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## On the Ergodic Capacity of MIMO Triply Selective Rayleigh Fading Channels

Chengshan Xiao, Senior Member, IEEE, and Yahong Rosa Zheng, Senior Member, IEEE

Abstract—The ergodic capacity is investigated for triply selective MIMO Rayleigh fading channels. A mathematical formula is derived for the ergodic capacity in the case when the channel state information is known to the receiver but unknown to the transmitter. A closed-form formula is derived that quantifies the effect of the frequency-selective fading on the ergodic capacity into an intersymbol interference (ISI) degradation factor. Different from the existing conclusion that the frequency-selective fading channel has the same ergodic capacity as the frequency flat fading channel, we show that the discrete-time inter-tap correlated frequency-selective fading channel has smaller ergodic capacity than the frequency flat fading channel. Only in the special case when the fading does not have ISI inter-tap correlations will the ergodic capacity be the same as that of the frequency flat channel. Theoretical derivation and computer simulation demonstrate that the inter-tap correlations can have more significant impact on the ergodic capacity than the spatial correlations.

Index Terms—Ergodic capacity, frequency selective fading, MIMO channel, Rayleigh fading, triply selective fading.

#### I. INTRODUCTION

**M**ULTIPLE Input Multiple Output (MIMO) wireless communication has received significant attention due to its enormous channel capacity potential in rich scattering environment [1],[2]. The ergodic capacity results have been well established for MIMO Rayleigh fading channels which are spatially correlated (including spatially uncorrelated), time quasi-static, and frequency nonselective, see [3]-[14] and the references therein. These capacity results are based on the assumption that the MIMO channels have neither Doppler spread nor delay spread, which is not the case in many moderate and high mobility, and high date rate mobile communication applications.

The capacity studies for MIMO frequency selective Rayleigh fading channels have also received some attention [15]-[18]. Specifically, in [16], it was reported that OFDMbased MIMO frequency selective (delay spread) channels will in general provide advantages over frequency flat fading channels not only in terms of outage capacity but also in terms of ergodic capacity. However, in [18], it was reported that frequency selectivity does not affect the ergodic capacity of

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wide-band MIMO channels, which is agreeable with the SISO ergodic capacity results in [19]. Both [18] and [19] are based on the assumption that the discrete-time sampled channel impulse response has no inter-tap correlation. Recently, it was independently reported by Xiao *et al* [20] and Paulraj *et al* [10] that the sampled fading channel taps are in general inter-tap correlated due to the convolution of the transmit pulse-shaping filer, the air-link physical fading channel, and the receive matched filter.

In this paper, we consider the ergodic capacity of a MIMO system that undergoes inter-tap correlated (including intertap uncorrelated as a special case) frequency selective, timevarying and spatially correlated fading, which is referred to as triply selective fading [20]. Due to the time variation, we assume that the channel state information is unknown to the transmitter but perfectly known to the receiver. Therefore, the equal power allocation scheme is used at the transmitter. New results for the ergodic capacity are derived for MIMO triply selective Rayleigh fading channels. We find that the intertap correlations of frequency selective fading channels can have significant impact on the ergodic capacity. This impact is quantified into an ISI degradation factor in a closed-form formula. In a general frequency selective fading channel, the ergodic capacity is reduced by the ISI degradation factor. In the special case when the ISI has no inter-tap correlations, the ISI degradation factor is one, and the ergodic capacity is the same as that of the frequency flat channel. The theoretical results are verified via extensive simulations using improved Jakes' Rayleigh fading simulator [20], [21].

#### II. CHANNEL MODEL AND PRELIMINARIES

Consider a wideband MIMO wireless channel shown in Fig. 1. Assume that the transmit pulse shaping filter  $p_{\tau}(\tau)$  and the receive matched filter  $p_{R}(\tau)$  are normalized with unit energy. Assume also that each physical fading subchannel  $g_{m,n}(t,\tau)$  is wide-sense stationary uncorrelated scattering (WSSUS) [22] Rayleigh fading with normalized unit average power. When the maximum Doppler is much smaller than the signal bandwidth, the continuous-time MIMO channel depicted in Fig. 1 can be accurately converted to the following discrete-time MIMO fading channel model with proper delay [20],[23]

$$\mathbf{y}(k) = \sum_{l=0}^{L-1} \mathbf{H}(l,k) \cdot \mathbf{x}(k-l) + \mathbf{v}(k), \quad k = 0, 1, \cdots, \infty \quad (1)$$

where the input  $\mathbf{x}(k) = [x_1(k), x_2(k), \cdots, x_N(k)]^t$ , the noise  $\mathbf{v}(k) = [v_1(k), v_2(k), \cdots, v_M(k)]^t$ , and the output  $\mathbf{y}(k) = [y_1(k), y_2(k), \cdots, y_M(k)]^t$ , with the superscript  $(\cdot)^t$  being the

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Fig. 1. The baseband block diagram of a MIMO wireless channel, which consists of the transmit filter  $p_T(\tau)$ , the physical fading impulse response  $g_{m,n}(t,\tau)$ , and the receive filter  $p_R(\tau)$  for the (m,n)th-subchannel. The composite MIMO channel impulse response is  $h_{m,n}(t,\tau) = p_R(\tau) \otimes g_{m,n}(t,\tau) \otimes p_T(\tau)$ . It can be accurately converted to a discrete-time channel model represented by an L-tap FIR  $h_{m,n}(l,k)$ ,  $l = 0, 1, \dots, L-1$ .

transpose; L is the channel length which is depending on the transmit filter, delay spread power profiles and receive filter; and the matrix  $\mathbf{H}(l,k)$  is the  $lT_s$ -delayed channel matrix at time instant k, defined by

$$\mathbf{H}(l,k) = \begin{bmatrix} h_{1,1}(l,k) & h_{1,2}(l,k) & \cdots & h_{1,N}(l,k) \\ h_{2,1}(l,k) & h_{2,2}(l,k) & \cdots & h_{2,N}(l,k) \\ \vdots & \ddots & \ddots & \vdots \\ h_{M,1}(l,k) & h_{M,2}(l,k) & \cdots & h_{M,N}(l,k) \end{bmatrix}$$
(2)

where  $h_{m,n}(l,k)$  is the (m,n)th subchannel's *l*th tap coefficient with time-varying index *k*.

If we adopt the commonly used assumption that the spatial correlation matrices of the transmit and receive antennas are in Kronecker product form [3], and keep in mind that the physical fading subchannel  $g_{m,n}(t,\tau)$  has been assumed as WSSUS Rayleigh fading, then the composite discrete-time fading channel coefficients  $h_{m,n}(l,k)$  are zero-mean complex-valued Gaussian random variables. The correlation function between the channel coefficients  $h_{m,n}(l,k)$  and  $h_{p,q}(l,k)$  is given by [20],[23]

$$\mathcal{E}\left[h_{m,n}(l_1,k_1) \cdot h_{p,q}^*(l_2,k_2)\right] = \Psi_{_{RX}}(m,p) \cdot \Psi_{_{TX}}(n,q)$$
$$\cdot \Psi_{_{ISI}}(l_1,l_2) \cdot \Psi_{_{DPR}}(k_1,k_2) \tag{3}$$

where the superscript \* denotes the conjugate,  $\mathcal{E}[\cdot]$  denotes the expectation. The matrices  $\Psi_{RX}$ ,  $\Psi_{TX}$ ,  $\Psi_{ISI}$  and  $\Psi_{DPR}$  are the receive correlation coefficient matrix, the transmit correlation coefficient matrix, the intersymbol interference (ISI) inter-tap correlation coefficient matrix, and the temporal correlation coefficient matrix, respectively.

We give three specific remarks on the elements of these four matrices. First,  $\Psi_{RX}(m,p)$  is the receive correlation coefficient between receive antennas m and p related to angle spread at the receiver with  $0 \leq |\Psi_{RX}(m,p)| \leq \Psi_{RX}(m,m) = 1$ , and  $\Psi_{TX}(n,q)$  is the transmit correlation coefficient between transmit antennas n and q related to angle spread at the transmitter with  $0 \leq |\Psi_{TX}(n,q)| \leq \Psi_{TX}(n,n) = 1$ . Second, the coefficient  $\Psi_{ISI}(l_1,l_2)$  is related to the channel fading power delay profile, the transmit filter, and the receive filter.

Its calculation is given by (17) of [20]. Even if the physical channel  $g_{m,n}(t,\tau)$  is WSSUS channel which means no interpath correlation, the discrete-time sampled channel  $h_{m,n}(l,k)$  will generally have inter-tap correlations [20], [10] because of the convolution between  $p_T(t)$ ,  $g_{m,n}(t,\tau)$  and  $p_T(t)$ . Our third remark goes to  $\Psi_{DPR}$ . Different fading model will have different  $\Psi_{DPR}$ . For the commonly used Clarke's 2-D isotropic scattering model-based Rayleigh fading,  $\Psi_{DPR}(k_1,k_2) = J_0(2\pi F_d(k_1-k_2)T_s)$ , with  $J_0(\cdot)$  being the zero-order Bessel function of the first kind,  $F_d$  the maximum Doppler frequency, and  $T_s$  the symbol period. The first three matrices satisfy  $tr(\Psi_{RX}) = M$ ,  $tr(\Psi_{TX}) = N$ , and  $tr(\Psi_{ISI}) = 1$  [20] due to normalizations.

This discrete-time MIMO channel model (3) is a generalized model describing triply selective MIMO channels. It contains many existing channel models as special cases. For example, 1), if L = 1 and  $F_d = 0$ , then the channel model becomes the spatially correlated, time quasi-static, and frequency flat model [4]. 2) If L = 1,  $F_d = 0$ ,  $\Psi_{TX} = \mathbf{I}_N$ , and  $\Psi_{RX} = \mathbf{I}_M$ , then the model becomes the spatially uncorrelated, time quasi-static, and frequency flat model [1]. 3) If M = 1 and N = 1, then our model becomes the doubly selective fading model for SISO systems [24]. 4) If L = 1 and  $\Psi_{DPR}$  is an identity matrix, then this model becomes a symbol-wise temporally independent fading model [1].

When the channel has intersymbol interference (frequencyselective), the channel capacity has to be analyzed based on a block of K output symbols  $\{\mathbf{y}(k+1), \mathbf{y}(k+2), \cdots, \mathbf{y}(k+K)\}\$ at the receiver. The MIMO channel with ISI is then represented by

$$\mathbf{Y}_{K} = \mathcal{H} \mathbf{X}_{K+L-1} + \mathbf{V}_{K} \tag{4}$$

where  $\mathbf{Y}_{\kappa} = [\mathbf{y}^t(k+1), \mathbf{y}^t(k+2), \cdots, \mathbf{y}^t(k+K)]^t$ , the input vector  $\mathbf{X}_{K+L-1}$  is circularly symmetric complex Gaussian (with padded zeros to clear out the ISI memory), and the noise vector  $\mathbf{V}_{\kappa}$  is the additive white complex Gaussian random vector whose entries are independent and identically distributed (i.i.d.) and circularly symmetric, and

 $\mathcal{H} = \begin{bmatrix} \mathbf{H}(L - 1, k + 1) & \mathbf{H}(L - 2, k + 1) & \cdots & \mathbf{H}(0, k + 1) & 0 & 0 \\ 0 & \mathbf{H}(L - 1, k + 2) & \ddots & \mathbf{H}(1, k + 2) & \ddots & 0 \\ 0 & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & \mathbf{H}(L - 1, k + K) & \cdots & \mathbf{H}(0, k + K) \end{bmatrix}$ 

When the channel matrix  $\mathcal{H}$  is perfectly known to the receiver but unknown to the transmitter, the equal power allocation scheme is employed. Then the instantaneous mutual information (per input symbol) is defined as

$$\mathcal{I}_{\kappa}(k) = \frac{1}{K + L - 1} \left[ \log_2 \det \left( \mathbf{I}_{\kappa M} + \frac{\gamma}{N} \mathcal{H} \mathcal{H}^{\dagger} \right) \right], \quad \text{b/s/Hz} \quad (5)$$

where  $\gamma = \frac{\mathcal{P}}{\sigma^2}$  is the normalized SNR with  $\sigma^2$  being the receive noise power at each receive antenna and  $\mathcal{P}$  being the average total transmission power over the N antennas per symbol interval, and the superscript  $(\cdot)^{\dagger}$  denotes the conjugate transpose. For a large  $K \gg L$ , the factor 1/(K+L-1) in (5) can be approximated by 1/K. The ergodic capacity  $\mathcal{C}_{_{MIMO}}^{av}$  is given by

$$\mathcal{C}_{MIMO}^{av} = \lim_{K \to \infty} \frac{1}{K} \left\{ \mathcal{E}_{\mathcal{H}} \left[ \log_2 \det \left( \mathbf{I}_{KM} + \frac{\gamma}{N} \mathcal{H} \mathcal{H}^{\dagger} \right) \right] \right\}.$$
(6)

**Remark 1:** Strictly speaking (6) will be only an achievable information rate if the transmitter knows that channel distribution information. However, if the transmitter does not know the channel state or channel distribution information, then (6) is indeed the ergodic capacity. For convenient discussion, we use the latter term in this paper.

**Remark 2:** It is well known that for the special MIMO channel with time quasi-static and frequency flat fading, which corresponds to the block length K = 1 and the number of ISI taps L = 1 in the channel model (4), the channel matrix  $\mathcal{H}$  can be simplified and decomposed [3], [4] directly into  $\mathcal{H} = \mathbf{H}(0, k+1) = \Psi_{RX}^{1/2} \mathbf{H}_W \Psi_{TX}^{1/2}$ , where  $\mathbf{H}_W$  is a random matrix with  $M \times N$  i.i.d. complex Gaussian random variables. Unfortunately, a similar form of decomposition does not exist for the triply selective MIMO fading channel with a general channel matrix  $\mathcal{H}$  ( $K \neq 1$  and  $L \neq 1$ ). Therefore, the Wishart (random) matrix theory [25], [26] can not be directly employed to study the triply selective fading channel capacity (6).

#### **III. NEW RESULTS FOR ERGODIC CAPACITY**

In this section, we first present an explicit formula for the ergodic capacity of a SISO doubly selective (*i.e.*, time-varying and frequency-selective) Rayleigh fading channel. Then we extend the SISO channel capacity result to SIMO, MISO and MIMO doubly selective Rayleigh fading channels. Finally, we present the ergodic capacity results for SIMO, MISO and MIMO *triply selective* Rayleigh fading channels.

#### A. Doubly Selective Rayleigh Fading Channels

**Proposition 1:** For a SISO doubly selective Rayleigh fading channel, whose coefficients are assumed to be known at the receiver but unknown to the transmitter, then the ergodic capacity of this channel is given by

$$\mathcal{C}_{SISO}^{av} = \int_0^\infty \left\{ \frac{1}{2\pi} \int_0^{2\pi} \log_2 \left[ 1 + \gamma \cdot f(\omega) \cdot \lambda \right] d\omega \right\} \cdot e^{-\lambda} \cdot d\lambda \quad (7)$$

where  $f(\omega)$  is given by

$$f(\omega) = 1 + 2\sum_{i=1}^{L-1} a_i \cos(i\omega), \quad a_i = \sum_{l=0}^{L-1-i} \Psi_{ISI}(l, l+i).$$
(8)

#### **Proof:** See Appendix A.

Remark 3: The significance of Proposition 1 is that the effect of the frequency selectivity (or the ISI fading) on the ergodic capacity is quantified into a frequency dependent function  $f(\omega)$  whose mean over the frequency range  $[0, 2\pi)$  is 1. For the special case that the fading channel has no inter-tap correlations,  $\Psi_{ISI}$  is a diagonal matrix, and  $f(\omega) \equiv 1$ , then the double-integral formula (7) becomes a single-integral formula and the frequency-selective Rayleigh fading channel has the same ergodic capacity as the frequency flat Rayleigh fading channel. This special case is in agreement with the existing results [19, p.366] and [18, p.2515]. However, for the general case that the fading channel has inter-tap correlations,  $f(\omega)$ is a frequency-dependent function with  $\frac{1}{2\pi} \int_{0}^{2\pi} f(\omega) d\omega = 1$ . According to Jensen's inequality [27], the ergodic capacity of the inter tar correlations. of the inter-tap-correlated frequency-selective Rayleigh fading channel is smaller than that of the frequency flat Rayleigh fading channel.

**Proposition 2:** The ergodic capacity given by (7) can be accurately approximated by

$$\mathcal{C}_{_{SISO}}^{^{av}} \doteq \int_{0}^{\infty} \log_2 \left( 1 + \gamma \cdot \gamma_{_{ISI}} \cdot \lambda \right) \cdot e^{-\lambda} \cdot d\lambda \tag{9}$$

where  $\gamma_{\rm ISI}$  is the ISI degradation factor due to the channel ISI inter-tap correlations, determined by

$$\gamma_{\rm \scriptscriptstyle ISI} = (2^{C\gamma} - 1)/\gamma \tag{10}$$

with

$$C_{\gamma} = \frac{1}{2\pi} \int_0^{2\pi} \log_2 \left[ 1 + \gamma \cdot f(\omega) \right] d\omega. \tag{11}$$

**Proof:** See Appendix A.

**Remark 4:** It should be pointed out that we employed a great number of examples to do the numerical integrations for (7) and (9), we got exactly the same results, however, due to lack of two-dimensional mean-value theorem of double integrals, we make a conservative statement by using "accurately approximated" in Proposition 2.

**Remark 5:** Proposition 2 simplifies the computation of SISO doubly selective fading channel capacity from a doubleintegral to a single-integral, which is a commonly used expression for fading channel capacity. In the general case where the ISI taps have inter-tap correlations,  $\gamma_{ISI}$  is always smaller than one, *i.e.*,  $\gamma_{ISI} < 1$ , this can be proved from (8)-(10) by utilizing Jensen's inequality. In the special case where the ISI taps have no inter-tap correlations,  $\gamma_{ISI} = 1$ , then our result becomes the same as that of [19]. It is also noted that if the inter-tap correlation increases, then  $\gamma_{ISI}$  usually decreases.

We are now in a position to extend our SISO results to SIMO, MISO and MIMO channels with doubly selective Rayleigh fading.

**Proposition 3:** For doubly selective SIMO and MISO Rayleigh fading channels, if the individual subchannels are

spatially uncorrelated, *i.e.*,  $\Psi_{TX}$  and  $\Psi_{RX}$  are identity matrices, then the ergodic capacities are given by

$$\mathcal{C}_{SIMO}^{av} = \frac{1}{(M-1)!} \int_0^\infty \left\{ \frac{1}{2\pi} \int_0^{2\pi} \log_2 \left[ 1 + \gamma \cdot f(\omega) \cdot \lambda \right] d\omega \right\} \cdot \lambda^{M-1} e^{-\lambda} d\lambda \tag{12}$$

$$\mathcal{C}_{_{MISO}}^{^{av}} = \frac{1}{(N-1)!} \int_{0}^{\infty} \left\{ \frac{1}{2\pi} \int_{0}^{2\pi} \log_2 \left[ 1 + \frac{\gamma}{N} \cdot f(\omega) \cdot \lambda \right] d\omega \right\} \\ \cdot \lambda^{^{N-1}} e^{-\lambda} d\lambda \tag{13}$$

and these capacity results can be accurately approximated by

$$\mathcal{C}_{_{SIMO}}^{^{av}} \doteq \frac{1}{(M-1)!} \int_{0}^{\infty} \log_2\left(1 + \gamma \cdot \gamma_{_{ISI}} \cdot \lambda\right) \lambda^{^{M-1}} e^{-\lambda} d\lambda \quad (14)$$

$$\mathcal{C}_{_{MISO}}^{^{av}} \doteq \frac{1}{(N-1)!} \int_{0}^{\infty} \log_2 \left( 1 + \frac{\gamma}{N} \cdot \gamma_{_{ISI}} \cdot \lambda \right) \lambda^{^{N-1}} e^{-\lambda} d\lambda$$
(15)

where M is the number of receive antennas of the SIMO system, N is the number of transmit antennas of the MISO system, and  $\gamma_{\rm ISI}$  is defined in the same way as that in Proposition 2.

**Proposition 4:** For a doubly selective MIMO Rayleigh fading channel, if the channel is spatially uncorrelated, meaning that  $\Psi_{TX} = \mathbf{I}_N$  and  $\Psi_{RX} = \mathbf{I}_M$ , then the ergodic capacity can be accurately approximated by

$$\mathcal{C}_{_{MIMO}}^{^{av}} \doteq \int_{0}^{\infty} \log_2 \left( 1 + \frac{\gamma}{N} \cdot \gamma_{_{ISI}} \cdot \lambda \right) \\ \cdot \sum_{i=0}^{m-1} \frac{i!}{(i+n-m)!} \left[ L_i^{n-m}(\lambda) \right]^2 \lambda^{n-m} e^{-\lambda} d\lambda \quad (16)$$

where  $m = \min\{M, N\}$ ,  $n = \max\{M, N\}$ ,  $L_i^j(\cdot)$  is the associated Laguerre polynomial [1] of order *i*, and  $\gamma_{\scriptscriptstyle ISI}$  is the ISI degradation factor due to the channel ISI inter-tap correlations.

**Remark 6:** Propositions 3 and 4 show that the capacity results of the SISO doubly selective Rayleigh fading channel can be extended to SIMO, MISO and MIMO doubly selective fading channels. The proofs of these two propositions are similar to those of Propositions 1 and 2. Details are omitted for brevity. It should be noted that the difference between (16) and Telatar's result [1] lies in the ISI degradation factor  $\gamma_{tst}$ .

**Remark 7:** The ergodic capacity formula given by (16) can be approximated with high accuracy when m is large and  $M \ge N$ , as follows

$$\mathcal{C}_{_{MIMO}}^{^{av}} \approx \mathcal{C}_{_{MIMO}}^{^{apprx}} = \frac{N}{2\pi} \int_{a}^{b} \log_{2} \left(1 + \gamma \cdot \gamma_{_{ISI}} \cdot \lambda\right) \\ \cdot \frac{\sqrt{(\lambda - a)(b - \lambda)}}{\lambda} d\lambda \qquad (17)$$

where  $a = (\sqrt{M/N} - 1)^2$  and  $b = (\sqrt{M/N} + 1)^2$ .

This implies that if the number of antennas increases with a fixed ratio  $\frac{N}{M}$ , then the ergodic capacity increases linearly with N (or M).

#### B. Triply Selective Rayleigh Fading Channels

We are now in a position to present the ergodic capacity results for MIMO triply selective Rayleigh fading channels.

**Theorem 1:** For the triply selective fading MIMO channel characterized by equations (1)-(3), the ergodic capacity defined by (6) is equivalent to the following expression

$$\mathcal{C}_{_{MIMO}}^{^{av}} = \frac{1}{2\pi} \int_{0}^{2\pi} \mathcal{E}_{\mathbf{H}_{W}} \left\{ \log_{2} \det \left[ \mathbf{I}_{_{M}} + \frac{\gamma}{N} \cdot f(\omega) \right. \right. \\ \left. \cdot \mathbf{\Psi}_{_{RX}} \mathbf{H}_{_{W}} \mathbf{\Psi}_{_{TX}} \mathbf{H}_{_{W}}^{\dagger} \right] \right\} d\omega, \quad \text{b/s/Hz}$$
(18)

where  $\mathbf{H}_{w}$  is an  $(M \times N)$  matrix whose elements are normalized i.i.d. complex Gaussian random variables, and  $f(\omega)$ is the channel power spectrum function determined solely by  $\Psi_{ISI}$  as follows

$$f(\omega) = 1 + 2\sum_{i=1}^{L-1} a_i \cos(i\omega), \quad a_i = \sum_{l=0}^{L-1-i} \Psi_{ISI}(l, l+i).$$

**Proof:** See Appendix B.

As a straightforward extension of Propositions 2–4, one can obtain the following result.

**Proposition 5:** The triply selective fading MIMO channel ergodic capacity given by (18) can be accurately approximated by

$$\mathcal{C}_{_{MIMO}}^{^{av}} \doteq \mathcal{E}_{\mathbf{H}_{W}} \left\{ \log_{2} \det \left[ \mathbf{I}_{_{M}} + \frac{\gamma}{N} \cdot \gamma_{_{ISI}} \cdot \boldsymbol{\Psi}_{_{RX}} \mathbf{H}_{_{W}} \boldsymbol{\Psi}_{_{TX}} \mathbf{H}_{_{W}}^{\dagger} \right] \right\} (19)$$

where  $\gamma_{\rm ISI}$  is defined by the same way as that of Proposition 2.

**Remark 8:** The advantage of (18) over the capacity definition (6) is that the infinite sized channel matrix  $\mathcal{H}$  in (6) is reduced into finite and small sized (*i.e.*,  $M \times N$ ) random matrix  $\mathbf{H}_{W}$  in (18). Furthermore, the  $L \times L$  ISI inter-tap correlation matrix  $\Psi_{ISI}$  is also converted to a scalar function  $f(\omega)$  under the condition that the multiple subchannels share the same ISI fading characteristics [20]. This condition is met if the base station antenna separations are much smaller than the distance between the base station and the mobile station, which is usually the case in practice. This salient feature of (18) is obtained through the decomposition property (3). It makes the capacity analysis of triply selective fading channels mathematically manageable. Proposition 5 makes one step forward to simplify the computation of the underline ergodic capacity.

**Remark 9:** It is noted that for spatially semicorrelated cases, *i.e.*,  $\Psi_{TX} = \mathbf{I}_N$  or  $\Psi_{RX} = \mathbf{I}_M$ , the capacity formula (19) can be derived to have deterministic expression by utilizing the techniques proposed in [7] and [9] for frequency flat fading channels. For the case that both  $\Psi_{TX}$  and  $\Psi_{RX}$  are non-identity matrices, very tight upper bound and lower bound can be derived for (19) by employing the procedure presented in [14] for frequency flat fading channels, details are omitted for brevity.

**Remark 10:** The ergodic capacity formula given by (19) can be significantly simplified for SIMO and MISO systems as shown below.

**Proposition 6:** For SIMO triply selective Rayleigh fading channels, the individual subchannels are spatially correlated,

*i.e.*,  $\Psi_{\scriptscriptstyle RX}$  is not an identity matrix. Since N = 1 and  $\Psi_{\scriptscriptstyle TX} = 1$ , the ergodic capacity (19) can be simplified to be

$$\mathcal{C}_{_{SIMO}}^{^{av}} \doteq \int_{0}^{\infty} \log_2 \left(1 + \gamma \cdot \gamma_{_{ISI}} \cdot \lambda\right) \cdot p_{_{\lambda}}(\lambda) \cdot d\lambda \quad (20)$$

where

 $p_{\lambda}(\lambda) = \sum_{k=1}^{M} \frac{\beta_{k}}{\sigma_{k}} \exp\left(-\frac{\lambda}{\sigma_{k}}\right)$  $\beta_k = \prod_{i=1, i \neq k}^{M} \frac{\sigma_k}{\sigma_k - \sigma_i} \text{ and } \sigma_i \text{ being the } i\text{th eigenvalue}$ of  $\Psi_{_{RX}}$ .

**Proposition 7:** For MISO triply selective Rayleigh fading channels, the individual subchannels are also spatially correlated, *i.e.*,  $\Psi_{TX}$  is not an identity matrix. Since M = 1 and  $\Psi_{RX} = 1$ , the ergodic capacity (19) can be simplified to be

$$\mathcal{C}_{_{MISO}}^{^{av}} \doteq \int_{0}^{\infty} \log_2 \left( 1 + \frac{\gamma}{N} \cdot \gamma_{_{ISI}} \cdot \lambda \right) \cdot p_{_{\lambda}}(\lambda) \cdot d\lambda$$
(21)

where

$$p_{\lambda}(\lambda) = \sum_{k=1}^{N} \frac{\beta_k}{\sigma_k} \exp\left(-\frac{\lambda}{\sigma_k}\right)$$
 with

with

 $\beta_k = \prod_{i=1, i \neq k}^{N} \frac{\sigma_k}{\sigma_k - \sigma_i} \text{ and } \sigma_i \text{ being the } i\text{th eigenvalue}$ 

The proof of Propositions 6 and 7 are omitted for brevity.

#### **IV. NUMERICAL RESULTS**

To verify the theoretical ergodic capacity results presented in Section III, we have conducted extensive simulations which employs the discrete-time time-varying frequency-selective Rayleigh fading MIMO channel model described in Section II with different channel conditions such as Doppler spread  $F_d$ , channel length L, block length K, and antenna numbers M and N. To keep the paper within the length limit, we only present three representative examples here.

Our first example is on the ergodic capacity of three SISO Rayleigh fading channels, which is presented in Fig. 2. The fading channels are 1) frequency flat Rayleigh fading channel, 2) frequency-selective Rayleigh fading channels with four uncorrelated taps, and 3) frequency-selective Rayleigh fading channels with Hilly Terrain (HT) profile as defined in [28], whose discrete-time delay line taps are inter-tap correlated, and the matrix  $\Psi_{ISI}$  can be calculated by the algorithm in [20]. As a reference, the AWGN channel capacity is also shown in Fig. 2. It is clear that the inter-tap-uncorrelated frequencyselective fading channel has the same ergodic capacity as that of the frequency flat fading channel, and the inter-tap correlated HT frequency-selective fading channel has smaller ergodic capacity than that of the frequency flat fading channel.

As the second example, Fig. 3 depicts the capacity of a  $4 \times 1$ (SIMO) system and an  $1 \times 4$  (MISO) system over four different Rayleigh fading channels: 1) spatially-uncorrelated frequency flat Rayleigh fading channel; 2) spatially-correlated frequency flat Rayleigh fading channel with the spatial correlation matrix being  $\Psi_{_{RX}}(i,j) = 0.9^{|i-j|}$  for SIMO system or  $\Psi_{_{TX}}(i,j) =$  $0.9^{|i-j|}$  for MISO system; 3) spatially-uncorrelated HT fading channel; 4) spatially-correlated HT fading channel with the spatial correlation matrix being  $\Psi_{_{RX}}(i,j) = 0.9^{|i-j|}$ 



Fig. 2. Ergodic capacity of SISO Rayleigh fading channels.

or  $\Psi_{_{TX}}(i,j) = 0.9^{|i-j|}$ . As can be seen from this figure, both spatial correlations and inter-tap correlations reduce the ergodic capacity. Moreover, the inter-tap correlations can have larger impact on the ergodic capacity than the spatial correlations.



Fig. 3. Ergodic capacity of  $4 \times 1$  (SIMO) and  $1 \times 4$  (MISO) Rayleigh fading channels.

As the third example, Fig. 4 plots the ergodic capacity for  $2 \times 2$  and  $4 \times 4$  systems over three fading channel conditions: 1) spatially-uncorrelated frequency flat fading; 2) spatiallyuncorrelated frequency-selective fading with  $\Psi_{ISI}(i,j) =$  $\frac{0.95^{|i-j|}}{4}$ ; 3) triply selective fading with  $\Psi_{TX}(i,j) = 0.7^{|i-j|}$ ,  $\Psi_{_{RX}}(i,j) = 0.7^{|i-j|}$  and  $\Psi_{_{ISI}}(i,j) = \frac{0.95^{|i-j|}}{4}$ . Again, all the simulation results are in excellent agreement with the theoretical results obtained from Theorem 1 and Propositions 4 and 5. It is also indicated that the spatial correlation and the ISI inter-tap correlation reduce the ergodic capacity for equal power allocation at the transmitter.

It is worthwhile to note that in our simulations, we employed the improved Jakes simulator [21] to incorporate



Fig. 4. Ergodic Capacity vs SNR for  $2 \times 2$  and  $4 \times 4$  systems.

different time variations by choosing different mobile speeds such as 3 km/h, 30 km/h and 120 km/h, we obtained the same capacity results.

#### V. CONCLUSION

The ergodic capacity has been investigated for triply selective (spatially-correlated, time-varying and frequency-selective) MIMO Rayleigh fading channels. A closed-form formula has been derived that quantifies the effect of the ISI fading on the ergodic capacity into an ISI degradation factor  $\gamma_{ISI}$ . In the special case when the ISI fading does not have inter-tap correlations,  $\gamma_{ISI} = 1$ , and the ergodic capacity is the same as that of the frequency flat channel. In the more general cases of frequency selective MIMO channels,  $\gamma_{ISI} < 1$ , and the inter-tap correlations of the ISI fading will reduce the ergodic capacity. A set of simplified results has been derived for SIMO and MISO systems. The new formulae have been mathematically proved and experimentally verified via Monte-Carlo simulations.

#### APPENDIX A: PROOF OF PROPOSITIONS 1 AND 2

We prove Proposition 1 by considering the slowly timevarying scenario, in which the channel matrix  $\mathcal{H}$  becomes a Toeplitz matrix as follows

$$\mathcal{H} = \begin{bmatrix} h_{L-1} & h_{L-2} & \cdots & h_0 & 0 & 0 & 0\\ 0 & h_{L-1} & \cdots & \cdots & h_0 & 0 & 0\\ 0 & 0 & \ddots & \cdots & \ddots & 0\\ 0 & 0 & 0 & h_{L-1} & \cdots & \cdots & h_0 \end{bmatrix}. \quad (22)$$

According to the property of finite-order (band-limited) Toeplitz matrix shown in [29], the ergodic capacity defined by (6) can be derived to be

$$\mathcal{C}_{SISO}^{av} = \mathcal{E}\left\{\frac{1}{2\pi} \int_0^{2\pi} \log_2\left[1 + \gamma \cdot |h(\omega, k)|^2\right] d\omega\right\}$$
(23)

where

$$h(\omega, k) = \sum_{l=0}^{L-1} h_l(k) \cdot \exp\left(\sqrt{-1} \cdot l \cdot \omega\right).$$
 (24)

For Rayleigh fading channels, the channel coefficients  $h_l(k)$  are zero mean circularly symmetric complex Gaussian random variables for all l and k. For a given value  $\omega$ , the function  $h(\omega, k)$  is also a zero mean circularly symmetric complex Gaussian random variable. Thus  $|h(\omega, k)|^2$  is exponentially distributed at the given value  $\omega$ . Moreover, the variance of  $h(\omega, k)$  is given by

$$f(\omega) = \mathcal{E}\left[h(\omega, k) \cdot h^{*}(\omega, k)\right]$$
  
=  $\sum_{l_{1}=0}^{L-1} \sum_{l_{2}=0}^{L-1} \mathcal{E}\left[h_{l_{1}}(k) \cdot h_{l_{2}}^{*}(k)\right] \cdot \exp\left[\sqrt{-1} \cdot (l_{1}-l_{2}) \cdot \omega\right]$   
=  $\sum_{l_{1}=0}^{L-1} \sum_{l_{2}=0}^{L-1} \Psi_{ISI}(l_{1}, l_{2}) \cdot \exp\left[\sqrt{-1} \cdot (l_{1}-l_{2}) \cdot \omega\right]$   
=  $\sum_{l=0}^{L-1} \Psi_{ISI}(l, l) + 2 \sum_{k=1}^{L-1} a_{k} \cos(k \cdot \omega)$   
=  $1 + 2 \sum_{k=1}^{L-1} a_{k} \cos(k \cdot \omega)$  (25)

where  $a_k = \sum_{l=0}^{L-1-k} \Psi_{\rm \scriptscriptstyle ISI}(l,l+k)$ . Therefore, the PDF of  $|h(\omega,k)|^2$  is equivalent to the PDF of  $(\lambda \cdot f(\omega))$  with  $\lambda$  being a unit variance exponentially distributed random variable, and the ergodic capacity is given by

$$\mathcal{C}_{SISO}^{av} = \int_0^\infty \left\{ \frac{1}{2\pi} \int_0^{2\pi} \log_2 \left[ 1 + \gamma \cdot f(\omega) \cdot \lambda \right] d\omega \right\} \cdot e^{-\lambda} \cdot d\lambda.$$
 (26)

This completes the proof for Proposition 1.

We are now in a position to prove Proposition 2.

Based on the double integration (26), utilizing the meanvalue theorem of integrals [30], one can obtain

$$\mathcal{C}_{_{SISO}}^{^{av}} = \int_{0}^{\infty} \log_2 \left[ 1 + \gamma \cdot f(\xi) \cdot \lambda \right] \cdot e^{-\lambda} \cdot d\lambda.$$
(27)

where  $\xi \in [0, 2\pi)$ .

It is noted here that the mean-value theorem has been commonly used for single integration. For our double integration case, the parameter  $\xi$  does depend on the distributions of  $\lambda$ and  $\omega$ . However, since  $f(\omega)$  is a periodic function in the range  $[0, 2\pi)$  and  $\lambda$  has unit mean value, the parameter  $\xi$  has a value within  $[0, 2\pi)$ .

To approximate  $f(\xi)$ , we fix  $\lambda$  at its mean value which is 1, then calculate the following integral at a given SNR  $\gamma$  as follows

$$C_{\gamma} = \frac{1}{2\pi} \int_0^{2\pi} \log_2 \left[ 1 + \gamma \cdot f(\omega) \right] d\omega.$$
 (28)

Based on  $C_{\gamma} = \log_2 [1 + \gamma \cdot f(\xi)]$ , we can find  $f(\xi)$  and define it as the ISI degradation factor

$$f(\xi) = \frac{2^{c_{\gamma}} - 1}{\gamma} = \gamma_{\scriptscriptstyle ISI}.$$
(29)

This completes the proof for Proposition 2.

#### APPENDIX B: PROOF OF THEOREM 1

The principle of proving this theorem is similar to that of Proposition 1. Some details are outlined here. Since the time variation does not affect the ergodic capacity for ergodic fading channels, we prove the theorem by considering the slowly time-varying scenario. The channel matrix  $\mathcal{H}$  becomes a block Toeplitz matrix with the form as follows

$$\mathcal{H} = \begin{bmatrix} \mathbf{H}(L-1,k) & \mathbf{H}(L-2,k) & \cdots & \mathbf{H}(0,k) & 0 & 0 & 0 \\ 0 & \mathbf{H}(L-1,k) & \ddots & \mathbf{H}(1,k) & \mathbf{H}(0,k) & \ddots & 0 \\ 0 & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & \mathbf{H}(L-1,k) & \mathbf{H}(L-2,k) & \cdots & \mathbf{H}(0,k) \end{bmatrix}$$

Based on the property of the block Toeplitz matrix [29], we can reduce the large dimension  $(KM \times KM)$  random matrix to a much smaller dimension  $(M \times M)$  one as

$$\mathcal{C}_{MIMO}^{av} = \lim_{K \to \infty} \frac{1}{K} \left\{ \mathcal{E}_{\mathcal{H}} \left[ \log_2 \det \left( \mathbf{I}_{KM} + \frac{\gamma}{N} \mathcal{H} \mathcal{H}^{\dagger} \right) \right] \right\} \\ = \mathcal{E} \left\{ \frac{1}{2\pi} \int_0^{2\pi} \log_2 \det \left[ \mathbf{I}_M + \frac{\gamma}{N} \cdot H(\omega) \cdot H^{\dagger}(\omega) \right] d\omega \right\} (30)$$

where

$$H(\omega) = \sum_{l=0}^{L-1} \mathbf{H}(l,k) \cdot \exp\left(\sqrt{-1} \cdot l \cdot \omega\right).$$
(31)

Let  $H_{mn}(\omega)$  be the (m, n)th element of  $H(\omega)$ , utilizing the de-coupling property (3), we have the cross-correlation between  $H_{mn}(\omega)$  and  $H_{pq}(\omega)$  as follows

$$\mathcal{E}\left[H_{mn}(\omega) \cdot H_{pq}^{*}(\omega)\right] = \mathcal{E}\left[\sum_{l_{1}=0}^{L-1} h_{mn}(l_{1},k) \cdot e^{(\sqrt{-1}l_{1}\omega)} \\ \cdot \sum_{l_{2}=0}^{L-1} h_{pq}^{*}(l_{2},k) \cdot e^{(-\sqrt{-1}l_{2}\omega)}\right]$$
$$= \sum_{l_{1}=0}^{L-1} \sum_{l_{2}=0}^{L-1} \mathcal{E}\left[h_{mn}(l_{1},k) \cdot h_{pq}^{*}(l_{2},k)\right] \cdot e^{[\sqrt{-1}(l_{1}-l_{2})\omega]}$$
$$= \sum_{l_{1}=0}^{L-1} \sum_{l_{2}=0}^{L-1} \Psi_{RX}(m,p) \Psi_{TX}(n,q) \Psi_{ISI}(l_{1},l_{2}) e^{[\sqrt{-1}(l_{1}-l_{2})\omega]}$$
$$= \Psi_{RX}(m,p) \cdot \Psi_{TX}(n,q) \cdot f(\omega)$$
(32)

where  $f(\omega) = 1 + 2\sum_{k=1}^{L-1} a_k \cos\left(k \cdot \omega\right)$  with  $a_k = \sum_{l=0}^{L-1-k} \Psi_{\scriptscriptstyle ISI}(l, l+k).$ 

For MIMO Rayleigh fading channels, all the channel coefficients  $h_{mn}(l,k)$  are zero mean circularly symmetric complex Gaussian random variables. For a given value  $\omega$ , the functions  $H_{ij}(\omega)$  are also zero mean circularly symmetric complex Gaussian random variables for all *i* and *j*. Moreover, based on (32) with some algebraic and statistics manipulations, we can prove that the statistical properties of  $H(\omega)$  are identical to those of the product matrix  $\sqrt{f(\omega)} \cdot \Psi_{RX}^{\frac{1}{2}} \mathbf{H}_{W} \Psi_{TX}^{\frac{h}{2}}$ , where  $\mathbf{H}_{W}$  is an  $M \times N$  matrix with all elements being i.i.d. zero mean circularly symmetric complex Gaussian random variables. Therefore, the expression (30) of the ergodic capacity  $\mathcal{C}_{MIMO}^{av}$ 

becomes

$$\begin{aligned} \mathcal{C}_{_{MIMO}}^{^{av}} &= \frac{1}{2\pi} \int_{0}^{2\pi} \mathcal{E} \left\{ \log_2 \det \left[ \mathbf{I}_{_M} + \frac{\gamma}{N} \cdot \sqrt{f(\omega)} \right. \\ & \left. \cdot \mathbf{\Psi}_{_{RX}}^{\frac{1}{2}} \mathbf{H}_{_W} \mathbf{\Psi}_{_{TX}}^{\frac{h}{2}} \cdot \sqrt{f(\omega)} \cdot \mathbf{\Psi}_{_{TX}}^{\frac{1}{2}} \mathbf{H}_{_W}^{h} \mathbf{\Psi}_{_{RX}}^{\frac{h}{2}} \right] \right\} d\omega \\ &= \frac{1}{2\pi} \int_{0}^{2\pi} \mathcal{E} \left\{ \log_2 \det \left[ \mathbf{I}_{_M} + \frac{\gamma}{N} f(\omega) \mathbf{\Psi}_{_{RX}} \mathbf{H}_{_W} \mathbf{\Psi}_{_{TX}} \mathbf{H}_{_W}^{h} \right] \right\} d\omega. \end{aligned}$$

This completes the proof.

#### References

- I. E. Telatar, "Capacity of multi-antenna Gaussian channels," *Eur. Trans. Telecom.*, vol. 10, pp. 585-595, Nov. 1999. Also in *AT&T Bell Lab. Tech. Memo*, June 1995.
- [2] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Commun.*, vol. 6, pp. 311-335, 1998.
- [3] D. S. Shiu, G. J. Foschini, M. J. Gans, and J. M. Kahn, "Fading correlation and its effect on the capacity of multielement antenna systems," *IEEE Trans. Commun.*, vol. 48, pp. 502-513, Mar. 2000.
- [4] C. N. Chuah, D. N. C. Tse, J. M. Kahn, and R. A. Valenzuela, "Capacity scaling in MIMO wireless systems under correlated fading," *IEEE Trans. Inform. Theory*, vol. 48, pp. 637-650, Mar. 2002.
- [5] M. Kang and M. Alouini, "Largest eigenvalue of complex wishart matrices and performance analysis of MIMO MRC systems," *IEEE J. Select. Areas Commun.*, vol. 21, pp. 418-426, Apr. 2003.
- [6] A. Goldsmith, S. A. Jafar, N. Jindal, and S. Vishwanath, "Capacity limits of MIMO channels," *IEEE J. Select. Areas Commun.*, vol. 21, pp. 684-702, June 2003.
- [7] M. Chiani, M. Z. Win, and A. Zanella, "On the capacity of spatially correlated MIMO Rayleigh-fading channels," *IEEE Trans. Inform. Theory*, vol. 49, pp. 2363-2371, Oct. 2003.
- [8] H. Shin and J. H. Lee, "Capacity of multiple-antenna fading channels: spatial fading correlations, double scattering, and keyhole," *IEEE Trans. Inform. Theory*, vol. 49, pp. 2636-2647, Oct. 2003.
- [9] P. J. Smith, S. Roy, and M. Shafi, "Capacity of MIMO systems with semicorrelated flat fading," *IEEE Trans. Inform. Theory*, vol. 49, pp. 2781-2788, Oct. 2003.
- [10] A. J. Paulraj, R. Nabar, and D. Gore, *Introduction to Space-Time Wireless Communications*. Cambridge University Press, 2003.
- [11] Special issue on MIMO systems and applications, *IEEE J. Select. Areas Commun.*, vol. 21, Apr. and June 2003.
- [12] Special issue on space-time transmission, reception, coding and signal processing, *IEEE Trans. Inform. Theory*, vol. 49, no. 10, Oct. 2003.
- [13] Special issue on MIMO wireless communications, *IEEE Trans. Signal Processing*, vol. 51, no. 11, Nov. 2003.
- [14] Q. T. Zhang, X. W. Cui, and X. M. Li, "Very tight capacity bounds for MIMO-correlated Rayleigh-fading channels," *IEEE Trans. Wireless Commun.*, vol. 4, pp. 681-688, Mar. 2005.
- [15] A. Lozano and C. Papadias, "Layered space-time receivers for frequency-selective wireless channels," *IEEE Trans. Commun.*, vol. 50, pp. 65-73, Jan. 2002.
- [16] H. Bolcskei, D. Gesbert, and A. J. Paulraj, "On the capacity of OFDMbased spatial multiplexing systems," *IEEE Trans. Commun.*, vol. 50, pp. 225-234, Feb. 2002.
- [17] Z. Zhang and T. M. Duman, "Achievable information rates of multiantenna systems over frequency-selective fading channels with constrained inputs," *IEEE Commun. Lett.*, vol. 7, pp. 260-262, June 2003.
- [18] K. Liu, V. Raghavan, and A. M. Sayeed, "Capacity scaling and spectral efficiency in wide-band correlated MIMO channels," *IEEE Trans. Inform. Theory*, vol. 49, pp. 2504-2526, Oct. 2003.
- [19] L. H. Ozarow, S. Shamai, and A. D. Wyner, "Information theoretic considerations for cellular mobile radio," *IEEE Trans. Veh. Technol.*, vol. 43, pp. 359-378, May 1994.
- [20] C. Xiao, J. Wu, S.-Y. Leong, Y. R. Zheng, and K. B. Letaief, "A discretetime model for triply selective MIMO Rayleigh fading channels," *IEEE Trans. Wireless Commun.*, vol. 3, pp. 1678-1688, Sept. 2004.
- [21] C. Xiao, Y. R. Zheng, and N. C. Beaulieu, "Novel sum-of-sinusoids simulation models for Rayleigh and Rician fading channels," *IEEE Trans. Wireless Commun.*, vol. 5, pp. 3667-3679, Dec. 2006.
- [22] P. A. Bello, "Characterization of randomly time-variant linear channels," *IEEE Trans. Commun. Sys.*, vol. 11, pp. 360-393, Dec. 1963.
- [23] C. Sgraja and C. Xiao, "On discrete-time modeling of time-varying WSSUS fading channels," in *Proc. 2006 IEEE ICC'06*, vol. 12, pp. 5486-5490, Istanbul, Turkey, June 2006

- [24] X. Ma and G. B. Giannakis, "Maximum-diversity transmissions over doubly-selective wireless channels," *IEEE Trans. Inform. Theory*, vol. 49, no. 7, pp. 1832-1840, July 2003.
- [25] A. Edelman, "Eigenvalues and condition numbers of random matrices," Ph.D. thesis, M.I.T. Press, Cambridge, MA, USA, May 1989.
- [26] V. L. Girko, Theory of Random Determinants. Kluwer, 1990.
- [27] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. Wiley, 1991.
- [28] ETSI. GSM 05.05, "Radio transmission and reception," ETSI EN 300 910 V8.5.1, Nov. 2000.
- [29] R. M. Gray, "On the asymptotic eigenvalue distribution of Toeplitz matrices," *IEEE Trans. Inform. Theory*, vol.IT-18, pp. 725-730, Nov. 1972.
- [30] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products, 6th Ed.* Edited by A. Jeffrey, Academic Press, 2000.



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