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# Reliability Modeling for the Advanced Electric Power Grid\*

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**Abstract.** The advanced electric power grid promises a self-healing infrastructure using distributed, coordinated, power electronics control. One promising power electronics device, the Flexible AC Transmission System (FACTS), can modify power flow locally within a grid. Embedded computers within the FACTS devices, along with the links connecting them, form a communication and control network that can dynamically change the power grid to achieve higher dependability. The goal is to reroute power in the event of transmission line failure. Such a system, over a widespread area, is a cyber-physical system. The overall reliability of the grid is a function of the respective reliabilities of its two major subsystems, namely, the FACTS network and the physical components that comprise the infrastructure. This paper presents a mathematical model, based on the Markov chain imbeddable structure, for the overall reliability of the grid. The model utilizes a priori knowledge of reliability estimates for the FACTS devices and the communications links among them to predict the overall reliability of the power grid.

**Keywords:** Reliability, cyber-physical, embedded, FACTS, power grid.

## 1 Introduction

Providing reliable power delivery has always been an essential requirement in the design and maintenance of the power generation and distribution system. In its simplest form, the power grid consists of generators, transmission lines, and corresponding loads. Increased load and a greater number of power transfers, caused by deregulation, stress the power grid to an unprecedented extent. Continued provision of reliable power delivery necessitates distributed, intelligent control of the grid.

The advanced electric power grid, as proposed by the US Department of Energy [1], promises a self-healing infrastructure. To achieve this vision requires a mechanism for maintaining functionality of the grid and ensuring its adaptability to changes in load. Coordinated power electronics can be used to this end,

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by controlling power flow to prevent failures, or to mitigate the effect of failures when they occur.

A Flexible AC Transmission System (FACTS) controller is a system based on power electronics that enable control of one or more AC transmission system parameters [2]. FACTS devices modify specific parameters of the transmission line, such as the line impedance, to control power flow. Correct modification of these parameters necessitates communication among the FACTS devices, to maintain balance in the grid and to harden it against contingencies.

This need for communication and control creates a cyber-physical system with two parallel networks, a cyber network comprised of FACTS devices and the communication links among them, and a physical network comprised of generators, loads, and transmission lines. These two networks interact; however, there is no one-to-one correspondence between their respective components.

Cascading failures have particular significance in our reliability analysis of the grid, as component failures can no longer be assumed independent. One such scenario occurred in North America on August 14, 2003. A power plant failure in Ohio, combined with the tripping of a transmission line, caused a series of cascading failures that eventually led to a complete blackout. Eight states in the Northeastern United States and the province of Ontario were affected, and over 50 million people were left without power [3]. In order to prevent similar blackouts in the future, the power grid and the network of FACTS devices need to be carefully designed and maintained to provide reliable delivery of power.

In this paper, we present a system-level reliability model for the power grid, which includes FACTS devices and the interactions among them. Our effort towards producing a mathematical representation is guided by actual failure scenarios from the field. Exhaustive analysis of all possible failures in a cyber-physical system is infeasible; hence, our model takes into consideration the most common failure scenarios for the grid. This information can aid in determination of the expected frequency of failures, as well as identification of areas of the system where adding redundancy will have the greatest impact on fault tolerance.

## 2 Related Literature

Reliability modeling has been the subject of numerous studies. In this section, we discuss a select few that are most relevant to this paper. One such study is [4], which describes an early effort in hierarchical modeling of system reliability. The method proposed is useful in cases where the system is too large to be analyzed as a whole, and is instead studied as a sum of its parts. The importance of individual components in determining system reliability was first examined in [5]. The studies presented in [6], [7] and [8] further explore the topic of component importance, introducing metrics and indices that can be used in assessing the importance of components in a system. None of those studies, however, utilizes this knowledge of importance in implementing redundancy or otherwise improving the reliability of the system.

In our research, the specific system being analyzed for dependability is the power grid. Relevant studies include [9], which numerically estimates the reliability of the power grid based on component attributes such as failure rate, outage time, and unavailability. Other research investigates methods for increasing fault tolerance of the grid, including [10], which proposes the use of hybrid cars as local generators of electric power.

As described in Sect. 1, FACTS devices are used to increase the reliability of power grids and facilitate reliable transmission of power, even in the presence of failures in the grid. Use of the max flow algorithm and prudent positioning of FACTS devices are discussed in [11]. This work is further extended in [12], where a distributed version of the max flow algorithm is introduced. FACTS devices are particularly important in reducing the risk of cascading failures. A number of related case studies appear in [13], and [14], which demonstrates the use of FACTS devices in mitigating the risk of cascading failures.

In subsequent sections of this paper, we will use the results of [13] and [14] to develop a model for system reliability. Our work is different from other studies presented in this section, as we are modeling the reliability of the power grid as a whole, including both the physical network, i.e., the grid itself, and the cyber network of FACTS devices.

### 3 Reliability Modeling of Complex Networks

The power grid can be represented as a network comprised of two sets of components; nodes and links. Nodes can represent buses or transformers, while links represent the transmission lines among them. The two main models that we use to represent power networks are the mesh network and the bipartite network. This section provides descriptions of both models.

#### 3.1 The Mesh Network Model

Figure 1(a) depicts the mesh network model for a system of 4 nodes and 6 links. A direct link between every pair of nodes provides redundancy. For a fully functional system, all 4 nodes and all 6 links should be operational; however, the system can still function well despite the failure of one or two of the links, provided that a path still exists between any two nodes. Node failure is more harmful than link failure, but it is possible for the system to remain functional, albeit in a degraded mode, despite the failure of one or more nodes. The number of component failures that the system can tolerate depends on the application for which it is being used.

#### 3.2 The Bipartite Network Model

Figure 1(b) depicts the bipartite network model. Similar to the case of a mesh network, both the links and the nodes of a bipartite network are prone to failure. However, a distinction between this model and the previous one is the presence

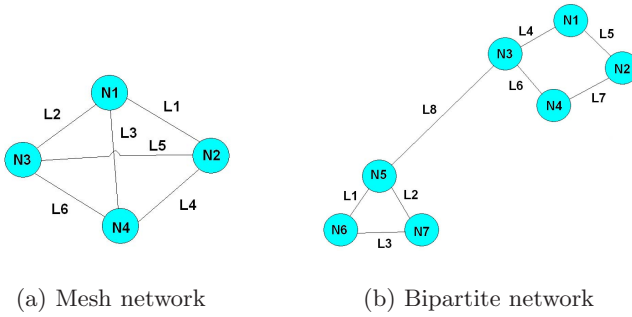


Fig. 1. The two network models used to represent the power grid

of link L8, which connects two separate subsystems. When this link fails, the two subsystems are disconnected from each other, and even if each subsystem can function separately, the overall system will operate in a degraded mode. We acknowledge the critical location of L8 by assigning a higher importance value to this link, and therefore, a greater contribution to the overall reliability.

### 3.3 The Markov Chain Imbeddable Structure

Our model for the reliability of a network of components is based on a Markov chain Imbeddable Structure (MIS). In this section, we provide a brief introduction to the MIS, and illustrate its application to the two network models described in Sects. 3.1 and 3.2.

The state of a system composed of  $n$  components can be represented by an  $n$ -dimensional binary vector,  $\mathbf{S}$ . Each of the  $2^n$  possible states of this vector represents one combination of component failures for the system.

Let  $\mathbf{II}_0$  denote a vector of probabilities, where  $\Pr(Y_0 = \mathbf{S}_i)$  is the probability of the system initially being in state  $\mathbf{S}_i$ . In a normal system, the initial state would be  $\mathbf{S}_0$ , which represents a fully functional network with no component failures.

$$\mathbf{II}_0 = [\Pr(Y_0 = \mathbf{S}_0), \Pr(Y_0 = \mathbf{S}_1), \dots, \Pr(Y_0 = \mathbf{S}_N)]^T . \tag{1}$$

Also, for a given component,  $l$ , we define a matrix  $\mathbf{A}_l$  that represents the state transition probabilities of the system as a function of  $l$ . In other words, each element  $p_{ij}(l)$  in the matrix  $\mathbf{A}_l$  represents the probability that the system would switch from state  $\mathbf{S}_i$  to another state  $\mathbf{S}_j$  due to the failure of component  $l$ .

Finally we define  $\mathbf{u}$ , which is a vector of length equal to the number of states, where each element has a value of 1 if the corresponding state is considered a “good” state for the system, and 0 otherwise. In this sense, the system is in a “good” (i.e., acceptable) state if it exceeds the minimum threshold for a given quality of service parameter.

The overall reliability of the  $n$ -component system can now be expressed as:

$$R_n = (\mathbf{I}\mathbf{I}_0)^T \left( \prod_{l=1}^n \mathbf{A}_l \right) \mathbf{u} . \quad (2)$$

The MIS technique can now be applied to the mesh and bipartite networks. For brevity, we illustrate the technique only for the more general bipartite network.

### 3.4 Application of MIS to the Bipartite Network Model

To further refine the state of the system, we now define five levels of operation and explain each in detail. This refinement allows the system to endure a finite number of component failures; as long as it delivers some fraction of its expected functionality, it is still considered operational. The system is assumed to initially be in the fully functional state, but transitions to an increasingly degraded operational level with each additional component failure. The five levels are described below.

**Level 1: Fully functional system.** All components are functional, i.e., the system is in state  $\mathbf{S}_0$ .

**Level 2: Degraded mode.** A number of links have failed, but all system nodes are still able to communicate with each other, i.e., no node is isolated from the others.

**Level 3: Barely acceptable.** A single node is isolated from the rest of the system. This could happen as a result of the node failing, or when the failure of a sufficient number of links isolates the node from the remainder of the network.

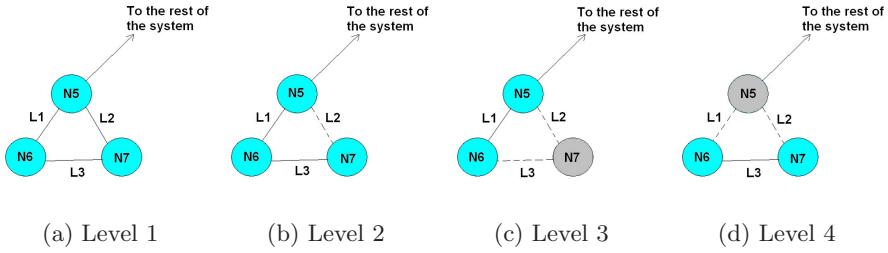
**Level 4: Separated subsystems level.** The link connecting the two subsystems (L8) has failed. The two subsystems are still functioning separately, but communication between them is lost.

**Level 5: Unacceptable operation.** Any failures beyond those described in Levels 1-4 bring the system to this state. This level of functionality is considered unacceptable.

Mathematically, a different reliability function corresponds to each level of functionality. Recall that the state vector  $\mathbf{u}$  represents each “acceptable” state of the system with a 1. Any state where the system is not in Level 5 is considered an acceptable state.

The bipartite network representing a power grid is typically quite large, and as the system grows in size, the reliability analysis becomes increasingly complex. The matrix  $\mathbf{A}$ , which represents the state transition probabilities, doubles in size with the addition of one component. To simplify the analysis, we divide the system into two subsystems, each of which is analyzed separately with the MIS technique. The results are then combined to produce the system-level reliability equations.

For the example of Fig. 1(b), the portion depicted in Fig. 2(a) is labeled “Subsystem 1,” and analyzed first. Here, the subsystem is considered fully functional.



**Fig. 2.** Operational states for Subsystem 1 of the bipartite network of Fig. 1(b)

Level 2 results from the failure of a single link, e.g., L2 in Fig. 2(b). The failure of two links, or equivalently, a single node, further degrades the subsystem operation to Level 3. The failed components depicted in Fig. 2(c) are links L2 and L3, or node N7. Level 4, which causes the separated subsystems state, appears in Fig. 2(d), where links L1 and L2, or node N5 have failed.

### 3.5 Derivation of Reliability Expressions

In the bipartite network, for each of the two subsystems, equations were calculated using the MIS technique for the different operational levels defined. The overall system reliability can then be calculated as:

$$R_{sys} = R_{sub1} * R_{sub2} * R_{L8} , \tag{3}$$

where  $R_{sys}$  is the system reliability,  $R_{sub1}$  and  $R_{sub2}$  are the reliabilities of subsystems 1 and 2, respectively, and  $R_{L8}$  is the reliability of link L8, which connects the two subsystems.

In order to derive an expression for each of the operational levels, different combinations of  $R_{sub1}$  and  $R_{sub2}$  are used to reflect the overall operational level of the system.

Let  $R_{subi_j}$  denote the reliability expression corresponding to the  $j^{th}$  level of operation in Subsystem  $i$ . The equations for overall system reliability can be calculated as follows:

**Operational Level 1.** For this level, none of the components in either subsystem can fail; therefore, the corresponding expressions for each subsystem are  $R_{sub1_1}$  and  $R_{sub2_1}$ , respectively, and the overall system reliability becomes:

$$R_1 = R_{sys} = R_{sub1_1} * R_{sub2_1} * R_{L8} . \tag{4}$$

**Operational Level 2.** For this level, only a single link is allowed to fail; this link could be in either subsystem. This raises two cases, if the failed link is in Subsystem 1, then:

$$R_{sys_1} = R_{sub1_2} * R_{sub2_1} * R_{L8} . \tag{5}$$

If the failed link is in Subsystem 2, then:

$$R_{sys_2} = R_{sub1_1} * R_{sub2_2} * R_{L8} . \quad (6)$$

Assuming that the two cases are equally likely, the mean of (5) and (6) represents the overall system reliability:

$$R_2 = R_{sys} = 0.5 * (R_{sys_1} + R_{sys_2}) . \quad (7)$$

**Operational Level 3.** In this level, the failure of a single node can be tolerated in one of the subsystems, and the failure of a single link in the other subsystem. This again leads to two cases, based on the respective subsystems of the failed node and link. If a node fails in Subsystem 1 and a link in Subsystem 2, then:

$$R_{sys_1} = R_{sub1_3} * R_{sub2_2} * R_{L8} . \quad (8)$$

If it is the other way around, then:

$$R_{sys_2} = R_{sub1_2} * R_{sub2_3} * R_{L8} . \quad (9)$$

Symmetry, and the assumption that all states are equally likely, leads to the following expression for system reliability at Level 3.

$$R_3 = R_{sys} = 0.5 * (R_{sys_1} + R_{sys_2}) . \quad (10)$$

**Operational Level 4.** This level of operation can occur in any of three cases; if link L8 between the subsystems fails, or if one of nodes N3 or node N5 fails. This can be translated into the following expressions:

$$R_{sys_1} = R_{sub1_1} * R_{sub2_4} * R_{L8} . \quad (11)$$

$$R_{sys_2} = R_{sub1_4} * R_{sub2_1} * R_{L8} . \quad (12)$$

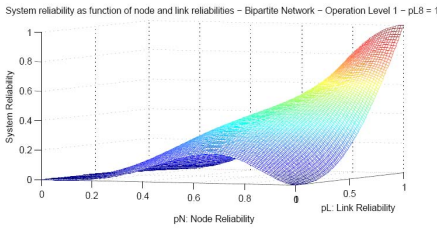
$$R_{sys_3} = R_{sub1_1} * R_{sub2_1} * (1 - R_{L8}) . \quad (13)$$

Again, assuming that all cases are equally likely:

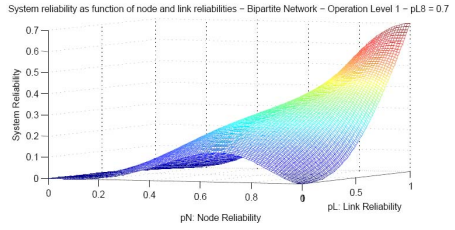
$$R_4 = R_{sys} = (1/3) * (R_{sys_1} + R_{sys_2} + R_{sys_3}) . \quad (14)$$

Figures. 3(a), 3(b), and 3(c) depict the overall system reliability as a function of node and link reliability. In Fig. 3(a), the value of  $R_{L8}$  is assumed to be 1, corresponding to no chance of failure for the critical link L8. Figure 3(b) assumes  $R_{L8}$  to be 0.7. It is clear that this link has a direct effect on system reliability, as the decrease of  $R_{L8}$  drastically affects the rest of the system. Finally, Fig. 3(c) represents the reliability of the bipartite network in Operational Level 2. In the next section, we will apply the generalized model for system reliability derived thus far to the specific case of the power grid.

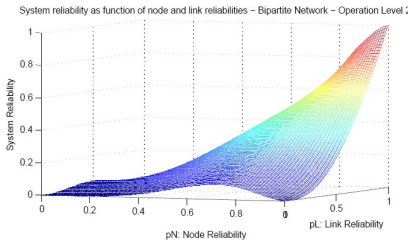




(a) Bipartite network, Level 1,  $R_{L8} = 1$



(b) Bipartite network, Level 1,  $R_{L8} = 0.7$



(c) Bipartite network, Level 2

**Fig. 3.** Reliability of networks in Fig. 1 as a function of node and link reliability

## 4 Reliability of Power Transmission Lines

Several components constitute the power grid, including power generators, transmission lines, and transformers. In addition, we also consider the presence of FACTS devices, as they significantly affect the overall system reliability.

In general, all devices are prone to failure; however, our main interest will be in the failures of transmission lines. Even though generators and transformers can fail, there are usually sufficient backup units installed in the system to compensate for their failures, leaving transmission lines as the main contributors to system failures. Analysis of the reliability of transmission lines is the subject of this section.

### 4.1 Known Failure Scenarios, No FACTS Devices Installed

Several cascading failure scenarios in the IEEE 118 bus system have been studied and described in detail in [13]. Cascading failures show that links are not independent, negating a common assumption in reliability modeling. Our model does not assume independence of the links.

The following definitions and notations will be used for the remainder of this paper.

$F_A$ : The event that link A remains functional for a specified period of time.

$OV_A$ : The event that link A is overloaded.

$NOV_A$ : The event that link A is not overloaded.

$R(A)$ : Reliability of link A

$Q(A)$ : Unreliability of link A, which is equal to  $1 - R(A)$

$Pr(\cdot)$ : Probability that the specified event will happen.

The reliability of link A is defined as the probability that link A remains functional for a specified period of time. Using conditional probability notation,

$$R(A) = Pr(F_A) = Pr(F_A|OV_A) * Pr(OV_A) + Pr(F_A|NOV_A) * Pr(NOV_A) . \tag{15}$$

In theory, lines that are not overloaded are expected to function properly for an indefinite period of time. Under normal operating conditions, the failure of a line can only be attributed to accidents, such as inclement weather or physical disconnection. Therefore, the probability denoted as  $Pr(F_A|NOV_A)$  is the probability that no such accidents occur for the transmission line during normal non-overloaded conditions.

However, when a line is overloaded, a different situation occurs. Depending on the amount of overload, the duration of time before the line fails can range between 0.15 seconds for an overload of 2000% of the line capacity, and 4.2 seconds for an overload of 150% [15].

In our analysis, we assume that no repair will be carried out for overloaded lines. According to this assumption, if the line becomes overloaded, it will eventually fail. In that case, the term  $Pr(F_A|OV_A)$  will be reduced to 0, and (15) becomes:

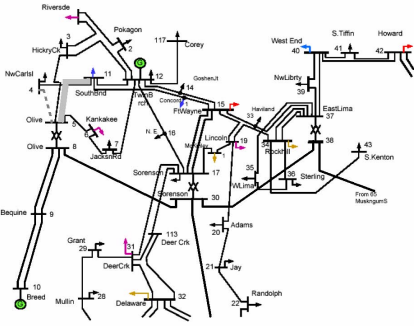
$$R(A) = Pr(F_A) = Pr(F_A|NOV_A) * Pr(NOV_A) . \tag{16}$$

**Outage of Line 4-5.** As described in [13], the outage of line (4-5) leads to several other outages that lead to a failure in the system. The problem starts when the line fails, as line (5-11) becomes overloaded in the process. Taking out line (5-11) will force power to be diverted through lines (5-6), (6-7), and (7-12). Line (7-12) becomes overloaded, and its failure causes overload in lines (3-5) and (16-17). As line (3-5) fails, power is diverted through line (8-30). This causes overload in lines (15-17), (14-15), (15-19), (15-33), and (33-37). Eventually, line (14-15) breaks and causes the system to fail. This sequence of events is depicted in Figs. 4(a)-4(e) From the information available about this scenario, we can now construct reliability expressions for the links involved. For example, line (5-11) becomes overloaded if line (4-5) fails. Thus:

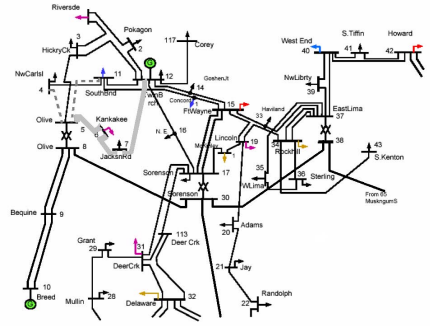
$$R(5 - 11) = Pr(F_{(5-11)}) = Pr(F_{(5-11)}|NOV_{(5-11)}) * Pr(NOV_{(5-11)}) . \tag{17}$$

The probability  $Pr(F_{(5-11)}|NOV_{(5-11)})$  is the probability of link (5-11) staying functional in non-overloading conditions, as described above. It is readily apparent that the probability of link (5-11) is the same as the probability of link (4-5) failing, or link (4-5)'s unreliability,  $Q(4-5)$ , which is equal to  $1 - R(4-5)$ . Therefore,  $Pr(NOV_{(5-11)}) = 1 - Q(4 - 5) = R(4 - 5)$ , and the equation for  $R(5-11)$  becomes:

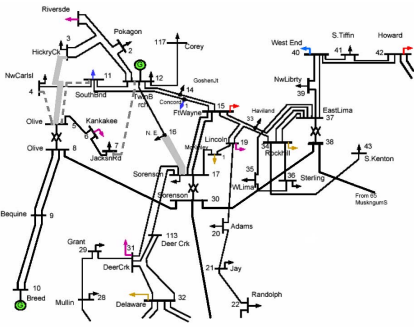
$$R(5 - 11) = Pr(F_{(5-11)}) = Pr(F_{(5-11)}|NOV_{(5-11)}) * R(4 - 5) . \tag{18}$$



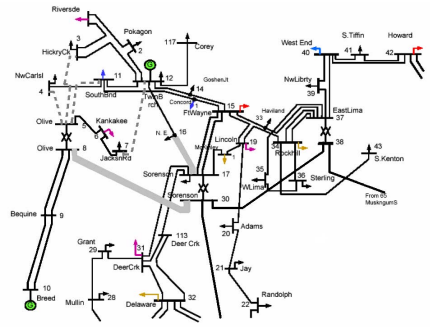
(a) Cascade stage 1



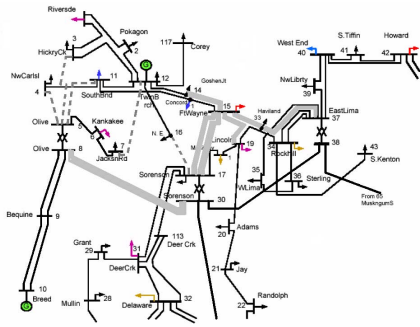
(b) Cascade stage 2



(c) Cascade stage 3



(d) Cascade stage 4



(e) Cascade stage 5

Fig. 4. Cascading failure resulting from the outage of line (4-5)

Taking another example from the same cascading scenario, the reliability of line (7-12) can be found using the expression:

$$R(7 - 12) = Pr(F_{(7-12)}) = Pr(F_{(7-12)}|NOV_{(7-12)}) * Pr(NOV_{(7-12)}) . \quad (19)$$

Again,  $Pr(NOV_{(7-12)})$  is a function of the reliability of line (5-11), whose failure causes line (7-12) to overload. Hence, (19) becomes:

$$R(7 - 12) = Pr(F_{(7-12)}) = Pr(F_{(7-12)}|NOV_{(7-12)}) * R(5 - 11) . \quad (20)$$

Substituting (18) into (20) yields:

$$R(7 - 12) = Pr(F_{(7-12)}|NOV_{(7-12)}) * Pr(F_{(5-11)}|NOV_{(5-11)}) * R(4 - 5) . \quad (21)$$

We can further simplify the notation, by defining  $R_{NOV}(A)$  as the non-overloading, or nominal, reliability of link A to be equal to  $Pr(F_A|NOV_A)$ . Equations (18) and (21) will therefore become, respectively:

$$R(5 - 11) = R_{NOV}(5 - 11) * R(4 - 5) . \quad (22)$$

$$R(7 - 12) = R_{NOV}(7 - 12) * R_{NOV}(5 - 11) * R(4 - 5) . \quad (23)$$

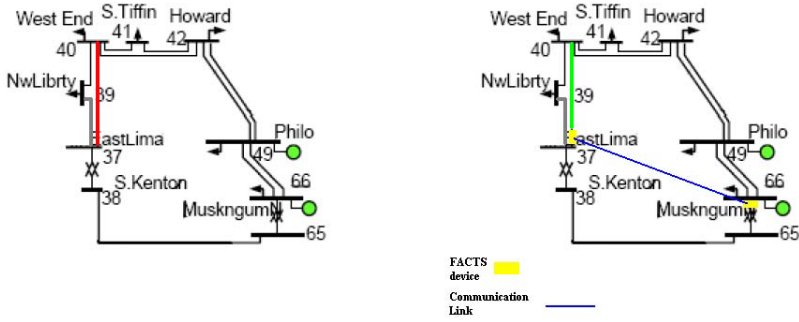
In a similar fashion, reliability equations can be derived for every other link.

## 4.2 Known Failure Scenarios, FACTS Devices Installed

As an improvement to the situation described in Sect. 4.1, FACTS devices are installed in the system in order to prevent cascading failures from happening. However, if these devices fail or lose communication with each other, failures can still occur and cascades can happen.

Figs. 5(a) and 5(b) show a scenario where the outage of line (37-39) can lead to a cascading failure. After this line fails, line (37-40) becomes overloaded and eventually fails. Installing FACTS devices on lines (37-40) and (66-65) can alleviate the problem, by rerouting enough power through lines (37-38), (38-65), and through transformer (65-66) to prevent line (37-40) from becoming overloaded. A problem arises if one or more of the installed FACTS devices, or a the communication links among them fail. If the FACTS device on line (37-40) fails, line (37-39) will fail, causing (37-40) to become overloaded and eventually fail. In the case of a failure in the communication line, the FACTS devices can still function on their own; however, the effect on line (37-40) will depend on how they choose to operate. A number of possible alternatives are summarized below.

- Bypass the FACTS devices and leave the system to function according to the laws of physics. This choice will obviously cause the system to fail whenever an outage in line (37-39) occurs.
- Set the power flow value to the capacity of the line. This could cause a trouble only if nearby lines are of lower capacity, but in that case the FACTS device would probably have been installed on the lower capacity line. This choice should normally keep the system stable.



(a) Potential cascading failure

(b) Mitigation performed by FACTS devices

**Fig. 5.** Outage of line (37-39)

- Maintain the power flow values as they were the last time the max flow algorithm was run before the outage of the communication link occurred. Depending on the system, this may or may not be a good choice. In our case, this choice leaves overloads on lines (24-72), (34-43) and (70-75), so it is not the best choice.
- Allow the power flow to change in the system according to the laws of physics, as long as they stay within  $\pm 20\%$  of the line capacity, otherwise stop the power flow from increasing any further. Again, depending on the system, this may or may not be a good choice.

In general, one of two cases arises, depending on our choice of action when the communication link fails.

**Case 1.** The reliability of line (37-40) depends on the reliability of both the FACTS devices at (37-39) and (65-66) and the communication link between them.

**Case 2.** The reliability of line (37-40) depends only on the reliability of the FACTS device at (37-40), and does not depend on the communication link or the other FACTS device.

In Case 1, the reliability equation for line (37-40) will be:

$$R(37 - 40) = R_{NOV}(37 - 40) * Pr(NO_{V(37-40)}) \quad (24)$$

In order to compute the probability of overload for line (37-40), we define the following events.

- F1: The event that the FACTS device at line (37-40) remains functional.
- F2: The event that the FACTS device at transformer (65-66) remains functional.
- CL1: The event that the communication line between the FACTS devices stays functional.
- L(37-39): The event that line (37-39) remains functional.

The probability of no overloading in line (37-40) can now be defined as:

$$Pr(NOV_{(37-40)}) = Pr \left[ L(37-39) \cup \overline{[L(37-39)]} \cap [F1 \cap F2 \cap CL1] \right] . \quad (25)$$

This can be translated into:

$$Pr(NOV_{(37-40)}) = R(37-39) + Q(37-39) * R(F1) * R(F2) * R(CL1) , \quad (26)$$

and the equation for the reliability of link (37-40) becomes:

$$R(37-40) = R_{NOV}(37-40) * (R(37-39) + Q(37-39) * R(F1) * R(F2) * R(CL1)) . \quad (27)$$

For Case 2, we are only concerned with the reliability of the FACTS device installed on line (37-40). The equation then reduces to:

$$R(37-40) = R_{NOV}(37-40) * (R(37-39) + Q(37-39) * R(F1)) . \quad (28)$$

## 5 Conclusions and Future Work

The objective of the research described in this paper is the development of a model for the reliability of the advanced electric power grid. The model takes into consideration the FACTS network used to regulate power flow and lessen the likelihood of cascading failures. We used the Markov chain imbeddable structure to find expressions for system reliability at predefined levels of degraded operation. On the link level, we demonstrated the sequence of events leading to a cascading failure. The occurrence of cascading failures illustrates that links do not fail independently, meaning that the reliability of each link is dependent on the reliabilities of surrounding links. Using actual failure scenarios, we developed a model for link-level reliability.

The limited amount of information available on failure scenarios is the main challenge in accurate modeling of any aspect of dependability for the electric power grid. In future extensions to this research, we plan to use statistical techniques to investigate the confidence levels achievable for reliability models based on this limited sample set. We also plan to validate the proposed reliability model through large-scale simulation of the power grid, which will undoubtedly lead to refinements to the model.

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