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Elastic wave confinement and absorption in a dissipative metamaterial

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In this paper, we report on the theoretical investigation of the elastic wave confinement and absorption in a dissipative metamaterial, which is constituted of two-dimensional phononic crystals with binary composite material defect composed of aluminum discs hemmed around by damped rubber. Based on an efficient finite element method in combination with a super cell technique, the dispersion relations and the power transmission spectra of the proposed dissipative metamaterials have been calculated. Numerical results show that the proposed dissipative metamaterials can yield complete band gap as well as defect states. Elastic waves of the specific frequency of the defect models in the range of gap frequencies have been confined and dissipated simultaneously in the point defect or along the line defects. In contrast to the traditional damper for vibration energy dissipation, the proposed dissipative metamaterials can be equivalent to elastic wave energy attractor and possess significant higher energy dissipation rate for ambient distributed vibration. These elastic wave confinement and dissipation properties of the proposed dissipative metamaterials can potentially be utilized to generate vibration absorbers as well as optimization design of damped structure.

Keywords: Dissipative metamaterial, Elastic wave, Defect state, Finite element method

1 Introduction

Over the last two decades, a great deal of researchers have paid considerable attention to the propagation of acoustic and elastic waves in periodic composite materials, known as phononic crystals (PCs) due to their abundant physics and potential engineering applications[1-4].

Earlier studies on PCs are mainly concentrated on the formation mechanisms and the influence factors [illegible] to the bandgap characteristics of the perfect PCs, increasing attention has been focused on the investigation of the defect state characteristics[9-16]. Sigalas[9] studied the defect states in two-dimensional PCs composed of solid cylinders in air by using the plane wave expansion method. They indicated that, with the introduction of defect structures into perfect PCs, defect states and bands appeared, elastic waves of specific frequencies were localized in the point defects or along the line defects, respectively. Khelif *et al.*[10-12] demonstrated experimentally the guiding and the bending of acoustic waves in highly confined waveguides formed by removing rods from a periodic two-dimensional lattice of steel cylinders immersed in water. Wu *et al.*[13] studied the point defect states of acoustic wave in two-dimensional PCs constituting of water cylinders with circular cross section in mercury host. The effect of the defect geometry on the defect band modes was investigated. Zhang *et al.*[14] [illegible] eated by introducing a bend-shaped linear defect in a two-dimensional acoustic band-gap material consisting of water cylinders in mercury background by using the super cell plane wave method. They found that the waves were localized in the bend-shaped linear defect. Li *et al.*[15] theoretical investigated the acoustic confinement and wave guiding in two-dimensional PCs with material defect states. In contrast to the typical formation pattern of defect states, the proposed material defect states are created by replacing single cylinder in the core center or one row of cylinders in the perfect PC with different material cylinders. Jiang *et al.*[16]

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investigated the characteristics of band gaps and defect states in a PC structure consisting of multiple square stubs deposited on both sides of a thin homogeneous plate.

Obviously, traditional PCs demonstrate unique phononic band gaps and defect state properties, possess low-absorption properties due to neglecting damped dissipation energy, and are widely used in low frequencies for vibration and noise isolation[17-20], and in high frequencies for filters and waveguides[21-24], whereas relatively little attention has been paid to the investigation of the PC vibration absorbers with low-reflection, low-transmission and high-absorption properties. In addition to the band gap properties of PCs without damping, increasing attention has been concentrated on the propagation and absorption properties of acoustic waves in viscoelastic PCs[25-30]. Zhao *et al.*[25,26] considered the absorptive properties of three-dimensional PC composed of steel spheres arranged in viscoelastic rubber. Merheb *et al.*[27] theoretically and experimentally investigated the transmission of acoustic waves through centimeter-scale elastic and viscoelastic two-dimensional silicone rubber/air PC structures by using a finite difference time domain method. They indicated that the difference in transmission between elastic and viscoelastic rubber/ air crystals resulted from attenuation of transmission over a very wide frequency range, leaving only narrow passing bands at very low frequencies. Zhao *et al.*[28] studied the band gap of one dimensional PC with viscoelastic host material by using the plane wave expansion method and the Bloch-Floquet wave theory. Hussein *et al.*[29,30] presented a theory for Bloch wave propagation in damped elastic media and showed that damping qualitatively alters the shape of the dispersion curves.

Significantly, current research is mainly concentrated on the effects of visco elastic damping on the band structures and low-frequency acoustic absorption, the elastic wave confinement and absorption in PCs with damped material defects has been scarcely studied. In this paper, we propose a distinct dissipative metamaterial, which is constituted of two-dimensional PCs with binary composite material defect composed of aluminum discs hemmed around by damped rubber and present the theoretical investigation of elastic wave confinement and absorption in the proposed dissipative metamaterial.

2 Model and Method of Calculation

The considered PC structure is composed of periodic arrangement of lead rods embedded in rubber matrix as shown in Fig. 1(a). In the unit cell, the lattice constant of the PC and the radius of scatters are denoted by a = 10 mm and r = 3.5 mm, respectively. The filling fraction is defined as $F=\pi r^2/a^2$=38.48%. The infinite system of the perfect PC is formed by repeating the unit cell periodically along the x- and y-directions. Figure 1(b) illustrates the 5×5 super cell structure of perfect PC. The dissipative metamaterial structures are formed by replacing single or one row of cylinders in the perfect PC with binary composite material defect composed of aluminum discs hemmed around by damped rubber as shown in Figs 1(c) and (d), respectively. The radiuses of aluminum discs and damped rubber are denoted by R=0.45 mm and r_0=3.5 mm, respectively. The material parameters are chosen as; the density ρ_A=1300 kg/m^3, the Young's modulus E_A=117.5 KPa and the Poisson's ratio υ_A=0.47 for rubber; the density ρ_B=11600 kg/m^3, the Young's modulus E_B=40.8 GPa, and the Poisson's ratio υ_B=0.369 for lead; the density ρ_c=2730 kg/m^3, the Young's modulus E_c=66.9 GPa and the Poisson's ratio υ_c=0.35 for rubber; the density ρ_d=1300 kg/m^3, the Young's modulus E_d=117.5 KPa, the Poisson's ratio υ_d=0.47 and the damping ratio ξ_d=0.1 for damped rubber.

To investigate the elastic wave confinement and absorption in the proposed dissipative metamaterials, the dispersion relations and the transmission spectra are calculated based on an efficient finite element

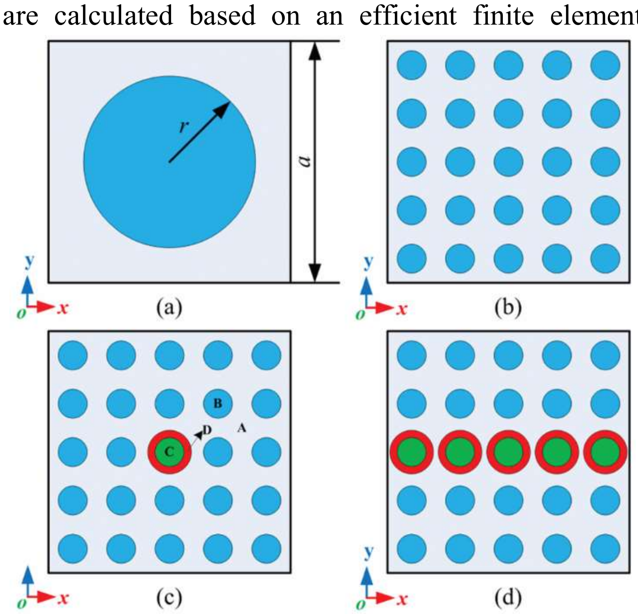

Fig. 1 — Schematics of the proposed dissipative metamaterial (a) schematics of the unit cell model of perfect PC, (b) schematics of the 5×5 supercell structure of perfect PC, (c) schematics of the 5×5 supercell structure with damped composite material point defect and (d) schematics of the 5×5 supercell structure with damped composite material line defect.

method in combination with a supercell technique. The structural mechanics module operating under the two-dimensional stress-strain application mode (smps) in COMSOL Multiphysics[31] is applied for the calculations of damped Eigen frequency of the proposed dissipative metamaterials. The loss factor damping is taken into account in the finite element models. Since the infinite system is periodic along the *x*- and *y*- directions simultaneously, only the super cell needs to be considered. According to Bloch-Floquet theorem, periodic boundary conditions are applied on the interfaces among all adjacent super cell, and the Bloch wave vector *k* is introduced and only needs to have a value along the border line of irreducible brillouin zone, because of the periodicity of the PC in both *x*- and *y*- directions and the symmetry of the super cell. With a given value of Bloch wave vector, a group of Eigen values and Eigen modes can be calculated by solving the Eigen value problem. Letting the value of Bloch wave vector along the boundary of the first brillouin zone and we can obtain the dispersion relations of the perfect PC as well as the proposed dissipative metamaterials.

In addition, the structural mechanics module operating under the two-dimensional stress-strain application mode (smps) in COMSOL multiphysics is applied for the calculations of the transmission spectrum. We consider a finite array structure composed of *N* units in the *x*-direction, while the periodic boundary conditions are still applied in the *y*-direction to represent the infinite units. The acceleration excitation source with single-frequency incidents from the left side of the finite array propagates along the *x*-direction. The transmitted acceleration value is detected and recorded on the right side of the structure. The transmission spectra are defined as the ratio of the transmitted power through the *N* layered finite system to the incident power. By varying the excitation frequency of the incident waves, the transmission spectra can be obtained.

3 Numerical Results and Discussion

To illustrate the elastic wave confinement and energy dissipation in the proposed metamaterials, some numerical calculations are carried out based on the finite element method in combination with a super cell technique. Figure 2(a,b) shows the band structures of the unit cell model and the 5×5 super cell perfect PC model, respectively. It can be found that one complete band gap extended from 434 to 736 Hz in the frequency range from 0 to 900 Hz. The dispersion relations of the perfect phononic crystal structure calculated by the super cell model are in good agreement with that calculated by the unit cell model, which validates the correction and efficiency of the finite element method in combination with a supercell technique. Figure 2(c,d) represents the dispersion relations of the 5×5 supercell model of PC with undamped and damped composite material point defect, respectively. The dispersion relations of the 5×5 super cell model of PC with undamped and damped composite material line defect are illustrated in Fig. 2(e,f), respectively. One can observe that, with the introduction of composite material point defect and line defect into the perfect PC, the location and width of the band gap keep basically unchanged; several defect bands appear in the

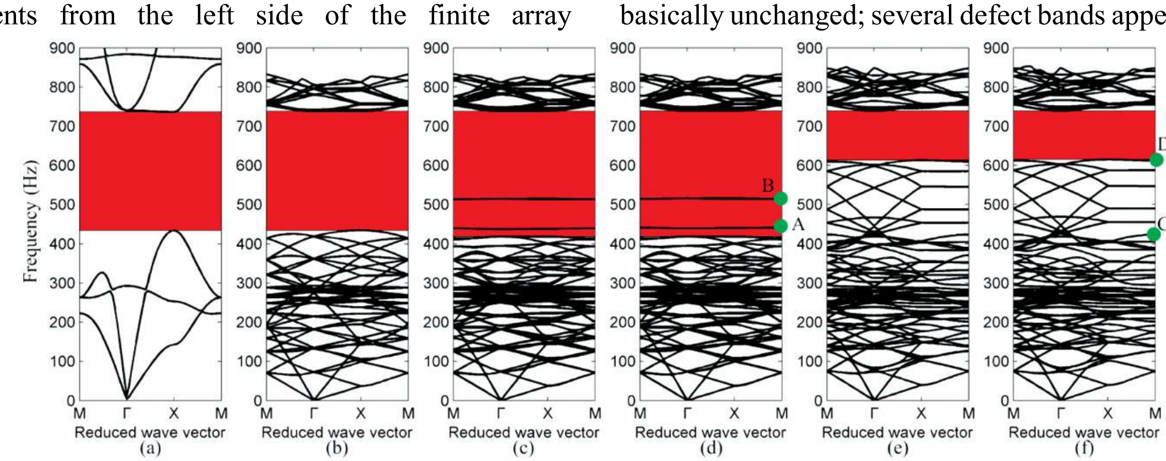

Fig. 2 — The dispersion relations of two-dimensional PC composed of lead cylinders embedded in rubber matrix: (a) the unit cell model of perfect PC with *a*=10 mm and *r*=3.5 mm, (b) the 5×5 supercell model of perfect PC, (c) the 5×5 supercell model of PC with undamped composite material point defect, (d) the 5×5 supercell model of PC with damped composite material point defect, (e) the 5×5 supercell model of PC with undamped composite material line defect and (f) the 5×5 supercell model of PC with damped composite material line defect.

range of gap frequencies, resulting in the occurrence of defect states as well as waveguide. In addition, we can find that the damped rubbers in the proposed metamaterials have little effect on the dispersion relations as the damped rubbers are only a tiny part in the proposed metamaterials.

To further intuitively illustrate the propagation characteristics of elastic wave in the proposed metamaterial structures, the displacement fields of the Eigen modes in the defect states marked in Fig. 2 are carried out. Figure 3(a,b) shows the displacement fields of the Eigen modes of point defect state at the frequency 441 Hz and 513 Hz, respectively. It can be found that, with the introduction of composite material point defect, different defect states occur, the displacement fields are concentrated in the composite material and manifest the local resonance characteristics of the defect cavity respectively corresponding to the defect states, elastic waves of specific frequencies are localized in the point defects. Figure 3(c,d) illustrates the displacement fields of the Eigen modes of line defect state at the frequency 423.5 Hz and 611.8 Hz. One can observe that, with the introduction of line defects, the coupled interaction between the defect bands results in the propagation of elastic waves along the line defect. It can be concluded that, with the introduction of composite material defects into the perfect PCs, elastic waves at the frequencies of defect states in the range of gap frequency can be confined in the point defect or along the line defects.

In order to theoretically investigate the elastic wave confinement and energy dissipation property in the proposed metamaterials, the transmission power spectra for a finite structure are calculated by using the finite element method as shown in Fig. 4. We consider four different finite array structures in the *x*-direction, while the periodic boundary conditions are still applied in the *y*-direction to represent the infinite units. Perfectly matched layers (PMLs) are applied as extended domains at the two ends of the finite structure in the *x*-direction to prevent the reflections. The acceleration excitation source with single-frequency incidents from the left side of the finite array propagates along the *x*-direction. The transmitted acceleration values are detected on the right side of the proposed metamaterials, respectively. On the one hand, one can observe from Fig. 4 that the location and width of the complete band gaps exhibit very good agreement between the dispersion relations and the transmission power spectra. In addition, the location of point defect state and the width of the line waveguide in the

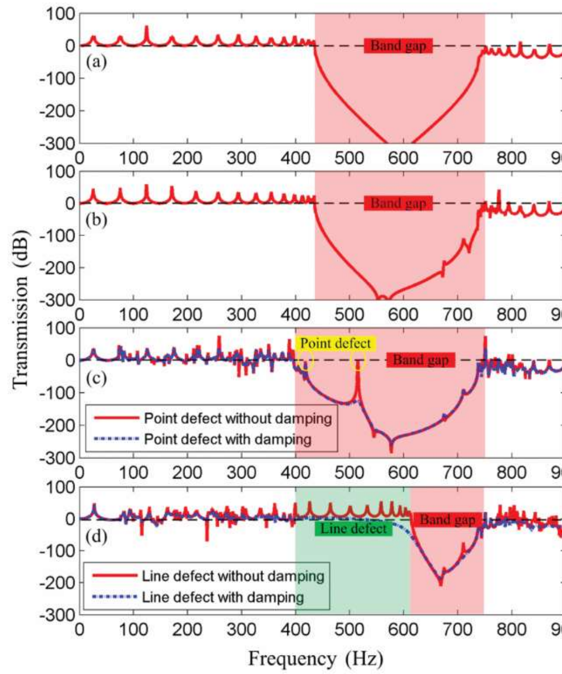

Fig. 4 — The transmission power spectra for waves propagating along the ΓX direction: (a) the 15×1 finite array structures of the perfect PC, (b) the 15×5 finite array structures of the perfect PC, (c) the 15×5 finite array structures of PC with composite material point defect and (d) the 15×5 finite array structures of PC with composite material line defect.

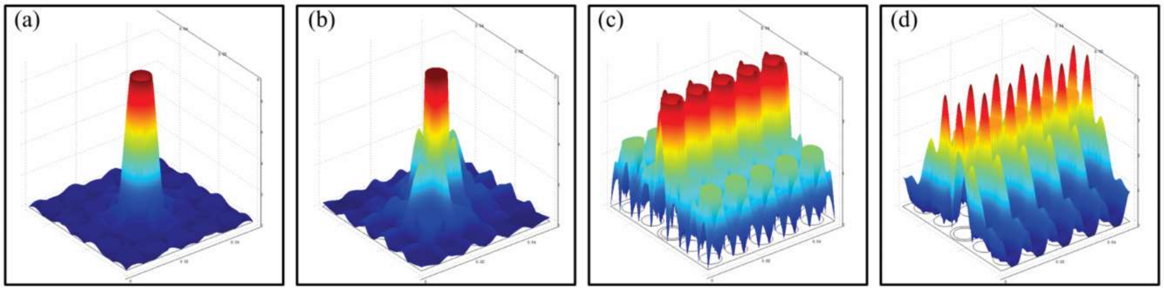

Fig. 3 — Displacement fields of the Eigen modes: (a) the point defect state at the frequency 441 Hz, (b) the point defect state at the frequency 513 Hz, (c) the line defect state at the frequency 423.5 Hz and (d) the line defect state at the frequency 611.8 Hz.

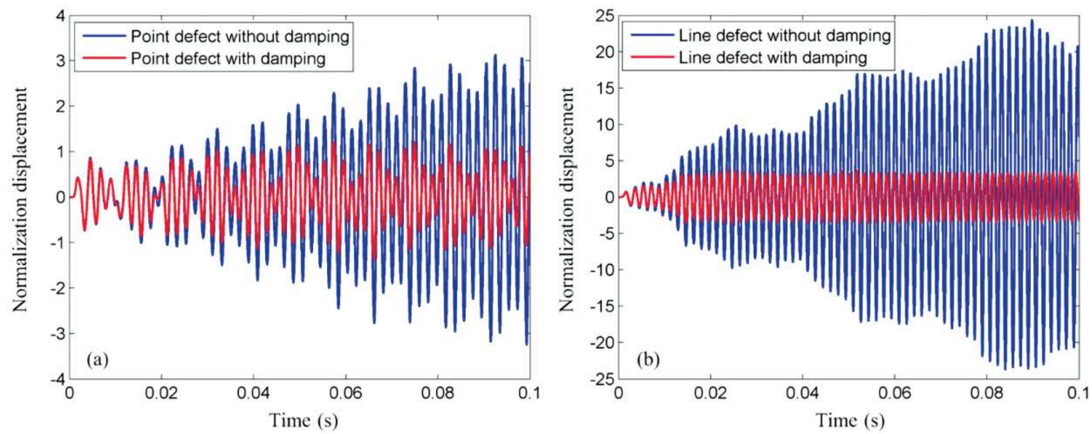

Fig. 5 — The time histories of vibration response: (a) the point defect state at the frequency 513 Hz and (b) the line defect state at the frequency 611.8 Hz.

transmission power spectra are in good agreement with that in the dispersion relations. On the other hand, we can find from Fig. 4(c,d) that, with the introduction of damped composite material defects, the frequency response amplitudes in the defect states reduce remarkably, the vibration energy of elastic waves confined in the defects are dissipated effectively. In contrast to the traditional damper for vibration energy dissipation, the proposed dissipative metamaterials can be equivalent to elastic wave energy attractor and possess significant higher energy dissipation rate for ambient distributed vibration. The proposed dissipative metamaterial structures can potentially be utilized to generate vibration absorbers for narrowband and broadband excitations, respectively.

In order to further intuitively demonstrate the process of elastic wave confinement and energy dissipation in the proposed dissipative metamaterials, the time histories of vibration response at the frequencies of defect states for a finite structure are calculated by using the finite element method. We consider a 5×5 super cell finite array structure in the *x*-direction, while the periodic boundary conditions are still applied in the *y*-direction to represent the infinite units. The displacement excitation source with single-frequency incidents from the left side of the finite array propagate along the *x*-direction. The transmitted displacement values are detected in the middle of the 5×5 super cell for the finite array structures, respectively. Figure 5(a,b) presents the time histories of vibration displacement response at the frequency of point defect state (513 Hz) and line defect state (611.8 Hz). The normalization displacement is defined as the ratio of the transmitted displacement through the finite system to the incident displacement. One can observe that, when the undamped composite material defects are introduced into the perfect PCs, the normalization displacements of the defects increase gradually, the elastic wave vibration energy is not only localized but also enhanced as the elastic waves propagate in the defect PCs. With the introduction of damped composite material defects, the vibration displacements reduce and the vibration energy confined in the defects dissipates significantly.

4 Conclusions

In this paper, we theoretically investigate the elastic wave confinement and energy dissipation in a dissipative metamaterial, which is constituted of two-dimensional phononic crystals with binary composite material defect composed of aluminum discs hemmed around by damped rubber. Based on the dispersion relations, the displacement fields of the Eigen modes as well as time-frequency analysis, we demonstrate that the proposed dissipative metamaterials can yield complete band gap as well as defect states, elastic waves of the specific frequency of the defect models in the range of gap frequencies are confined and dissipated simultaneously in the point defect or along the line defects. In contrast to the traditional dampe for vibration energy dissipation, the proposed dissipative metamaterials can be equivalent to elastic wave energy attractor and possess significant higher energy dissipation rate for ambient distributed vibration. These elastic wave confinement and dissipation properties of the proposed dissipative metamaterials can potentially be utilized to generate vibration absorbers as well as optimization design of damped structure.

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