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Marina Koledintseva Missouri University of Science and Technology, marinak@mst.edu

Poorna Chander Ravva

Richard E. DuBroff Missouri University of Science and Technology, red@mst.edu

James L. Drewniak Missouri University of Science and Technology, drewniak@mst.edu

et. al. For a complete list of authors, see https://scholarsmine.mst.edu/ele_comeng_facwork/1017

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Engineering of Composite Media for Shields at Microwave Frequencies

Marina Koledintseva, Poorna Chander Ravva, Richard DuBroff, James Drewniak EMC Laboratory, ECE Department, University of Missouri-Rolla, Rolla, MO, USA e-mail: marinak@umr.edu Konstantin Rozanov Institute for Theoretical and Applied Electromagnetics, Russian Academy of Sciences, Moscow, Russia e-mail: k_rozanov@mail.ru

Bruce Archambeault IBM Co. Research Triangle Park, NC, USA e-mail: barch@us.ibm.com

Abstract— Analytical and numerical modeling of composites with an isotropic dielectric base and multiphase conducting inclusions for the development of wideband microwave shields is considered. The model uses Maxwell Garnett formalism for multiphase mixtures. Such composites are required in many engineering applications, including electromagnetic compatibility.

Keywords- Maxwell Garnett formalism; shielding; conducting inclusions; dielectric base material

I. INTRODUCTION

Engineering of composite materials with the desired frequency characteristics is an important problem for various applications in radio frequency and microwave electronic devices, including design of filters and shields for electromagnetic compatibility purposes. Analytical and numerical modeling of frequency response for a desired composite material prior to manufacturing and testing of multiple materials will save resources and time for the development cycle.

To characterize electromagnetic properties of composite media, it is important to know the electromagnetic parameters of a host (matrix, base) material and inclusions. There are many effective media theories (EMT) allowing homogenization of composite media [1-4]. However, homogenization of a mixture is always an approximation. Macroscopic parameters, such as an effective permittivity, can be used only if the sources and fields are varying slowly, so that quasi-static approximation in the frequency-domain is employed.

Factors affecting frequency characteristics of composites are as follows:

- frequency dependence of constitutive parameters of a host material;
- frequency dependences of constitutive parameters of inclusions (permittivity, permeability, conductivity, etc.);
- shape of inclusions;
- volume fractions of a host material and inclusions (related to size of inclusions and packing density);

- orientation and alignment of electric and/or magnetic dipole moments of inclusions (due to their shape and crystallographic polarizability/magnetization);
- statistical distribution of the parameters of inclusions;
- morphology of the composite (contact between the inclusions, presence of clusters, etc. [5]).

The Maxwell Garnett (MG) model [6] is the simplest and the most widely used for description of composite media at comparatively low concentrations of inclusions. Another important feature of the MG formulation is its linearity with respect to frequency for a multiphase mixture. It allows frequency characteristics of media to be represented in the form of rational-fractional functions convenient for recursive procedures convolution in numerical time-domain electromagnetic codes, such as FDTD [7]. Herein, we apply the extended MG formalism for frequency-dependent dielectric properties of inclusions and host material and electrodynamically anisotropic mixtures, in particular, with a random in-plane distribution of particles in a thin layer.

II. MAXWELL GARNETT FORMALISM FOR MULTIPHASE MIXTURES

Assume that there is a multiphase mixture of metallic or dielectric particles in a homogeneous dielectric base that satisfies the following conditions:

- this mixture is electrodynamically isotropic;
- the mixture is linear, that is, none of its constitutive parameters depends on the intensity of electromagnetic field;
- the mixture is non-parametric, that is, its parameters do not change in time according to some law as a result of some external forces electrical, mechanical, etc.;
- inclusions are at the distances greater than their characteristic size;
- the characteristic size of inclusions is small compared to the wavelength in the effective medium;
- inclusions are arbitrary randomly oriented ellipsoids;
- if there are conducting inclusions, their concentration should be lower than the percolation threshold [8];

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- inclusions are static, that is, they do not move;
- the mixture is chemically stable, that is, inclusions do not dissolve in the host material, and do not interact chemically.

The generalized Maxwell Garnett mixing formula for multiphase mixtures with randomly oriented ellipsoidal inclusions is [9],

$$\varepsilon_{eff} = \varepsilon_b + \frac{\frac{1}{3} \sum_{i=1}^n f_i (\varepsilon_i - \varepsilon_b) \sum_{j=1}^3 \frac{\varepsilon_b}{\varepsilon_b + N_{ij} (\varepsilon_i - \varepsilon_b)}}{1 - \frac{1}{3} \sum_{i=1}^n f_i (\varepsilon_i - \varepsilon_b) \sum_{j=1}^3 \frac{N_{ij}}{\varepsilon_b + N_{ij} (\varepsilon_i - \varepsilon_b)}}$$
(1)

where ε_b is the relative permittivity of a base dielectric; ε_i is the relative permittivity of the *i*-th sort of inclusions; f_i is the volume fraction occupied by the inclusions of the *i*-th sort; N_{ij} are the depolarization factors of the *i*-th sort of inclusions, and the index j = 1,2,3 corresponds to x,y, and zCartesian coordinates.

Formulas for calculating depolarization factors of ellipsoids and the table of depolarization factors of canonical spheroids spheres, disks, and cylinders, can be found in [7]. Equation (1) allows that within the same phase (material of inclusions), particles can have different depolarization factors. However, in reality it is almost impossible to have perfect ellipsoidal or spheroidal particles, so, for any arbitrary shape a reasonable approximation is needed. If the inclusions are thin cylinders, their two depolarization factors are close to 1/2, and the third can be calculated as in [10], $N \approx \left(\frac{d}{l}\right)^2 \ln\left(\frac{l}{d}\right)$, where

l is the length of fibers, and d is their diameter.

Equation (1) admits that the permittivities of the base and inclusions can be complex functions of frequency in the form of

$$\varepsilon_{b}(j\omega) = \varepsilon_{\infty b} + \chi_{b}(j\omega);$$

$$\varepsilon_{i}(j\omega) = \varepsilon_{\infty i} + \chi_{i}(j\omega), \qquad (2)$$

where $\mathcal{E}_{\infty b,i}$ are the high-frequency permittivity values for the base material and for inclusions of the *i*-th type, respectively; and $\chi_{b,i}(j\omega)$ are the corresponding dielectric susceptibility functions. Since the MG formula is linear, the resultant effective permittivity of the mixture can be also represented through effective high-frequency permittivity and susceptibility function,

$$\varepsilon_{\text{eff}}(j\omega) = \varepsilon_{\text{weff}} + \chi_{\text{eff}}(j\omega).$$
⁽³⁾

Thus, the MG formalism can serve as a basis for engineering composite microwave materials. Their frequency responses can be synthesized taking the parameters to describe base and inclusion materials from experimental, manufacturer's, or published reference data. This means that mathematical modeling based on (1)-(3) will provide an efficient tool for the analysis of the frequency behavior of a composite depending on the physical properties and geometry of its constituents. Analytical and numerical modeling of mixtures with the desired frequency characteristics (as well as modeling coatings, filters, or other passive devices using these mixtures) prior to manufacturing and testing of multiple real materials will save resources and time for the development cycle.

As shown in [7], the effective susceptibility function of a mixture, in the simplest case can be approximated by a Debye or Lorentzian dependence; the arbitrary complex-shaped frequency dependences can then be approximated by a series of Debye-like terms (with the first-order poles),

$$\chi_{eff}(j\omega) = \sum_{k=1}^{N} \frac{A_k}{1+j\omega\tau_k}.$$
(4)

If inclusions are conducting (metallic) particles, their frequency characteristic in terms of relative permittivity is

$$\varepsilon_{i}(j\omega) = \varepsilon' - j\varepsilon'' = \varepsilon' - j\frac{\sigma}{\omega\varepsilon_{0}}$$
(5)

As mentioned above, the Maxwell Garnett mixing rule is applicable when the concentration of the conducting particles in the mixture is below the percolation threshold, $p \cong 4.5/a << 1$, where *a* is an aspect (axis) ratio for the inclusions in the form of highly prolate spheroids [8]. Otherwise, the different approximations from the general effective medium theories should be used, for example, McLachlan [11] or Ghosh-Fuchs approximations [12].

The base material might be quite transparent over the frequency range where high shielding effectiveness is desired. However, if there are conducting inclusions, the shielding effectiveness will be provided by absorption of electromagnetic energy due to conductivity loss and to the dimensional resonance in the particles. Presence of conductive particles will also increase reflection from composite layer. If there are magnetic inclusions, they might absorb electromagnetic energy due to the phenomenon of natural ferromagnetic resonance, like in hexagonal ferrite powders [13].

As an example, the real and imaginary parts of the relative effective permittivity of a mixture of carbon fibers and PMMA base are shown in Figures 1 and 2. PMMA is assumed to be a Debye dielectric [9] with the parameters $\varepsilon_S = 3.7$, $\varepsilon_{\infty} = 2.2$, and $\tau = 18 \cdot 10^{-11}$ s. For carbon inclusions, the relative

(**a**)

permeability is $\mu_r = 1$, and the d.c. conductivity is $\sigma = 6.78 \cdot 10^5$ S/m. Since $\varepsilon' \ll \sigma / (\alpha \varepsilon_0)$, the real part in (5) can be neglected. The volume fraction is chosen as $f_i = 0.025\%$. The length of fibers is l = 1 mm, while the diameter is about $d = 0.25 \,\mu$ m (aspect ratio a = l/d = 4000). This concentration is still below the percolation threshold [8].

It should be mentioned that with the frequency increase, due to skin effect not all the bulk of an inclusion affects the effective permittivity, but only a thin layer on its surface. However, for carbon inclusions at the frequencies of consideration (below 50 GHz) the skin effect can be neglected.



Figure 1. Real part of relative permittivity of carbon-fiber and PMMA mixture



Figure 2. Imaginary part of relative permittivity of carbon-fiber and PMMA mixture.

III. REFLECTION AND TRANSMISSION COEFFIENTS FOR A LAYER OF AN ISOTROPIC COMPOSITE MATERIAL

A. Normal incidence of a plane wave upon a composite isotropic layer

For a plane electromagnetic wave *normally incident* on the air-composite layer-air structure shown in Fig. 3, the reflection and transmission coefficients, as well as shielding effectiveness are calculated using a transmission-line approach.

Taking into account reflections between the boundaries, the overall reflection and transmission coefficient values can be calculated as

$$R = \frac{R_1 + R_2 e^{-2\gamma_m l}}{1 + R_1 R_2 e^{-2\gamma_m l}},$$
(6)

and

$$T = \frac{T_1 T_2 e^{-\gamma_m l}}{1 + R_1 R_2 e^{-2\gamma_m l}},$$
(7)

where γ_m is the complex propagation constant in the layer of thickness l, and the reflection and transmission coefficients at the boundaries z=0 and z=l are calculated in an ordinary way through the characteristic impedances of the corresponding media [14],

$$R_1 = \frac{Z_m - Z_0}{Z_m + Z_0}; \ R_2 = \frac{Z_0 - Z_m}{Z_0 + Z_m} \tag{8}$$

and

$$T_1 = \frac{2Z_m}{Z_m + Z_0}; \quad T_2 = \frac{2Z_0}{Z_0 + Z_m}$$
(9)



Figure 3. Air-composite layer-air geometry for calculating reflection and transmission coefficients.

In these formulas, index "1" corresponds to the first subscripts in the corresponding impedances, and "2" to the second ones.

The composite material is assumed to be nonmagnetic (its relative permeability is $\mu_r = 1$). The effective permittivity ε_{eff} is calculated through the mixing formula. The characteristic impedance of the air is $Z_0 = \sqrt{\mu_0 / \varepsilon_0}$, and characteristic impedance of the composite medium is $Z_m = Z_0 \sqrt{\mu_r / \varepsilon_{eff}}$. Both coefficients R and T are the complex values, since ε_{eff} is complex,

$$\varepsilon_{eff} = \varepsilon_{eff}' - j\varepsilon_{eff}'' = \left|\varepsilon_{eff}\right| e^{-j\delta}.$$
(10)

The propagation constant in the composite material is calculated as

$$\gamma_m = j\omega\sqrt{\mu_0\varepsilon_0}\sqrt{\varepsilon'_{eff} - j\varepsilon''_{eff}}$$
(11)

For a layer of composite media placed in air, the shielding effectiveness in dB is

$$S.E. = -20 \log_{10}(|T|).$$
 (12)



Figure 4. Transmission coefficient versus frequency for the 5-mm thick composite layer at various conductivities of inclusions. Normal incidence.





Examples of transmission and reflection coefficients for mixtures similar to that described in Section II, but with varying values of conductivity, are shown in Fig. 4 and 5.

B. Oblique incidence of a plane wave upon a composite isotropic layer

The same approach is used for *oblique incidence* of plane waves upon a composite layer. Equations (6) and (7) can be used for the overall reflection and transmission coefficients in the case of an air-composite layer-air structure. However, $R_{1,2}$ and $T_{1,2}$ will be different for parallel and perpendicular polarization [14].

$$R_{1\perp} = \frac{Z_m \cos \varphi_0 - Z_0 \cos \varphi_m}{Z_m \cos \varphi_0 + Z_0 \cos \varphi_m}; R_{2\perp} = \frac{Z_0 \cos \varphi_m - Z_m \cos \varphi_0}{Z_0 \cos \varphi_m + Z_m \cos \varphi_0} \quad (13)$$

$$T_{1\perp} = \frac{2Z_m \cos \varphi_0}{Z_m \cos \varphi_0 + Z_0 \cos \varphi_m}; T_{2\perp} = \frac{2Z_0 \cos \varphi_m}{Z_0 \cos \varphi_m + Z_m \cos \varphi_0}$$
(14)

$$R_{1\parallel} = \frac{Z_0 \cos \varphi_0 - Z_m \cos \varphi_m}{Z_0 \cos \varphi_0 + Z_m \cos \varphi_m}; \quad R_{2\parallel} = \frac{Z_m \cos \varphi_m - Z_0 \cos \varphi_0}{Z_m \cos \varphi_m + Z_0 \cos \varphi_0} \quad (15)$$

$$T_{1\parallel} = \frac{2Z_m \cos \varphi_0}{Z_0 \cos \varphi_0 + Z_m \cos \varphi_m}; T_{2\parallel} = \frac{2Z_0 \cos \varphi_m}{Z_m \cos \varphi_m + Z_0 \cos \varphi_0}$$
(16)

Transmission and reflection coefficients at different angles of incidence for a composite layer with carbon fibers ($\sigma = 6.78 \cdot 10^5$ S/m) and the same geometry as in Section II, are shown in Fig. 6 -9. The results differ for perpendicular and parallel polarizations.



Figure 6. Transmission coefficient versus frequency for the 5-mm thick composite layer at various angles of oblique incidence. Perpendicular polarization.



Figure 7. Reflection coefficient versus frequency for the 5-mm thick composite layer at various angles of oblique incidence. Perpendicular polarization.



Figure 8. Transmission coefficient versus frequency for the 5-mm thick composite layer at various angles of the oblique incidence. Parallel polarization.



Figure 9. Reflection coefficient versus frequency for the 5-mm thick composite layer at various angles of oblique incidence. Parallel polarization.

C. Normal incidence of a plane wave upon a thin composite anisotropic layer

An important case is a *thin layer of a composite material*. In many practical applications, the inclusions, like conducting fibers, are long compared to the thickness of a layer. A practical case is when all fibers are parallel to the plane of the layer. The layer of a composite material is then anisotropic.

If the material is non-texturized, i.e., dielectric dipole moments of inclusions are randomly oriented in-plane, this is a special two-dimensional ("2D") case, and the effective permittivity is then described by a tensor in the form of

$$\begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{22} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix},$$
(17)

where $\varepsilon_{11} = \varepsilon_{22} = 1.5\varepsilon_{eff}$ and $\varepsilon_{33} = \varepsilon_b$, ε_{eff} is calculated according to (1), and ε_b is the permittivity of the base material. The coefficient 1.5 comes from the re-distribution of dipole moments compared to the isotropic case, when there is an equal amount of dipole moments oriented along three axes (*x*,*y*, and *z*), and $\varepsilon_{11} = \varepsilon_{22} = \varepsilon_{33} = \varepsilon_{eff}$.

Consider a plane wave *normally incident* on a thin layer described by the tensor (17). Assuming that the wave propagates along the z-axis, with components $E_z = H_z=0$; and Ex, E_y , H_x , H_y , $H_z \neq 0$. Based on Maxwell's equations, it can be shown that the dispersion equation for the propagation constant is similar to that obtained in [15] for anisotropic ferrite medium,

$$\gamma_m^4 - \omega^2 \mu_0 \varepsilon_0 (\varepsilon_{11} + \varepsilon_{22}) \gamma_m^2 + \omega^4 \mu_0^2 \varepsilon_{11} \varepsilon_{22} = 0.$$
 (18)

The equation (18) yields a simple solution

$$\gamma_m = \pm \omega \sqrt{\frac{\mu \omega}{2}} \left(\sqrt{(\varepsilon_{11} + \varepsilon_{22}) \pm \sqrt{(\varepsilon_{11} + \varepsilon_{22})^2 + 4(\varepsilon_{21}\varepsilon_{12} - \varepsilon_{11}\varepsilon_{22})}} \right)$$
(19)

The first two \pm signs correspond to the positive and negative directions of the wave propagation along *z*, and the other two \pm signs correspond to the ordinary and extraordinary waves in anisotropic medium. However, since $\varepsilon_{11} = \varepsilon_{22}$, there is a single physically reasonable solution, like for the isotropic case. Its form is similar to (11), but instead of ε_{eff} the value $1.5\varepsilon_{eff}$ should be used. Then, for calculating the reflection and transmission coefficients in a single composite "2D" layer, (6)-(9) can be applied, as in the isotropic case, remembering that the characteristic impedances and propagation constants are related through

$$Z_0 = \frac{\omega \mu_0}{\gamma_0} ; \ Z_m = \frac{\omega \mu_0}{\gamma_m}$$
(20)



Figure 10. Shielding effectiveness for an isotropic and anisotropic composite layers; each is 5 mm thick.

S.E. levels for isotropic and "2D" anisotropic layers are shown in Fig. 10. The isotropic layer parameters are as in Section II. In the "2D" layer all the parameters are the same as in isotropic case, but the length of the inclusions is 8 mm, and

the concentration is reduced 8 times to keep the same volume fraction of carbon.

For the *oblique incidence* upon a thin composite "2D" layer, the solution is straightforward as soon as the corresponding dispersion equation for propagation constant is obtained and its roots are found. In this case the components E_z and $H_z \neq 0$. It is then necessary to consider corresponding polarizations; and the corresponding dispersion equation contains all three components of the tensor (17). For these reasons, the solution of the oblique incidence problem in anisotropic case is cumbersome.

IV. FDTD MODELING OF A SHIELDING ENCLOSURE

The frequency dependences of the composite material PMMA-carbon fibers considered in Section II can be approximated by the closest Debye dependence, using the genetic algorithm (GA) [7]. The parameters of this Debye dependence are $\varepsilon_S = 545.46$; $\varepsilon_{\infty} = 2.911$; $\sigma = 0.401$ S/m; and $\tau = 8.957 \cdot 10^{-11}$ s. These Debye parameters can be used to model a shielding enclosure made of this composite using the full-wave EZ-FDTD software, as is proposed in [7].

The size of the enclosure assumed in computations is 22 cm x 14.5 cm x 30 cm. A pseudo wire source placed at the center of the enclosure along the *y*-direction is used to excite the enclosure. An electric field probe, placed in the far zone from the source in the x-direction is used to "measure" the electric field. Figure 11 shows, that a composite enclosure reduces the far field intensity. The increase of the enclosure thickness leads to the lower far field intensity.



Figure 11. Spectrum of the electric field measured in far field

V. CONCLUSION

The Maxwell Garnett (MG) effective medium formalism allows engineering composite materials, that is, synthesizing their frequency responses, using parameters for the base and inclusion materials from experimental, manufacturer's, or published reference data.

The MG formulation is linear with respect to frequency for a multiphase mixture; it allows representing frequency characteristics of media in a form convenient for incorporating them into numerical time-domain electromagnetic codes, such as FDTD. It is shown that the MG formalism can be applied for frequency-dependent dielectric properties of inclusions and host materials and electrodynamically anisotropic mixtures, in particular, with a random in-plane distribution of particles in a thin layer.

Reflection and transmission coefficients of plane waves for both normal and oblique incidence on a layer are calculated on the basis of the MG formalism for composite isotropic materials.

Reflection and transmission coefficients at normal incidence upon a thin layer of material with in-plane random distribution of dipole moments are also considered. The oblique incidence case is straightforward, but more cumbersome.

A similar approach can be also applied for multilayer structures, using the cascading approach based on transmission matrix characterized by ABCD network parameters.

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