer diameter  $D_o$  of 6 mm and 40 turns; the strip width  $w$  and spacing  $s$  between the turns are 10  $\mu\text{m}$ . The fundamental resonance gives a pronounced transmission peak at  $\sim 75.84$  MHz for low and moderate rf powers (see Fig. 2 in Ref. 13). Up to  $-2$  dBm, the resonant peaks almost overlap, i.e., transmission  $|S_{21}(f)|$  is a Lorentzian function of frequency and does not

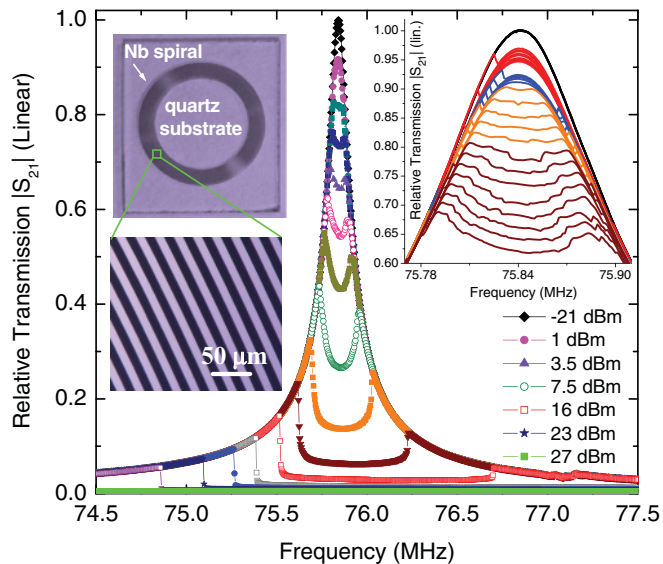


FIG. 1. (Color online) Global microwave transmission response from a  $D_o = 6$  mm spiral with 40 turns and  $w = s = 10 \mu\text{m}$  for a set of rf input powers at  $T = 4.4$  K. Right inset: a closer look at the linear and nonlinear regimes evolving with increasing input power. Black, Lorentzian-type curves corresponding to power values from  $-21$  to  $-2$  dBm overlap, progressively evolving into the red curves for power values between  $-1.5$  and  $+0.6$  dBm then blue curves for powers from  $+0.7$  to  $+1.1$  dBm, orange curves from  $+1.2$  to  $+2$  dBm, and brown curves from  $+2.2$  to  $+3.6$  dBm. Top left inset: an optical image of a spiral resonator with  $D_o = 6$  mm and 40 turns. Bottom left inset: zoom in the area marked in the top left inset showing the Nb spiral windings in detail. Dark strips are Nb turns in the picture.

show any significant difference in shape with increasing power (black curves in the right inset). Those curves falling into the linear-response regime correspond to a state where Nb is in the hot-spot-free superconducting phase. At a critical input power ( $-1.5$  dBm in this case),  $|S_{21}(f)|$  makes a sharp transition from one Lorentzian curve onto another with higher insertion loss and lower quality factor  $Q$  as frequency is scanned near resonance (the highest red curve in the right inset).

As the driving frequency is swept from lower values toward the resonance frequency  $f_0$  the circulating rf power builds up in the resonator and the circulating currents  $I_{\text{circ}}$  in the spiral will increase in magnitude, as  $I_{\text{circ}} = I_{\text{max}} / \sqrt{1 + [(f - f_0) / \delta f]^2}$ , where  $I_{\text{max}}$  is the maximum current induced in the resonator and  $\delta f$  is the 3 dB bandwidth. For low/medium rf input powers, no instantaneous power value causes excess dissipation because even the maximum current at resonance is low enough to maintain the linear characteristics. For the high-input power case, the cavity is overdriven in the vicinity of  $f_0$  and thermal effects cause the breakdown of superconductivity. The overstimulation of the resonator launches a new dissipation mechanism and the resonant response moves onto a different curve with lower  $Q$  corresponding to initial stages of a hot spot. With increasing input power this transition occurs at progressively lower frequencies where the dissipative mechanism is activated (the red curves in the right inset of Fig. 1). At still higher powers, a second discrete dissipative mechanism is activated, leading to another set of discontinuous curves (blue curves). At even more elevated powers, one has

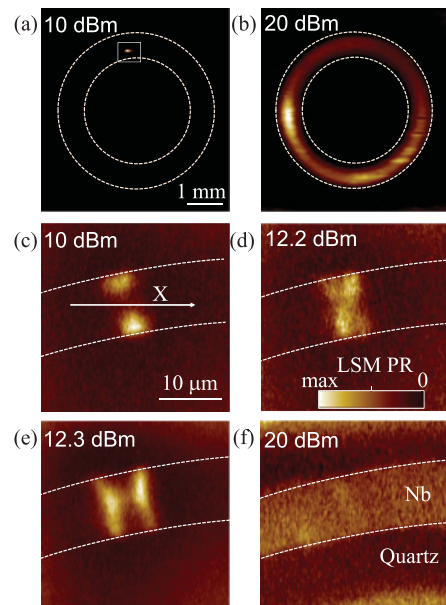


FIG. 2. (Color online) LSM PR images of the nucleation and power evolution of a hot spot. (a) Detection of the first microscopic defect causing the hot-spot formation on the Nb spiral resonator whose details are discussed in Fig. 1 at  $+10$  dBm,  $4.8$  K. (b) The hot spot shown in (a) heats the neighboring area and creates a resistive response coming from the entire spiral at  $+20$  dBm. The length scale bar in (a) is the same for (b). (c)–(f) Zoom in of the  $30 \times 30 \mu\text{m}^2$  area marked in (a) at progressively higher input powers. The color-scale bar in (d) applies to all LSM images shown in Fig. 2, the maximum LSM PR corresponds to  $0.3346$  mV for (a),  $0.1857$  mV for (b),  $3.6 \mu\text{V}$  for (c),  $11.4 \mu\text{V}$  for (d),  $23.2 \mu\text{V}$  for (e), and  $12.4 \mu\text{V}$  for (f). The length scale bar in (c) applies to the images in (c)–(f).

either a large number of discrete dissipation centers activated, or an expansion of the original hot spots, giving rise to a flattening of the transmission spectrum (orange curves), and eventually to a dip or crater (brown curves).

The  $f_0$  of the Nb spiral remains roughly constant with increasing power, unlike what is seen in the temperature-dependence data where the resonance shifts to smaller frequencies.<sup>7</sup> At the extreme case of input power, the dissipation is so large that superconductivity is quenched and the entire resonant feature is wiped away from the spectrum (see the data at  $+27$  dBm). Note that upon reducing the power, the resonant features are re-established.

## B. Laser Scanning Microscopy

To develop a microscopic understanding of the nonlinear resonant features in the transmission data, we examined the spiral resonators with the LSM technique.<sup>17,18</sup> For these measurements, the Nb spiral is stimulated by rf signals well below its transition temperature  $T_c$  and is scanned by a focused laser probe. The spiral resonator is supported on a sapphire disk along with a thermometer and heater located on a metallic platform nearby, inside an optical cryostat. The rf excitation has been done by two rf loops in a manner identical to that described above. A diode laser beam of  $5$  mW power and a wavelength of  $645$  nm illuminates the spiral surface. For the

large-scale LSM images (up to  $10 \times 10 \text{ mm}^2$  scanned area with  $500 \times 500$  points), a collimated laser beam is focused onto a spot with a diameter of  $20 \text{ }\mu\text{m}$  that produces an estimated temperature difference  $\delta T$  of 1.2 K at most relative to an area on the sample unexposed to the laser beam. For more detailed examination of the spiral surface (within a maximum scanned area of  $250 \times 250 \text{ }\mu\text{m}^2$ ), a  $1.6 \text{ }\mu\text{m}$  diameter laser beam is used. The raster scanning of the spot is done with  $0.2 \text{ }\mu\text{m}$  steps, much smaller than the laser spot diameter. However, the amplitude of the laser is modulated at a frequency of 100 kHz, which is significantly smaller than the inverse phonon escape time (on the order of nanoseconds), thus creating a steady-state temperature profile.<sup>18</sup> Moreover, the beam intensity is reduced by neutral-density optical filters down to  $1 \text{ mW/mm}^2$ , so the temperature rise is reduced to  $\delta T = 0.1 \text{ K}$ .

The LSM technique basically images photoresponse ( $PR$ ) that comes from the changes induced by periodically modulated laser power in the transmission characteristics  $\{[\partial|S_{21}(f)|/\partial T]\delta T\}$  of the spiral, synchronously detected with a lock-in amplifier. The response can be decomposed into inductive and resistive components.<sup>17</sup> Since the temperature is well below the  $T_c$  of Nb for the measurements shown here, the modulation of penetration depth  $\lambda$  (thus the inductive response) is small, so the contrast in the LSM images is mainly due to resistive  $PR$ , which is proportional to  $\delta[\int R_s \lambda^2 J_{RF}^2(x, y) dS]$ , where the integral is carried out over the area of laser-induced perturbation. The  $PR$  is therefore a convolution of changes in resistance below  $T_c$ ,  $\delta R_s$ , weighted by the local value of rf current density squared  $J_{RF}^2(x, y)$ .

Large-scale ( $7 \times 7 \text{ mm}^2$ ) low-temperature (4.8 K) LSM images of the Nb spiral presented in Fig. 1 are shown in Figs. 2(a) and 2(b). The excitation of the spiral is done at the fundamental frequency of  $\sim 73.6 \text{ MHz}$ <sup>19</sup> at which the first emerging hot spot is detected at +10 dBm [see Fig. 2(a)]. By the time the power is ramped up to +20 dBm, the entire sample is heated and the resistive response allows imaging of a convolution of  $\delta R_s$ , and the current distribution in the entire spiral [see Fig. 2(b)], showing a single half-wave-length of standing wave current spanning the length of the spiral. Most of the  $PR$  comes from the middle windings, and diminishes near the inner and outer edges.<sup>14</sup> The brighter areas observed on the left side and the lower half of the spiral in Fig. 2(b) are caused by the structural/morphological defects with enhanced  $\delta R_s$ .

The upper half of the spiral is free of physical imperfections (no defect is visible with  $0.5 \text{ }\mu\text{m}$  resolution reflected light microscope), thus we have concentrated on a small portion of that area to examine the appearance of the first microscopic defect. Figures 2(c)–2(f) are a detailed examination of the  $30 \times 30 \text{ }\mu\text{m}^2$  area on the Nb spiral marked in Fig. 2(a) and show the evolution of the first hot spot with increasing rf power at 4.8 K. At +10 dBm, we observe two bright spots in their initial phase at the opposite edges of a single strip in the spiral [see Fig. 2(c)], reminiscent of vortex-antivortex formation at the edges of the current-carrying superconducting stripes.<sup>20</sup> As the power is increased, the isolated spots expand and start to merge together [see Fig. 2(d)] where presumably phase-slip lines (i.e., isothermal nonequilibrium regions)<sup>21</sup> are formed. Further increase in rf power causes a change of the arrangement of these phase-slip lines [see Fig. 2(e)] due to the redistribution of current flow along the strip. This

follows from the nucleation of a normal domain across the strip interfacing with resistive superconducting domains [the bright regions established along the wings of a butterfly pattern seen in Fig. 2(e)].<sup>22</sup> With even further increase in power, the normal domain expands along the strip and finally, the entire strip shows a resistive response in Fig. 2(f).

The formation of a phase-slip line in the dc resistive state is signaled by the emergence of steplike features in the  $I(V)$  characteristics. Although the phase-slip lines discussed in this paper are generated by the combination of rf excitation and laser power, we see a very similar picture. The data presented in Fig. 1 with gradually degrading Q factor with increasing rf input power make a good analogy to the characteristics of the dc bias case with changing differential resistance as the voltage is increased.

We further studied the structure of the hot spot with another sub-100 MHz Nb spiral at 4.5 K. The spiral examined in Fig. 3 has 45 turns,  $D_o = 5 \text{ mm}$  and  $w = s = 10 \text{ }\mu\text{m}$ . Here, the laser beam passes through the hot spot on a  $20 \text{ }\mu\text{m}$  line scan [shown with “X” in Fig. 2(c)]. The frequency is swept through resonance ( $f_0 \sim 88.2 \text{ MHz}$ ) along the X-line cut, to create a position versus frequency image of the hot spot at a fixed rf input power. Figures 3(a)–3(d) show rf power evolution of the hot spot within a  $20 \text{ }\mu\text{m}$  line as frequency is scanned in a narrow band centered on  $f_0$ , while the corresponding  $PR$  profiles at a fixed position  $x_0$  near the center of the hot spot are shown in Figs. 3(e)–3(h) for both directions of frequency sweep. At 0 dBm, in the initial stages of the hot spot, one sees a strong Lorentzian  $PR(f, x_0)$  along with a linear  $|S_{21}(f)|^2$  [Figs. 3(a) and 3(e)], reminiscent of that at low power shown in Fig. 1. In the case of a spatially invariant  $\delta R_s$ , the dissipative term of  $PR(f)$  originates from the temperature dependence of  $J_{RF}^2(f)R_s(T)$ , meaning that it can be described by a simple difference between two  $|S_{21}(f)|$  characteristics  $\delta|S_{21}(f)| \sim J_{RF}^2(f)[R_s(T_2) - R_s(T_1)]$ , where  $T_1$  is the temperature of the nonradiated area while  $T_2$  is the temperature of the area radiated by the laser probe on the Nb film. At low rf powers, the temperature effect of the laser on transmission will be small, hence the  $PR$  traces out a Lorentzian-like curve, as seen in Fig. 3(e).

As the power increases, the hot spot produces a nonlinear behavior of  $\delta R_s(J_{RF}, T)$  causing peculiar jumps in  $|S_{21}(f)|$  whose trend is mimicked by  $PR(f)$ . At +4 dBm [see Figs. 3(b) and 3(f)], in the middle of the hot spot, one first sees a Lorentzian-like response at the onset of the frequency sweep, but then a breakdown occurs at a critical frequency, and a crater develops in the  $PR(f)$  trace similar to that seen in the higher power  $|S_{21}(f)|$  characteristics in Fig. 1. Note that these line cuts are not hysteretic; i.e., forward (blue curves) or backward (red curves) sweep of the frequency reproduce the same characteristics.

An important question arises: why does the LSM  $PR$  drop in the crater? Is it due to a decrease in  $\delta R_s$  or  $J_{RF}$ , or both? In fact, both mechanisms should contribute since LSM  $PR \sim (\partial R_s / \partial T)(\delta T) + (\partial J_{RF} / \partial T)(\delta T)$ . The dissipative component of LSM  $PR$  is proportional to both  $\delta R_s$  and changes in the normal (nonsuperconducting) component of  $J_{RF}$ ,  $\delta J_{RF}$ . The latter is reduced in the resistive state due to the diminished superfluid density. In this case, a craterlike pattern is formed due to time averaging of LSM  $PR$  coming from the localized



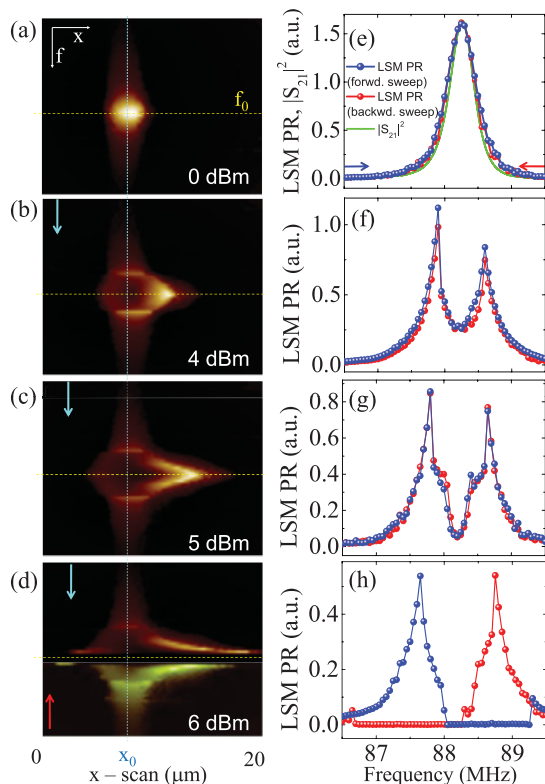


FIG. 3. (Color online) Frequency and space dependence of LSM  $PR$  at (a) 0, (b) +4, (c) +5, and (d) +6 dBm on a Nb spiral with  $D_o = 5$  mm, 45 turns,  $w = s = 10 \mu\text{m}$  over a  $20 \mu\text{m}$   $x$  scan. The frequency is swept between 86.5 and 89.5 MHz through the fundamental resonant frequency  $f_0 = 88.2$  MHz. The images in (a)–(c) show only forward frequency sweeps whose direction is indicated by blue arrows. In (d), different shade color maps point out both direction sweeps, shown with blue (forward) and red (backward) arrows. Corresponding  $PR(f, x)$  profiles at a fixed position,  $x = x_0$  of the laser probe are shown in (e)–(h). The blue dots are the data collected through forward frequency sweep whereas the red dots are those through backward sweep. The  $|S_{21}(f)|^2$  at low power, 0 dBm is shown in (e). The color-scale bar in Fig. 2(d) applies to all LSM images shown in Fig. 3 as well, the maximum LSM  $PR$  corresponds to (forward/backward sweep)–(1.61/1.62) mV for (a), (1.49/1.5) mV for (b), (1.54/1.55) mV for (c), and (1.12/1.4) mV for (d).

regions where resistive superconducting and normal domains are side by side, as in phase-slip processes.<sup>21,23</sup> When a fully normal domain is formed,  $\delta R_s \sim \delta R_N$ , where  $\delta R_N$  is the very small change in normal-state resistance with temperature, thus the LSM  $PR$  drops significantly, as seen in Fig. 3(g).

A higher input power (+5 dBm) LSM imaging of the same  $x$ -line scan is shown in Fig. 3(c). Note that there are now two shoulders and a central valley at resonance in the  $PR$  profile shown in Fig. 3(g). Our interpretation is that there is an rf critical (still partially superconducting) state with significant  $\delta R_s$  for excitation frequencies on the shoulders. In the  $PR$  valley, the material has become normal, mimicking what we have observed in Fig. 2(e).

With more power, at +6 dBm, the central valley at resonance becomes deeper in Fig. 3(d) and at some point the  $PR$  drops to zero in Fig. 3(h). Though away from the resonance, some recovery is observed, heating is so significant

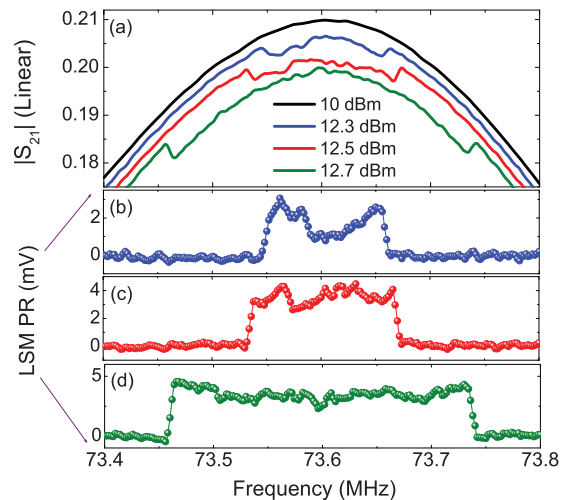


FIG. 4. (Color online) Evolution of global transmission with small increments in rf input power (a), corresponding LSM  $PR$  averaged over  $100 \mu\text{m}$  line scans across the center of a hot spot. Note that  $|S_{21}(f)|$  is measured simultaneously with LSM  $PR(f)$  when frequency is changed very slowly. A nearly exact correlation between  $|S_{21}(f)|$  and  $PR$  data is evident.

that back and forth sweeps of frequency can not reproduce the same  $PR$  profile. Note the hysteresis in in Fig. 3(h); the direction of the frequency scan is indicated by blue and red arrows. The size of the resistive domain expands along the  $x$  axis gradually with increasing power and reaches its maximum at +6 dBm [see Fig. 3(d)].

Line cuts at different  $x$  locations in Fig. 3 show a continuous variation of LSM  $PR(f)$  from Lorentzian-like, to flat-top, to cratered, which follows what is seen in the global  $|S_{21}(f)|$  measurements shown in Fig. 1. These data provide very clear observation of such a unique connection between local temperature change and global rf-power-dependent properties of a superconducting resonant system.

To further support the claims made above, Fig. 4 shows the evolution of global  $|S_{21}(f)|$  characteristics (a) and averaged  $100\text{-}\mu\text{m}$  line scans of LSM  $PR$  [see Figs. 4(b)–4(d)] with increasing power at 4.5 K on a 40-turn,  $D_o = 6$  mm spiral. The linear  $|S_{21}(f)|$  characteristics occurring at +10 dBm [black curve in Fig. 4(a)] turns into nonlinear curves with small increments in rf power. The curves are shifted with equal offset in the vertical direction for clarity. With increasing input power, the LSM  $PR$  appears over progressively broader ranges of frequency around resonance [see Figs. 4(b)–4(d)]. The development of  $PR$  contrast at the hot spot corresponds almost exactly to the deviations of the global  $|S_{21}(f)|$  response from the linear/Lorentzian behavior. This demonstrates quite clearly the direct correspondence between hot-spot formation and nonlinear response in superconducting resonators.

#### IV. CONCLUSIONS

In conclusion, we presented experimental evidence linking the global nonlinear microwave transmission characteristics of a superconducting resonator with hot spots induced inside the material making up the resonator. The hot spots are imaged with LSM technique at various stages of development

and their influence on the global transmission characteristics is documented.

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