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Maciej Jan Zawodniok
Missouri University of Science and Technology, mjzx9c@mst.edu

Q. Shang

Jagannathan Sarangapani
Missouri University of Science and Technology, sarangap@mst.edu

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Distributed Power Control for Cellular Networks in the Presence of Channel Uncertainties

S. Jagannathan, *Senior Member, IEEE*, Maciej Zawodniok, *Member, IEEE*, and Q. Shang, *Member, IEEE*

***Abstract*— In this paper, a novel distributed power control (DPC) scheme for cellular network in the presence of radio channel uncertainties such as path loss, shadowing, and Rayleigh fading is presented. Since these uncertainties can attenuate the received signal strength and can cause variations in the received Signal-to-Interference ratio (SIR), a new DPC scheme, which can estimate the slowly varying channel uncertainty, is proposed so that a target SIR at the receiver can be maintained. Further, the standard assumption of a constant interference during a link's power update used in other works in the literature is relaxed.**

A CDMA-based cellular network environment has been developed to compare the proposed scheme with earlier approaches. The results show that our DPC scheme can converge faster than others by adapting to the channel variations. In the presence of channel uncertainties, our DPC scheme renders lower outage probability while consuming significantly low power per active mobile user compared with other schemes that are available in the literature.

***Index Terms*— Channel uncertainties, wireless network, distributed power control, signal-to-interference ratio (SIR), outage probability.**

I. Introduction

THE objectives of transmitter power control include minimizing power consumption while increasing the network capacity, and prolonging the battery life of mobile units, by managing mutual interference so that each mobile unit can meet its signal-to-interference ratio (SIR) and other quality of service (QoS) requirements. Early work on power control [1] [2] focused on balancing the signal-to-interference (SIR) ratios on all radio links using centralized power control. Later, distributed SIR–balancing schemes [3][4] were developed to maintain quality of service requirements of each link. Foschini and Miljanic [5] proposed a more general and realistic model in which a positive receiver noise and a user-specific target SIR were taken into account. This distributed algorithm was proven to converge either synchronously [5] or asynchronously [6] to a fixed point of a feasible system. Based on this, Grandhi and Zander suggested a distributed constrained power control (DCPC) algorithm [7], in which a ceiling was imposed on the transmit power of each user. Another distributed algorithm was proposed by Bambos et al. [8], which aimed at protecting the active links from performance degradation when new users try to access the channel. Second-order power control (CSOPC) [9] and state space-based control design (SSCD) [10, 17] and its optimization [17] are respectively proposed. In [10], active link protection and admission control with power control in the presence of path loss is described.

Earlier works in distributed power control (DPC) neglect the changes observed in radio channel. In fact, they all assume that: 1) only path loss component is present, 2) no other uncertainty exists in the channel, and 3) the interference is held constant. Consequently, the slower rate of convergence of these algorithms and the associated power updates are of an issue in a highly dynamic wireless environment. The proposed work overcomes these limitations.

In this paper, we propose a novel DPC scheme for the next generation wireless networks with channel uncertainties. The rate of convergence of the proposed DPC scheme is faster compared with the existing schemes. The proposed algorithm can estimate the variations in the slowly varying channel, and the power update is then selected so that a desired SIR is maintained. The algorithms being highly distributive in nature doesn't require inter-link communication, centralized computation, and reciprocity assumption as required in a centrally controlled wireless environment. In addition, the proposed DPC scheme converges in the presence of channel uncertainties. As the necessity of inter-link communication is eliminated, network capacity increases and easy controlled recovery from error events is possible.

Section II describes wireless channel with uncertainties. In Section III, a suite of available DPC schemes is studied and a novel DPC scheme is introduced along with convergence proofs in Section IV. Section V presents the simulations whereas Section VI carries the conclusions.

II. Radio Channel with Uncertainties

The radio channel places fundamental limitations on wireless communication systems. The path between the transmitter and the receiver can vary from simple line-of-sight to one that is severely obstructed by buildings, mountains, and foliage. Unlike wired channels that are stationary and predictable, radio channels involve many uncertain factors, so they are extremely random and do not offer easy analysis. In wireless networks, channel uncertainties such as path loss, the shadowing, Rayleigh fading can attenuate the power of the signal at the receiver and thus cause variations in the received SIR and therefore degrading the performance of any distributed power control (DPC) scheme. It is important to understand these

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S. Jagannathan and Maciej Zawodniok are with the Department of Electrical and Computer Engineering, University of Missouri-Rolla, Rolla, MO 65409 USA (e-mail: mjzx9c@umr.edu; sarangap@umr.edu).

Miss Q. Shang is currently a student in the Department of Electrical and Computer Engineering, Vanderbilt University, Nashville, Tennessee USA (e-mail: *qingyang*.*shang*@vanderbilt.edu).

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uncertainties before the development of a DPC scheme. We focus our effort on these main channel uncertainties, such as path loss, shadowing and Rayleigh fading. They are discussed next.

A. Path loss

If only path loss is considered, the power attenuation is taken to follow the inverse fourth power law [11]:

$$g_{ij} = \frac{\overline{g}}{d_{ij}^{n}} \tag{1}$$

where $\overline{g}$ is a constant usually equal to 1 and d_{ij} is the distance between the transmitter of the j^{th} link to the receiver of the i^{th} link and n is the path loss exponent. A number of values for n have been proposed for different propagation environments, depending on the characteristics of the communication medium. A value of n=4 is taken in our simulations, which is commonly used to model path loss in an urban environment. Further, without user mobility, g_{ij} is a constant.

B. Shadowing

High building, mountains and other objects block the wireless signals. Blind area is often formed behind a high building or in the middle of two buildings. This is often seen especially in large urban areas. The term $10^{0.1\varsigma}$ is often used to model the attenuation of the shadowing to the received power [12][19], where ς is assumed to be a Gaussian random variable.

C. Rayleigh fading

In mobile radio channels, the Rayleigh distribution is commonly used to describe the statistical time varying nature of the received envelope of a flat fading signal, or the envelope of an individual multipath component. The Rayleigh distribution has a probability density function (pdf) given by [11]

$$p(x) = \begin{cases} \dfrac{x}{\sigma^2}\exp\left(-\dfrac{x^2}{2\sigma^2}\right) & (0 \le x \le \infty) \\ 0 & (x < 0) \end{cases} \tag{2}$$

where x is a random variable, and σ^2 is known as the fading envelope of the Rayleigh distribution.

Since the channel uncertainties can distort the transmitted signals, therefore, the effect of these uncertainties is represented via a channel loss (gain) factor typically multiples the transmitter power. Then, the channel gain or loss, g, can be expressed as [12]

$$g = f(d, n, X, \zeta) = d^{-n} \cdot 10^{0.1\zeta} \cdot X^2 \tag{3}$$

where d^{-n} is the effect of path loss, $10^{0.1\zeta}$ corresponds to the effect of shadowing. For Rayleigh fading, it is typical to model the power attenuation as X^2 , where X is a random variable with Rayleigh distribution. Typically the channel gain, g, is a function of time.

III. Existing Distributed Power Control (DPC) Schemes

"Distributed" implies per individual link. Each receiver of the link measures the interference it is faced and communicates this information to its transmitter. Each link decides autonomously how to adjust its power based on information collected on it exclusively. Therefore, the decision-making is fully distributed at the link level. The overhead due to the feedback control is minimal in DPC compared with its counterpart when centralized operations are used.

The goal of power control is to maintain a required SIR threshold for each network link while the transmitter power is adjusted so that the least possible power is consumed in the presence of channel uncertainties. Suppose there are $N \in Z_+$ links in the network. Let g_{ij} be the power loss (gain) from the transmitter of the j^{th} link to the receiver of the i^{th} link. It involves the free space loss, multi-path fading, shadowing, and other radio wave propagation effects, as well as the spreading/processing gain of CDMA transmissions. The power attenuation is considered to follow the relationship given in equation (3). In the presence of such uncertainties, our objective is to propose a novel DPC and to compare its performance with others.

The channel uncertainties will appear in the power loss (gain) coefficient of all transmitter receiver pairs. Calculation of SIR, $R_i(t)$, at the receiver of i^{th} link at the time instant t [13], is given by

$$R_i(t) = \frac{g_{ii}(t)\,P_i(t)}{I_i(t)} = \frac{g_{ii}(t)\,P_i(t)}{\sum\limits_{j \ne i} g_{ij}(t)\,P_j(t) + \eta_i(t)} \tag{4}$$

where $i, j \in \{1, 2, 3, \ldots, N\}$, $I_i(t)$ is the interference, $P_i(t)$ is the link's transmitter power, $P_j(t)$ are the transmitter powers of all other nodes, and $\eta_i(t) > 0$ is the variance power of the noise at its receiver node. For each link i there is a lower SIR threshold γ_i. Therefore, we require

$$\gamma_i \le R_i(t) \le \gamma_i^* \tag{5}$$

for every $i = 1, 2, 3, \ldots N$. The lower threshold value for all links can be taken equal to γ for convenience, reflecting a certain QoS the link has to maintain in order to operate properly. An upper SIR limit is also set, in order to decrease the interference due to its transmitter power at other receiver nodes. In the literature, several DPC schemes have been proposed. The most recent work include [8], CSOPC [9], SSCD and optimal [10][16-17] and they are discussed next.

A. Bambos power control

When equation (5) is not satisfied (i.e. $R_i(t) < \gamma_i$), each link independently increases its power if its current SIR is below its target γ_i , and decreases it otherwise using the power update [8]

$$P_i(l+1) = \frac{\gamma_i\, P_i(l)}{R_i(l)} \tag{6}$$

where $l = 1, 2, 3, \ldots$ and $t = l \cdot T$ where T is the sampling interval. When $P_i(l+1) > P_{\max}$, the new link is not added. Otherwise when $P_i(l+1) \ < P_{\min}$ (the minimum power needed to form a link), then $P_i(l+1) \ = P_{\min}$. The DPC algorithm updates the transmitter powers in steps (time slots) indexed by $l = 1, 2, 3, \ldots$, and so on. In this work only path loss component is considered.

B. Constrained Second Order power control

In [9], the SIR expressed in equation (4) is defined as a set of linear equations

$$\mathrm{AP} \ = \ \mu \tag{7}$$

where $A = I - H$, $P = (p_i)$, and $H = [h_{ij}]$ is defined as a Q x Q matrix, such that $h_{ij} = \dfrac{\gamma_i g_{ij}}{g_{ii}}$ for $i \neq j$ and $h_{ij} = 0$ for $i = j$. In addition $\mu = (\gamma_i v_i / g_{ii})$ is a vector of length Q. In addition, since the transmitter power is limited by an upper limit, the following condition is set as

$$0 \le P \le \overline{P}, \tag{8}$$

where $\overline{P} = (p_{\max})$ denotes the maximum transmission power level of each mobile. The algorithm assumes that there exists a unique power vector P^* , which would satisfy equation (8). Thus by feasible system, the matrix A is nonsingular and $0 \le P^* = A^{-1}\mu \le \overline{P}$. Iterative methods can be executed with local measurements to find the power vector P^* . Through some manipulations, following second-order iterative scheme is obtained as

$$p_i^{(l+1)} = \min\left\{\overline{p_i}, \max\left\{0,\ \frac{w^l \gamma_i p_i^l}{R_i^l} + (1 -\ w^l)\, p_i^{l-1}\right\}\right\} \tag{9}$$

where

$$w^l \ = \ 1 \ + \ 1/1.5^n \tag{10}$$

The min/max operators in equation (9) guarantee the allowable range of transmitter power.

C. State Space-Based Control Design (SSCD)

In [10,17], using state space approach [14], the SIR, $R_i(l)$, at the $(l+1)^{\text{th}}$ iteration is expressed as

$$R_i(l+1) = R_i(l) + v_i(l), \tag{11}$$

where by definition $R_i(l) = \dfrac{P_i(l)}{I_i(l)}$, and interference $I_i(l) = \left(\sum\limits_{j \neq i}^{n} P_j(l)\dfrac{g_{ij}}{g_{ii}} + \dfrac{\eta_i}{g_{ii}}\right)$, with N is the number of active links. The feedback input, $v_i(l)$, for each link should only depend upon the total interference produced by the other users. To maintain the SIR of each link above a desired target γ_i and to eliminate any steady-state errors, the error in SIR, $x_i(l) = R_i(l) - \gamma_i$, has to be minimal by appropriately selecting a power update for the i^{th} link. The closed-loop system is expressed as

$$x_i(l+1) = x_i(l) + v_i(l) \tag{12}$$

Theorem 1: Given the closed-loop SIR system in (12), and if the feedback for the i^{th} transmitter is chosen as $v_i(l) = -k_1 x_i(l) + \mu_i$ with k_1 and μ_i representing the feedback gain and the protection margin respectively, and the power is updated as $P_i(l+1) = P_i(l) + v_i(l) I_i(l)$, then the closed loop SIR system for each link is stable, active links are protected while some inactive links gain admission to the network.

Proof: See [10]. ■

Remark: The work of [10] extends the idea of [17] in protecting active links while inactive links gain admission. When the protection margin, $\mu_i = 0$, Theorem 1 is similar to the work of [17].

Remark: The transmission power is subject to the constraint $P_{\min} \le P_i \le P_{\max}$ where $P_{\min}$ is the minimum value needed to transmit, $P_{\max}$ is the maximum allowed power and $P_i(l)$ is the transmission power of the mobile i.

D. Optimal control algorithm

The above SSCD DPC scheme is modified to include a performance criteria defined as $\sum\limits_{i=1}^{\infty}\left(x_i^T T_i x + v_i^T Q_i v_i\right)$ so that an optimal DPC scheme can be stated as follows:

Theorem 2 (Optimal Control)[10,17]: Given the hypothesis presented in the previous theorem for DPC, with the feedback selected as $v_i(l) = -k_i x_i(l) + \mu_i$, where the feedback gains are taken as

$$k_i = (S_\infty + T_i)^{-1}\ S_\infty \tag{13}$$

where S_i is the unique positive definite solution of the Algebraic Ricatti Equation (ARE)

$$S_i = \left[S_i - S_i\,(S_i + T_i)^{-1}\,S_i\right] + Q_i \tag{14}$$

Then the resulting time invariant closed loop SIR system described by

$$x_i(l+1) = (I_i - k_i)\,x_i(l) + \mu_i, \tag{15}$$

is bounded. ■

IV. Proposed DPC algorithm

In the previous DPC schemes [1-10][13][15-18], only path loss uncertainty is considered. Moreover, the DPC algorithm with active link protection proposed in [8] appears to be slow in convergence compared to [10] for cellular networks and the outage probability is slightly higher. Nevertheless, in the presence of other channel uncertainties, the performance of these DPC schemes fails to render satisfactory performance as shown in simulations. The work, as presented in this paper, is aimed at demonstrating the performance in the presence of several channel uncertainties.

In the time domain, however, the channel is time-varying when channel uncertainties are considered and therefore $g_{ij}\,(t)$ is not considered a constant. In [15], a new DPC algorithm is presented where $g_{ii}\,(t)$ is treated as a time-varying function due to Raleigh fading by assuming that the interference $I_i\,(t)$ is held constant. Since this is a strong assumption, in this paper, a novel DPC scheme is given where both $g_{ii}\,(t)$ and the interference $I_i(t)$ are slow yet time-varying, and channel uncertainties are considered for all the mobile users. In other words, in all existing works [1-10][13][15-18], both $g_{ij}(t)$ and $I_j(t)$ are considered to be held constant between the updates, whereas in our work, this assumption is relaxed.

Considering SIR from (4) where the power attenuation $g_{ij}(t)$ is taken to follow the time-varying nature of the channel and differentiating (4) to get

$$R_i(t)' = \frac{(g_{ii}(t)P_i(t))' I(t) - (g_{ii}(t)P_i(t))I(t)'}{I_i^2(t)} \quad (16)$$

where $R_i(t)'$ is the derivative of $R_i(t)$ and $I_i(t)'$ is the derivative of $I_i(t)$.

To transform the differential equation into the discrete time domain, $x'(t)$ is expressed using Euler's formula as $\frac{x(l+1)-x(l)}{T}$, where T is the sampling interval. Equation (16) can be expressed in discrete time as

$$\begin{aligned} R_i(l)' &= \frac{(g_{ii}(l)P_i(l))' I(l) - (g_{ii}(l)P_i(l))I(l)'}{I_i^2(l)} \\ &= \frac{1}{I_i^2(l)}\Big[\left(g_{ii}'(l)P_i(l)\right)I(l) + \left(g_{ii}(l)P_i'(l)\right)I_i(l) \\ &\quad - (g_{ii}(l)P_i(l))\Big(\sum_{j\neq i} g_{ij}(l)P_j(l) + \eta_i(t)\Big)'\Big] \end{aligned} \quad (17)$$

In other words,

$$\begin{aligned} \frac{R_i(l+1)-R_i(l)}{T} &= \frac{1}{I_i(l)}\frac{[g_{ii}(l+1)-g_{ii}(l)]}{T}P_i(l) \\ &\quad + \frac{1}{I_i(l)}g_{ii}(l)\frac{[P_i(l+1)-P_i(l)]}{T} \\ &\quad - \frac{g_{ii}(l)P_i(l)}{I_i^2(l)}\sum_{j\neq i}\Big(\frac{g_{ij}(l+1)-g_{ij}(l)}{T}P_j(l) \\ &\quad + \frac{P_j(l+1)-P_j(l)}{T}g_{ij}(l)\Big) \end{aligned} \quad (18)$$

Canceling T on both sides and combining to get

$$\begin{aligned} R_i(l+1) &= \Big[\frac{g_{ii}(l+1)-g_{ii}(l)}{g_{ii}(l)} \\ &\quad - \frac{1}{I_i(l)}\sum_{j\neq i}\Big\{[g_{ij}(l+1)-g_{ij}(l)]P_j(l) \\ &\quad + [P_j(l+1)-P_j(l)]g_{ij}(l)\Big\}\Big]R_i(l) \\ &\quad + g_{ii}(l)\frac{P_i(l+1)}{I_i(l)} \end{aligned} \quad (19)$$

Now, define

$$\begin{aligned} \alpha_i(l) &= \frac{g_{ii}(l+1)-g_{ii}(l)}{g_{ii}(l)} \\ &\quad - \frac{1}{I_i(l)}\sum_{j\neq i}\{[g_{ij}(l+1)-g_{ij}(l)]P_j(l) \\ &\quad + [P_j(l+1)-P_j(l)]g_{ij}(l)\} \\ &= \frac{\Delta g_{ii}(l)}{g_{ii}(l)} - \frac{\sum_{j\neq i}[\Delta g_{ij}(l)P_j(l) + \Delta P_j(l)g_{ij}(l)]}{I_i(l)} \\ &= \frac{\Delta g_{ii}(l)}{g_{ii}(l)} - \frac{1}{I_i(l)}\sum_{j\neq i}[\Delta g_{ij}(l)I_i(l)R_j(l) \\ &\quad + \Delta I_j(l)\ \Delta R_j(l)g_{ij}(l)] \end{aligned} \quad (20)$$

where

$$\beta_i(l) = g_{ii}(l), \quad (21)$$

and

$$v_i(l) = \frac{P_i(l+1)}{I_i(l)} \quad (22)$$

Equation (19) can be expressed as

$$R_i(l+1) = \alpha_i(l)R_i(l) + \beta_i(l)v_i(l) \quad (23)$$

with the inclusion of noise, equation (19) is written as

$$R_i(l+1) = \alpha_i(l)R_i(l) + \beta_i(l)v_i(l) + r_i(l)\omega_i(l) \quad (24)$$

where $\omega(l)$ is the zero mean stationary stochastic channel noise with $r_i(l)$ is its coefficient.

The SIR of each link at time instant l is obtained using (24). Carefully observing (24), it is important to note that the SIR at the time instant $l+1$ is a function of channel variation from time instant l to $l+1$. The channel variation is not known beforehand and this makes the DPC scheme development difficult and challenging. Since α is not known, it has to be estimated for DPC development. Note available DPC schemes [1-10][17] ignore the channel variations and therefore they render unsatisfactory performance.

Now define $y_i(k) = R_i(k)$, then equation (24) can be expressed as

$$y_i(l+1) = \alpha_i(l)y_i(l) + \beta_i(l)v_i(l) + r_i(l)\omega_i(l) \quad (25)$$

The DPC development is given in two scenarios.

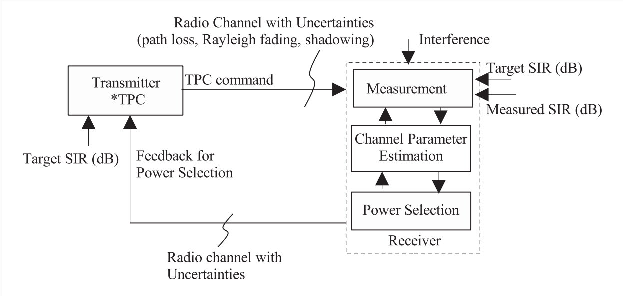
Case 1: α_i , β_i and r_i are considered known. In this scenario, one can select feedback as

$$v_i(l) = \frac{[\gamma - \alpha_i(l)y_i(l) - r_i(l)\omega_i(l) + k_v e_i(l)]}{\beta_i(l)} \quad (26)$$

where the error in SIR is defined as $e_i(l) = R_i(l) - \gamma$. This in turn results in

$$e_i(l+1) = k_v e_i(l) \quad (27)$$

By appropriately selecting k_v via placing the eigen values within a unit circle, it is easy to show that the closed-loop SIR



*TPC Transmission Power Control

Fig. 1. Block diagram representation of distributed power control.

system is asymptotically stable in the mean or asymptotically stable, $\lim_{l\to\infty} E\{e_i(l)\} = 0$. This renders that $y_i(l) \to \gamma$.

Remark: Case 1 inherently assumes that the nodes communicate their transmission powers and interferences to other nodes, which is a strong assumption. This causes significant overhead, which is not preferred in the literature. However, it clearly demonstrates that if the channel is known apriori and properly compensated, then the SIR errors asymptotically converge to zero.

Case 2: α_i , β_i and r_i are unknown. In this scenario, equation (25) can be expressed as

$$\begin{aligned} y_i(l+1) &= \begin{bmatrix}\alpha_i(l) r_i(l)\end{bmatrix} \begin{bmatrix} y_i(l) \\ \omega_i(l) \end{bmatrix} + \beta_i(l) v_i(l) \\ &= \theta_i^T(l)\psi_i(l) + \beta_i(l) v_i(l) \end{aligned} \tag{28}$$

where $\theta_i(l) = \begin{bmatrix}\alpha_i(l) & r_i(l)\end{bmatrix}$ is a vector of unknown parameters, and $\psi_i(l) = \begin{bmatrix} y_i(l) \\ \omega_i(l) \end{bmatrix}$ is the regression vector. Now selecting feedback for DPC as

$$v_i(l) = \beta_i^{-1}(l)\left[-\widehat{\theta}_i(l)\psi_i(l) + \gamma + k_v e_i(l)\right] \tag{29}$$

where $\widehat{\theta}_i(l)$ is the estimate of $\theta_i(l)$, then the SIR error system is expressed as

$$\begin{aligned} e_i(l+1) &= k_v e_i(l) + \theta_i^T(l)\psi_i(l) - \widehat{\theta}_i^T(l)\psi_i(l) \\ &= k_v e_i(l) + \widetilde{\theta}_i^T(l)\psi_i(l) \end{aligned} \tag{30}$$

where $\widetilde{\theta}_i(l) = \theta_i(l) - \widehat{\theta}_i(l)$ is the error in estimation of the channel parameters. From (30), it is clear that the closed-loop SIR error system is driven by channel estimation error. In the presence of errors in estimation, only boundedness of error in SIR can be shown. We can show that the actual SIR approaches the target (with some bounded error) provided the channel uncertainties are properly estimated. Figure 1 illustrates the block diagram representation of the proposed DPC where channel estimation and power selection are part of the receiver. To proceed further, Assumption 1 is required and therefore stated.

Assumption 1: The channel changes slowly compared to the parameters updates.

Theorem 3: Given the DPC above with channel uncertainty, if the feedback from DPC scheme is selected as (29), then the mean channel estimation error along with the mean SIR error converges to zero asymptotically, if the parameter updates are taken as

$$\widehat{\theta}_i(l+1) = \widehat{\theta}_i(l) + \sigma\psi_i(l)e_i^T(l+1) \tag{31}$$

provided

$$\sigma\|\psi_i(l)\|^2 < 1 \tag{32}$$

$$k_{v\max} < \frac{1}{\sqrt{\delta}} \tag{33}$$

where $\delta = \dfrac{1}{1-\sigma\|\psi_i(l)\|^2}$, and σ is the adaptation gain.

Proof: Define the Lyapunov function candidate

$$J_i = e_i^T(l)e_i(l) + \frac{1}{\sigma}\kappa\left[\widetilde{\theta}_i^T(l)\widetilde{\theta}_i(l)\right] \tag{34}$$

whose first difference is

$$\begin{aligned} \Delta J = \Delta J_1 + \Delta J_2 &= e_i^T(l+1)e_i(l+1) - e_i^T(l)e_i(l) \\ &+ \frac{1}{\sigma}\kappa\left[\widetilde{\theta}_i^T(l+1)\widetilde{\theta}_i(l+1) - \widetilde{\theta}_i^T(l)\widetilde{\theta}_i(l)\right] \end{aligned} \tag{35}$$

Consider ΔJ_1 from (35) and substituting (30) to get

$$\begin{aligned} \Delta J_1 &= e_i^T(l+1)e_i(l+1) - e_i^T(l)e_i(l) \\ &= \left(k_v e_i(l) + \widetilde{\theta}_i^T(l)\psi_i(l)\right)^T \left(k_v e_i(l) + \widetilde{\theta}_i^T(l)\psi_i(l)\right) \\ &- e_i^T(l)e_i(l) \end{aligned} \tag{36}$$

Taking the second term of the first difference from (35) and substituting (31) yields

$$\begin{aligned} \Delta J_2 &= \frac{1}{\sigma}\kappa\left[\widetilde{\theta}_i^T(l+1)\widetilde{\theta}_i(l+1) - \widetilde{\theta}_i^T(l)\widetilde{\theta}_i(l)\right] \\ &= -2\left[k_v e_i(l)\right]^T \widetilde{\theta}_i^T(l)\psi_i(l) - 2\left[\widetilde{\theta}_i^T(l)\psi_i(l)\right]^T \\ &\left[\widetilde{\theta}_i^T(l)\psi_i(l)\right] + \sigma\psi_i^T(l)\psi_i(l)\left[k_v e_i(l) + \widetilde{\theta}_i^T(l)\psi_i(l)\right]^T \\ &\left[k_v e_i(l) + \widetilde{\theta}_i^T(l)\psi_i(l)\right] \end{aligned} \tag{37}$$

Combining (36) and (37) to get

$$\begin{aligned} \Delta J &= -e_i^T(l)\left[I - \left(1 + \sigma\psi_i^T(l)\psi_i(l)k_v^T k_v\right)\right]e_i(l) \\ &+ 2\sigma\psi_i^T(l)\psi_i(l)\left[k_v e_i(l)\right]^T\left[\widetilde{\theta}_i^T(l)\psi_i(l)\right] \\ &- \left(1 - \sigma\psi_i^T(l)\psi_i(l)\right)\left[\widetilde{\theta}_i^T(l)\psi_i(l)\right]^T\left[\widetilde{\theta}_i^T(l)\psi_i(l)\right] \\ &\le -\left(1 - \delta k_{v\max}^2\right)\|e_i(l)\|^2 - \left(1 - \sigma\|\psi_i(l)\|^2\right) \\ &\cdot\left\|\widetilde{\theta}_i^T(l)\psi_i(l) - \frac{\sigma\|\psi_i(l)\|^2}{1-\sigma\|\psi_i(l)\|^2}k_v e_i(l)\right\|^2 \end{aligned} \tag{38}$$

where δ is given after (33). Taking now expectations on both sides yields

$$E(\Delta J) \le -E\Bigg(\left(1-\delta k_{v\max}^2\right)\|e_i(l)\|^2 - \left(1-\sigma\|\psi_i(l)\|^2\right)\cdot\left\|\widetilde{\theta}_i^T(l)\psi_i(l) + \frac{\sigma\|\psi_i(l)\|^2}{1-\sigma\|\psi_i(l)\|^2}k_v e_i(l)\right\|^2\Bigg) \tag{39}$$

Since $E(J)>0$ and $E(\Delta J)\le 0$, this shows the stability in the mean via sense of Lyapunov provided the conditions (32) and (33) hold, so $E[e_i(l)]$ and $E\left[\widetilde{\theta}_i(l)\right]$ (and hence $E\left[\widehat{\theta}_i(l)\right]$) are bounded in the mean if $E[e_i(l_0)]$ and $E\left[\widetilde{\theta}_i(l_0)\right]$ are bounded in a mean. Sum both sides of (39) and taking the limit $\lim_{l\to\infty} E(\Delta J)$, the SIR error $E[\|e_i(l)\|]\to 0$. ■

Consider now the closed-loop SIR error system with channel estimation error, $\varepsilon(l)$, as

$$e_i(l+1) = k_v e_i(l) + \widetilde{\theta}_i^T(l)\psi_i(l) + \varepsilon(l) \tag{40}$$

using the proposed DPC.

Theorem 4: Assume the hypothesis as given in Theorem 3, with the channel uncertainty (Path loss, Shadowing and Rayleigh fading) is now estimated by

$$\widehat{\theta}_i(l+1) = \widehat{\theta}_i(l) + \sigma\psi_i(l)e_i^T(l+1) \tag{41}$$

with $\varepsilon(l)$ is the error in estimation which is considered bounded above $\|\varepsilon(l)\|\le\varepsilon_N$, with ε_N a known constant. Then the mean error in SIR and the estimated parameters are bounded provided (32) and (33) hold.

Proof: Define a Lyapunov function candidate as in (34) whose first difference is given by (35). The first term ΔJ_1 and the second term ΔJ_2 can be obtained respectively as

$$\begin{aligned}\Delta J_1 = {} & e_i^T(l)k_v^T k_v e_i(l) + 2[k_v e_i(l)]^T\left[\widetilde{\theta}_i^T(l)\psi_i(l)\right] \\ & + \left[\widetilde{\theta}_i^T(l)\psi_i(l)\right]^T\left[\widetilde{\theta}_i(l)\psi_i(l)\right] + \varepsilon^T(l)\varepsilon(l) \\ & + 2[k_v e_i(l)]^T\varepsilon(l) + 2\varepsilon^T(l)e_i(l) - e_i^T(l)e_i^T(l)\end{aligned} \tag{42}$$

$$\begin{aligned}\Delta J_2 = {} & -2[k_v e_i(l)]^T\left[\widetilde{\theta}_i^T(l)\psi_i(l)\right] \\ & -2\left[\widetilde{\theta}_i^T(l)\psi_i(l)\right]^T\left[\widetilde{\theta}_i^T(l)\psi_i(l)\right] \\ & + \sigma\psi_i^T(l)\psi_i(l)\left[k_v e_i(l) + \widetilde{\theta}_i^T(l)\psi_i(l)\right]^T \\ & \cdot\left[k_v e_i(l) + \widetilde{\theta}_i^T(l)\psi_i(l)\right] \\ & -2\left[1-\sigma\psi_i^T(l)\psi_i(l)\right]e_i^T(l)\varepsilon(l) \\ & + 2\sigma\psi_i^T(l)\psi_i(l)[k_v e_i(l)]^T\varepsilon(l) \\ & + \sigma\psi_i^T(l)\psi_i(l)\varepsilon^T(l)\varepsilon(l)\end{aligned} \tag{43}$$

Using (48) and completing the squares for $\widetilde{\theta}_i^T(l)\psi_i(l)$ yields

$$\begin{aligned}\Delta J \le {} & -\left(1-\delta k_{v\max}^2\right)\Bigg(\|e_i(l)\|^2 - \frac{\delta}{1-\delta k_{v\max}^2}\varepsilon_N^2 \\ & - \frac{2\sigma k_{v\max}\|\psi_i(l)\|^2}{1-\delta k_{v\max}^2}\varepsilon_N\|e_i(l)\|\Bigg) - \left(1-\sigma\|\psi_i(l)\|^2\right) \\ & \cdot\left\|\widetilde{\theta}_i^T(l)\psi_i(l) - \frac{\sigma\|\psi_i(l)\|^2}{1-\sigma\|\psi_i(l)\|^2}\left(k_v e_i(l)+\varepsilon(l)\right)\right\|^2\end{aligned} \tag{44}$$

with δ is given after (33). Taking expectations on both sides to get

$$\begin{aligned}E(\Delta J) \le {} & -E\Bigg(\left(1-\delta k_{v\max}^2\right)\Bigg(\|e_i(l)\|^2 \\ & - \frac{\delta}{1-\delta k_{v\max}^2}\varepsilon_N^2 - \frac{2\sigma k_{v\max}\|\psi_i(l)\|^2}{1-\delta k_{v\max}^2}\varepsilon_N\|e_i(l)\|\Bigg) \\ & + \left(1-\sigma\|\psi_i(l)\|^2\right)\cdot\Bigg\|\widetilde{\theta}_i^T(l)\psi_i(l) \\ & - \frac{\sigma\|\psi_i(l)\|^2}{1-\sigma\|\psi_i(l)\|^2}\left(k_v e_i(l)+\varepsilon(l)\right)\Bigg\|\Bigg)\end{aligned} \tag{45}$$

as long as (32) and (33) hold, and

$$E[\|e_i(l)\|] > \frac{1}{\left(1-\sigma k_{v\max}^2\right)}\varepsilon_N\left(\sigma k_{v\max} + \sqrt{\sigma}\right) \tag{46}$$

This demonstrates that $E(\Delta J)$ is negative outside a compact set U. According to a standard Lyapunov extension, the SIR error $E[e_i(l)]$ is bounded for all $l\ge 0$. It is required to show that $\widehat{\theta}_i(l)$ or equivalently $\widetilde{\theta}_i(l)$ is bounded. The dynamics in error in the parameters are

$$\begin{aligned}\widetilde{\theta}_i(l+1) = {} & \left[I - \sigma\psi_i^T(l)\psi_i(l)\right]\widetilde{\theta}_i(l) \\ & - \sigma\psi_i(l)\left[k_v e_i(l) + \varepsilon(l)\right]^T\end{aligned} \tag{47}$$

where SIR error, $e_i(l)$, is bounded and estimation error, $\varepsilon(l)$, is bounded. Applying the persistency of excitation, one can show that $\widetilde{\theta}_i(l)$ is bounded. ■

Theorem 5: Assume the hypothesis as given in Theorem 4, with the channel uncertainty (Path loss, Shadowing and Rayleigh fading) is now estimated by

$$\begin{aligned}\widehat{\theta}_i(l+1) = {} & \widehat{\theta}_i(l) + \sigma\psi_i(l)e_i^T(l+1) \\ & - \|I - \psi_i^T(l)\psi_i(l)\|\widehat{\theta}_i(l)\end{aligned} \tag{48}$$

with $\varepsilon(l)$ is the error in estimation which is considered bounded above $\|\varepsilon(l)\|\le\varepsilon_N$, with ε_N a known constant. Then the mean error in SIR and the estimated parameters are bounded provided (32) and (33) hold. ■

V. Simulations

In the simulations, all the mobiles in the network have to achieve a desired target SIR value γ_i. The SIR value is a measure of quality of received signal, and can be used to determine the control action that needs to be taken. The SIR, γ_i can be expressed as

$$\gamma_i = \left((E_b/N_0)\,/\,(W/R)\right) \tag{49}$$

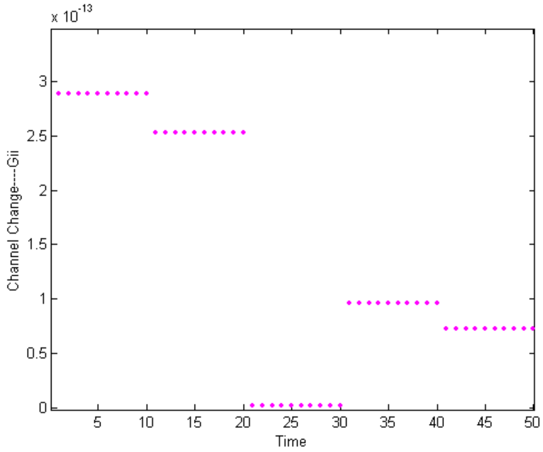


Fig. 2. Channel fluctuations.

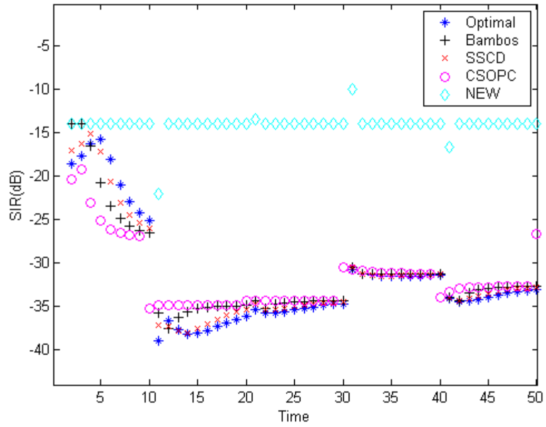


Fig. 3. SIR of a randomly selected user.

where E_b is the energy per bit of the received signal in watts, N_0 is the interference power in watts per Hertz, R is the bit rate in bits per second, and W is the radio channel bandwidth in Hertz.

A. *Performance metrics used in cellular networks*

In this paper metrics chosen to evaluate to performance of the power control scheme are outage probability and the total power consumed by the mobiles. Outage probability is defined as the probability of failing to achieve adequate reception of the signal due to co-channel interference. It is defined as the ratio of the number of disconnected or handed over users to that of the total number of users in the system. The total power consumed by the mobiles will be the sum of all the powers of each mobile user in the network.

B. *Results*

The cellular network is considered to be divided into 7 hexagonal cells covering an area of 10km x 10km (Figure 9). Each cell is serviced via a base station, which is located at the centre of the hexagonal cell. Mobile users in each cell are placed at random by using a Gaussian distribution. It is assumed that the power of for each mobile user is updated asynchronously. Consequently, the powers of all other mobile users do not change when link i^{th} 's power is updated. The receiver noise in the system η_i is taken as 10^{-12} with a standard deviation of 0.1%. The threshold SIR, γ , which each cell tries to achieve is 0.04 (-13.9794dB). The ratio of energy per bit to interference power per hertz E_b/N_0 is 5.12 dB. Bit rate R_b , is chosen at 9600 bits/second. Radio channel bandwidth B_c is considered to be 1.2288 MHz. The maximum power for each mobile $P_{\max}$ is selected as 1mw. Two cases are considered: channel changing sharply at a certain time instant and channel changing smoothly. The system is simulated with different DPC algorithms with 100 users. In the first few simulations, the users in each cell are placed at random and they are stationary. Later in the simulation section mobile users are considered.

1) Stationary Users: Case I: Constant But Abruptly Changing Channel: In this scenario, we select the parameters as $k_v = 0.01$ and $\sigma = 0.01$. Figure 2 shows the change of g_{ii} with time as a result of channel fluctuation, which obeys the Rayleigh fading and shadowing. Though channel changes sharply at every ten time units only, g_{ii} is changed once in every 10 time units and it is held constant otherwise. Figure 3 illustrates the plot of SIR of a randomly selected mobile user. From this figure it is clear that the proposed DPC scheme is the only scheme that maintains the target SIR in the presence of channel variations, while other can't. Moreover, from the SIR convergence, it is important to note that the proposed DPC is able to attain the target SIR for each mobile user in about 10 to 12 time steps due to the selection of $k_v = 0.01$ and $\sigma = 0.01$. These values were selected for all 100 mobile users and the SIR targets were also the same except the channel experienced by each user is different.

Figure 4 depicts the plot of total power consumed by all the mobiles in the network. The result shows that all the schemes consume similar power values (about 90mw) for the entire network, which is less than 1 mw per user. Figure 5 displays the plot of outage probability with time wherein the outage probability using the proposed DPC scheme is significantly less (near zero) when compared to all the other schemes (about 85%), i.e., our approach can accommodate more number of mobile users rendering high channel utilization or capacity in the presence of channel variations. This is mainly due to the presence of an accurate yet faster channel estimation scheme embedded into the DPC which generates suitable power for transmission. By contrast, other schemes do not use any accurate channel estimation scheme and their DPC scheme renders unsatisfactory performance during fading channels. Consequently, the power consumed per active user is less in the proposed scheme compared to other schemes. The individual power consumption plots are omitted due to space considerations. It is important to note that whenever the channel changes abruptly, the outage probability changes significantly further showing that the validity of the theoretical results.

2) Case II: Slowly Varying Channel : In this scenario, $k_v = 0.01$ and $\sigma = 3$. In this case, though the channel

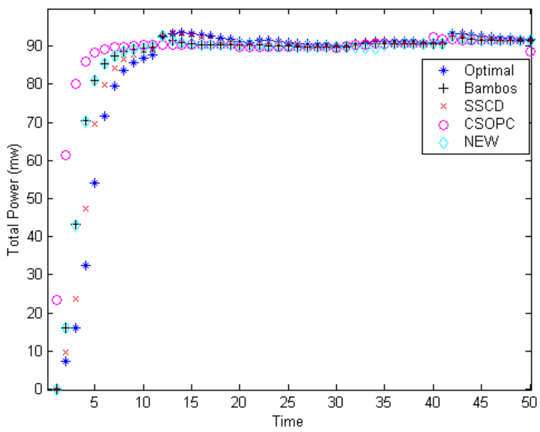


Fig. 4. Total power consumed.

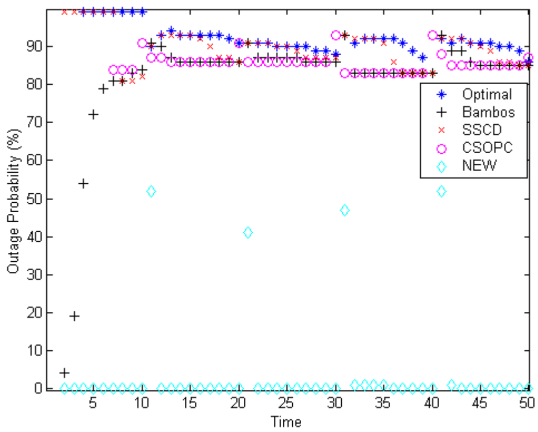


Fig. 5. Outage probability.

changes every 10 time units, it follows the Rayleigh fading and shadowing behavior. The channel variation is smoothed out using a linear function between the changes. In this case also, the proposed DPC scheme renders a low outage probability (about 30% as observed in Fig. 6) while consuming less power per active mobile since the proposed DPC scheme maintains the SIR of each link closer to its target compared to others. Other schemes result in about 85% in outage probability. The low outage probability for the proposed DPC scheme is the result of faster convergence, low SIR error while it consumes satisfactory power per active user. Moreover, it is important to note that the outage probability of the proposed DPC scheme increases linearly with channel state due to the delay incurred in the feedback for the compensation of the channel.

3) Performance Evaluation with Number of Users: When the total number of mobile users in the cellular network varies, we compare how the total power and outage probability varies. In this simulation scenario, the number of mobile users trying to gain admission increases by 25 from the previous value and the corresponding outage probability is calculated. Figures 7 and 8 present the performance of the DPC in terms of total power consumed and outage probability respectively when the channel changes slowly. As expected, the proposed scheme

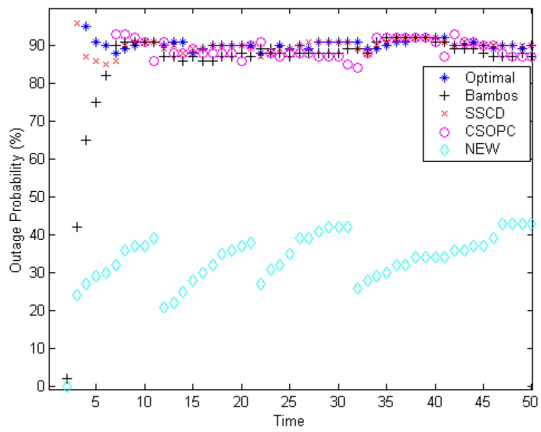


Fig. 6. Outage probability.

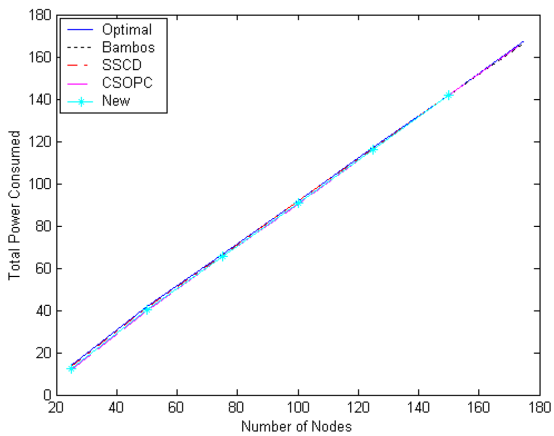


Fig. 7. Total power consumed with varying number of nodes.

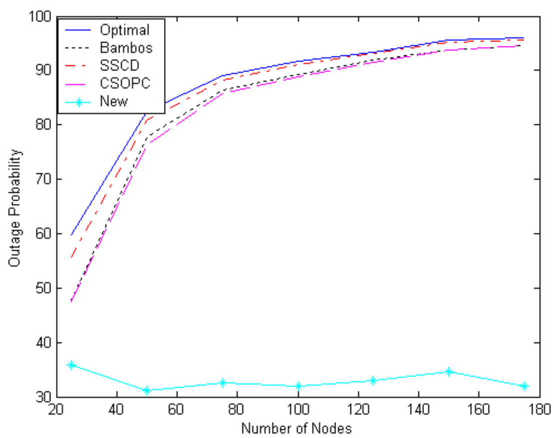


Fig. 8. Outage probability with varying number of nodes.

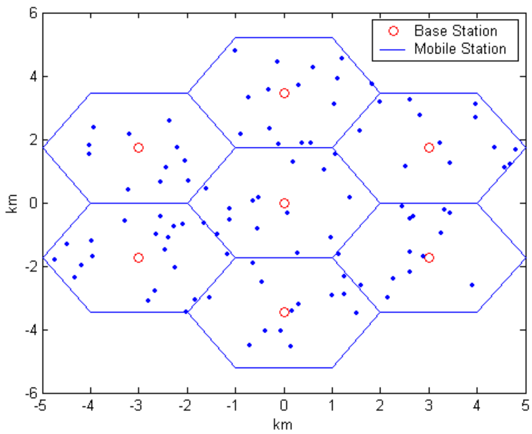


Fig. 9. Initial placement of users in cells.

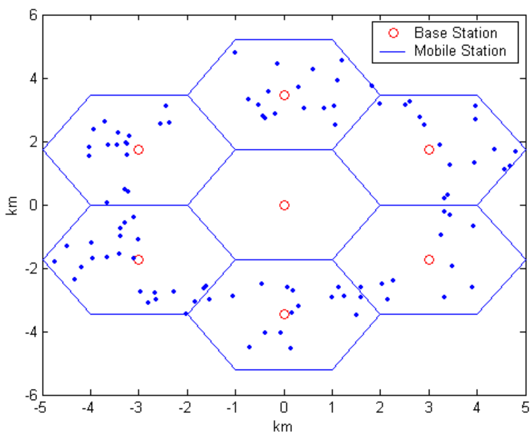


Fig. 10. Final location of users in cells.

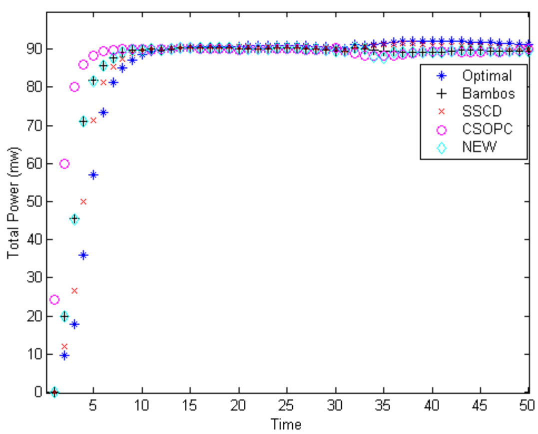


Fig. 11. Total power consumed.

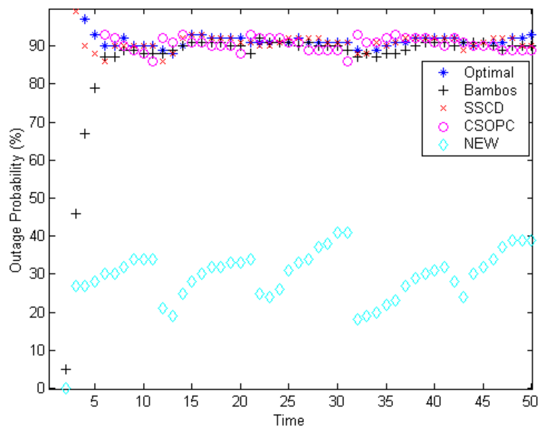


Fig. 12. Outage probability.

renders significantly low outage probability while ensuring low power per active mobile user compared to other schemes.

With more users gaining admission, interference experienced by each user increases besides the channel effects. Consequently, other DPC schemes have difficulty keeping the SIR close to the target due to slower convergence. The links drop out renders a high outage probability. By contrast, the proposed DPC was able to respond to the change in the interference and responds quicker. With more users, the power consumed varies linearly. It is important to note that with the proposed scheme, the users consume similar power values when other schemes are deployed. However, since the outage probability of the proposed is lower than others, the power consumed per user is significantly less with the proposed DPC compared to other schemes.

4) Mobile User Scenario: Since the users are mobile and in the presence of channel uncertainties, Fig. 9 illustrates the initial location of mobile users whereas Fig. 10 displays the final location. Users in the cellular network try to move in any one of the 8 pre-defined directions chosen at random at the beginning of the simulation. A user can move a maximum of 0.01 km per time unit. Since the time unit is small, the 0.01 km is a considerable amount of distance for any mobile user. Figures 11 and 12 depict the total power consumed and the corresponding outage probability. It is important to note that mobility plays a significant role in cellular networks due to hand offs and changes in the channel state. As evident from the result, the proposed DPC renders a lower outage probability (average of about 30%) compared to others (about 90%) while consuming low power per active mobile. Even with mobility, the response of the DPC scheme is around 10 time steps, which is quite satisfactory.

VI. Conclusions

In this paper, a suite of DPC schemes is presented capturing the essential dynamics of power control. It was seen that the proposed DPC scheme allows fully distributed power and has rendered better performance in the presence of radio channel uncertainties. The simulation results show that the proposed scheme converges faster than others, maintains a desired target SIR value for each link and can adapt to the channel variations in the radio channel better. In the presence of channel uncertainties, the proposed scheme can render lower outage, using significantly less transmitter power per active user compared to other DPC schemes. As a result, the proposed DPC scheme offers a superior performance in terms of convergence and it maximizes the network capacity compared to the available ones in the literature. Simulation results justify theoretical conclusion.

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S. Jagannathan was born in Madurai, India and he received the Bachelor's degree in Electrical Engineering from College of Engineering, Guindy at Anna University, Madras India in 1987, the Master of Science Degree in Electrical Engineering from the University of Saskatchewan, Saskatoon, Canada in 1989, and the Ph.D. degree in Electrical Engineering from the University of Texas in 1994. During 1986 to 1987, he was a junior engineer at Engineers India Limited, New Delhi, as a Research Associate and Instructor from 1990 to 1991, at the University of Manitoba, Winnipeg Canada, and worked at Systems and Controls Research Division, in Caterpillar Inc., Peoria as a consultant during 1994 to 1998. During 1998 to 2001 he was at the University of Texas at San Antonio, and since September 2001, he is at the University of Missouri-Rolla where he is currently a professor. He has coauthored more than 150 refereed conference and juried journal articles and several book chapters and one text book entitled "Neural network control of robot manipulators and nonlinear systems", published by Taylor & Francis, London in 1999 two more books in process entitled "Discrete-time neural network control of industrial nonlinear systems" and "Quality of service control in high speed and wireless networks". His research interests include adaptive and neural network control, MEMS and embedded sensor networks, computer/communication/sensor networks, prognostics, and autonomous systems/robotics.

Dr. Jagannathan received several gold medals and scholarships. He was the recipient of Teaching Commendation award in 2005, Presidential Award for Research Excellence at UTSA in 2001, NSF CAREER award in 2000, Faculty Research Award in 2000, Patent Award in 1996, and Sigma Xi "Doctoral Research Award" in 1994. He currently holds 17 patents and several are in process. He has served and currently serving on the program committees of several IEEE conferences. He is currently serving as the Associate Editor for the IEEE Transactions on Control Systems Technology and IEEE Transactions on Neural Networks. He is a member of Tau Beta Pi, Eta Kappa Nu, and Sigma Xi. He is a Senior Member of the IEEE.

Maciej Zawodniok is a PhD candidate in Computer Engineering at University of Missouri-Rolla. Mr. Zawodniok's research focuses on developing adaptive and energy-efficient protocols for wireless networks, network-centric systems and network security. Mr. Zawodniok is also involved in interdisciplinary projects related to application of RFID technology and wireless sensor networks in manufacturing.

Mr. Zawodniok graduated from the Silesian University of Technology with a Master of Science degree in Computer Science in 1999. Next, he worked for over three years for Ericsson AB as a consultant. During that time he delivered courses about cellular telecommunication for Ericsson company and its customers around the world. Then, for over a half year he worked as a Software Engineer in Motorola Polska. There, he was responsible for preparation and implementation of a novel automatic testing of cellular network's equipment. From 2003, Mr. Zawodniok is pursuing PhD at University of Missouri-Rolla. He is a student member of IEEE.