01 May 2008

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Recommended Citation
M. T. Ghasr and R. Zoughi, "Multimodal Solution for a Rectangular Waveguide Radiating into a Multilayered Dielectric Structure and its Application for Dielectric Property and Thickness Evaluation," *Proceedings of the IEEE Instrumentation and Measurement Technology Conference, 2008 (2008, Victoria, BC, Canada)*, pp. 552-556, Institute of Electrical and Electronics Engineers (IEEE), May 2008.
The definitive version is available at https://doi.org/10.1109/IMTC.2008.4547098

I[2]MTC 2008 – IEEE International Instrumentation and
Measurement Technology Conference
Victoria, Vancouver Island, Canada, May 12–15, 2008

Multimodal Solution for a Rectangular Waveguide Radiating into a Multilayered Dielectric Structure and its Application for Dielectric Property and Thickness Evaluation

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Abstract *–Open-ended rectangular waveguides are widely used for microwave and millimeter wave nondestructive testing applications. Applications have included detecting disbonds and delaminations in multilayered composite structures, thickness evaluation of dielectric sheets and coatings on metal substrates, etc. When inspecting a complex multilayered composite structure, made of generally lossy dielectric layers with arbitrary thicknesses and backing, the dielectric properties of a particular layer within the structure is of particular interest, such being health monitoring of structures such as radomes. The same is also true where one may be interested in the thickness or more importantly thickness variation of a particular layer within such structures. An essential tool for estimating the dielectric constant or thickness is an accurate model for simulating reflection coefficient at the aperture of the probing open-ended waveguide. One issue of interest is that radiation from open-ended rectangular waveguides into layered dielectric structures has been considered only when accounting for the dominant waveguide mode. However, when using these models for recalculating dielectric constant or thickness, the results may not be accurate (depending on the measurement requirements). To this end, this paper provides an accurate model for the reflection coefficient which also accounts for the effect of higher-order modes. Finally, the potential of this model for accurately estimating dielectric constant is shown.*

Keywords *– open-ended waveguide, stratified dielectric medium, higher-order modes, dielectric constan, thickness.*

I. INRODUCTION

Open-ended rectangular waveguides are the most widely used probes in near-field microwave and millimeter wave nondestructive testing (NDT) applications, imaging such as dielectric property measurement, thickness measurement of a dielectric slab, crack detection, and porosity level estimation to name a few [1]. When operating in the near-field, spatial resolution is a function of probe dimensions and hence obtaining high-resolution images are readily possible competing very well with other NDT methods [2]. In addition, when concerned with multilayered composites, thickness variation detection can be conducted with micrometer range accuracy at relatively low microwave frequencies [1, 3]. Furthermore, microwave NDT has the potential of providing information about electrical properties of dielectric structures. This is particularly important for inspecting structures such as radomes since the electrical properties of each layer in the structure directly affects whether a radome is transparent to electromagnetic radiation or not. Dielectric property measurement using open-ended rectangular waveguides has received significant attention both experimentally and from the modeling point of view. These works have included single layer or infinite half-space material evaluation [1, 4-6]. An issue of concern in multilayered composite characterization is when the effect of a particular layer on the reflection coefficient is small and any small error in modeling or measurement causes substantial error in evaluating the properties (thickness or dielectric) of another layer. In such cases the modeling error is commonly associated with ignoring the influence of higher-order modes while only taking the dominant mode into account. Subsequently, when the model is used in an inverse manner (or forward iterative) for measuring dielectric constant or thickness, slight to significant errors may be experienced. Some of these studies have utilized variational method, which results in an approximate solution [3], and some utilize more rigorous formulations including the effect of higher-order modes [4-6]. None of these studies except [4] offers a solution for reflection from a multilayered structure while taking higher-order modes into account. Furthermore, no study has shown the contribution of higher-order modes as function of various structural features of a multilayered structure such as thicknesses, dielectric profiles, and when the composite is backed by a dielectric infinite half-space or a conducting plate. As this paper will show, the effect of higher-order modes is much more prominent when analyzing multilayered structures.

This paper gives an exact formulation based on Fourier analysis for the reflection coefficient at the aperture of an open-ended rectangular waveguide irradiating a stratified dielectric structure. This formulation accounts for the contribution of TE and TM higher-order modes. Subsequently, an analysis of the effect of higher-order modes per structure features (thickness and dielectric constant) and number of higher-order modes required for convergence will be presented. Finally, the use of this model for extraction of dielectric constant will be presented through measuring reflection coefficient of lossy rubber and low-loss plexiglass sheets.

II. FORMULATION AND ANALYSIS

The reflection coefficient seen by a waveguide irradiating a stratified dielectric medium is formulated in a similar fashion to [5-6]. Yoshitomi *et. al.* [5] considered a waveguide with a lossy flange radiating into free-space for the purpose of examining the influence of the flange on the radiation pattern of open-ended waveguides, while and Bois *et. al.* [6] expanded these formulations for application to accurate extraction of the dielectric constant of a generally lossy infinite half-space of a dielectric material. This paper extends this formulation to a *general* case of a stratified dielectric medium possessing any number of layers (backed by an infinite half-space or a conductor) for the purpose of accurate extraction of dielectric constant or thickness of any layer. Fig. 1 shows a schematic of a waveguide radiating into a multilayered structure. The waveguide has a broad dimension of *2a* and a narrow dimension of *2b*. The waveguide has an infinite flange and radiating a multilayered dielectric structure backed by a conductor or an infinite half-space (IHS) of a dielectric. Each layer is defined by its complex permittivity ($\varepsilon_r = \varepsilon_r' - j\varepsilon_r''$) and permeability ($\mu_r = \mu_r' - j\mu_r''$) as well as its thickness, d.

The electric and magnetic fields in all regions may be derived from Hertzian vectors. Inside the waveguide, for the incident field a magnetic Hertzian vector with a TE_{10} mode distribution is used. On the other hand, the reflected waves are represented by electric and magnetic Hertzian vectors which represent a summation of all possible TM and TE waveguide modes. The definition for these Hertzian Vectors may be found in [5] and [6].

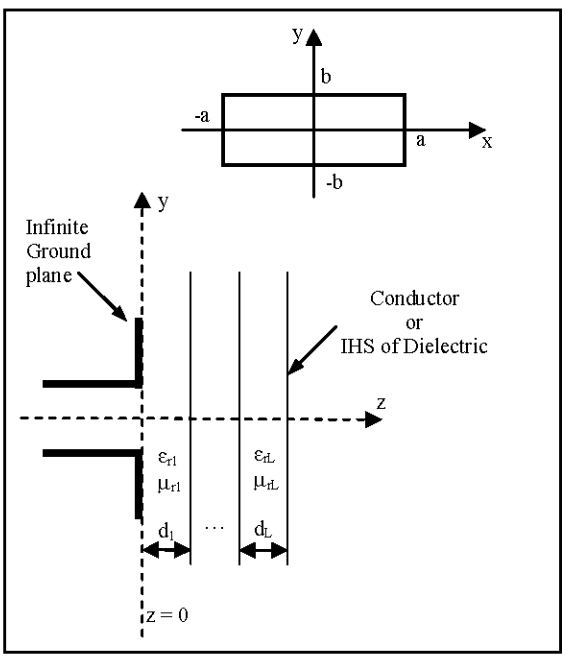

Fig. 1. The geometry of the problem.

In the multilayered dielectric structure, the electric and magnetic Hertzian vectors are defined as follows:

$$\hat{a}_z \cdot \prod\nolimits_l^e (x,y,z) = \frac{1}{4\pi^2 K_l^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[A_l^{e+}(\xi,\eta) e^{-j\zeta z} + A_l^{e-}(\xi,\eta) e^{j\zeta z} \right] e^{-j(\xi x + \eta y)} d\xi d\eta \quad (1)$$

$$\hat{a}_z \cdot \prod\nolimits_l^h (x,y,z) = \frac{1}{4\pi^2 K_l^2 Z_l} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[A_l^{h+}(\xi,\eta) e^{-j\zeta z} + A_l^{h-}(\xi,\eta) e^{j\zeta z} \right] e^{-j(\xi x + \eta y)} d\xi d\eta \quad (2)$$

where $K = \omega\sqrt{\varepsilon.\mu}$, $Z = \sqrt{\mu/\varepsilon}$, $\zeta = \sqrt{K^2 - \xi^2 - \eta^2}$, and $A^{e,h}(\xi,\eta)$ are the spectral domain wave functions representing radiated electromagnetic fields from the waveguide. These functions are unknown and are found by applying the boundary conditions. Since there are discontinuities along the z-direction, forward and backward propagating waves are presents in each layer which are donated by a + and – superscripts respectively. Furthermore, subscript *l* denotes the layer number. Expressions for converting Hertzian vectors into electric and magnetic fields, and Fourier relationships move the field between the spatial and spectral domains may be found in [5] and [6].

The solution is obtained by matching boundary conditions at all interfaces. This process starts at the last layer ($z = z_L$). For a conductor backed (CB) case, the tangential electric field is zero resulting in total reflection:

$$A_L^{e-} = A_L^{e+} e^{-2j\zeta_L z_L} \text{ , and } A_L^{h-} = -A_L^{h+} e^{-2j\zeta_L z_L} \quad (3)$$

On the other hand, for the IHS case, the backward traveling wave is zero, since there is no mechanism for reflections, therefore:

$$A_L^{e-} = 0 \text{ , and } A_L^{h-} = 0 \quad (4)$$

Matching the boundary conditions at intermediate layers results in the following relationships between the forward and backward traveling waves:

$$\frac{A_{l-1}^{e-}}{A_{l-1}^{e+}} = e^{-2j\zeta_{l-1} z_{l-1}} \frac{C_l^e e^{2j\zeta_l z_{l-1}} \left(1 + B_l^e\right) - e^{2j\zeta_l z_l} \left(1 - B_l^e\right)}{-C_l^e e^{2j\zeta_l z_{l-1}} \left(1 - B_l^e\right) + e^{2j\zeta_l z_l} \left(1 + B_l^e\right)} \quad (5)$$

where,

$$B_l^e = \frac{Z_{l-1} K_l \zeta_{l-1}}{Z_l K_{l-1} \zeta_l} \quad (6)$$

$$C_{l-1}^{e}=\frac{C_l^e e^{-2j\zeta_l d_l}\left(1+B_l^e\right)-\left(1-B_l^e\right)}{-C_l^e e^{-2j\zeta_l d_l}\left(1-B_l^e\right)+\left(1+B_l^e\right)} \tag{7}$$

Similar expressions are obtained for the magnetic field components:

$$B_l^h=\frac{Z_l K_l \zeta_{l-1}}{Z_{l-1}K_{l-1}\zeta_l} \tag{8}$$

$$C_{l-1}^{h}=\frac{C_l^h e^{-2j\zeta_l d_l}\left(1+B_l^h\right)-\left(1-B_l^h\right)}{-C_l^h e^{-2j\zeta_l d_l}\left(1-B_l^h\right)+\left(1+B_l^h\right)} \tag{9}$$

The $C^{e,h}$ coefficients are obtained iteratively starting from the last layer where $C_L^e=-C_L^h=1$ for a CB case, and $C_L^h=C_L^e=0$ for an IHS case.

The boundary conditions at the waveguide aperture ($z=0$), dictate the continuity of the total electric and magnetic field, as follows:

$$\overline{E}_{x,y}^{1}=\begin{cases}\overline{E}_{x,y}^{wg} & |x|\le a,|y|\le b\\ 0 & elsewhere\end{cases} \tag{10}$$

$$\overline{H}_{x,y}^{1}=\overline{H}_{x,y}^{wg} \qquad |x|\le a,|y|\le b \tag{11}$$

where the superscript *wg* denotes total fields in the waveguide and superscript 1 refers to the total fields in the first layer. Applying these boundary conditions and utilizing the orthogonal properties of the modes in the waveguide in similar fashion to [5] and [6] resulting in these two linear systems of equations:

$$\begin{aligned}&\sum_{m,n=1}^{\infty}\left[a_m I_1(m,n,p,q)+b_n I_2(m,n,p,q)\right]K_{mn}A_{mn}^{e}\\&+\sum_{\substack{m,n=0\\ m=n\neq 0}}^{\infty}\left[b_n I_1(m,n,p,q)-a_m I_2(m,n,p,q)\right]K_0 A_{mn}^{h}\\&+ab\left[A_{pq}^{e}K_0 b_q-A_{pq}^{h}K_{pq}a_p(1+\delta_{0q})\right]\frac{Z_1}{Z_0}\\&=\left[K_0 a_1 I_2(1,0,p,q)-2K_{10}a_1 ab\frac{Z_1}{Z_0}\delta_{1p}\delta_{0q}\right]A^{i}\end{aligned} \tag{12}$$

for $p=1, 2, 3, \ldots$, $q=0, 1, 2, 3, \ldots$, and

$$\begin{aligned}&\sum_{m,n=1}^{\infty}\left[a_m I_3(m,n,p,q)+b_n I_4(m,n,p,q)\right]K_{mn}A_{mn}^{e}\\&+\sum_{\substack{m,n=0\\ m=n\neq 0}}^{\infty}\left[b_n I_3(m,n,p,q)-a_m I_4(m,n,p,q)\right]K_0 A_{mn}^{h}\\&+ab\left[A_{pq}^{e}K_0 a_p+A_{pq}^{h}K_{pq}b_q(1+\delta_{0p})\right]\frac{Z_1}{Z_0}\\&=K_0 a_1 I_4(1,0,p,q)A^{i}\end{aligned} \tag{13}$$

for $p=0,1, 2, 3, \ldots$, $q=1, 2, 3, \ldots$,

In these equations A_{mn}^{e} and A_{mn}^{h} are the coefficients of the reflected and/or generated TM and TE modes at the waveguide aperture $a_m=\frac{m\pi}{2a}$, $b_n=\frac{n\pi}{2b}$, $K_{mn}=\sqrt{K_0{}^2-a_m^2-b_n^2}$, δ_{pq} is the Kronecker delta, and the integrals $I_{1,2,3,4}$ are:

$$I_1(m,n,p,q)=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}V_1 C_m^a(-\xi)S_n^b(-\eta)S_p^a(\xi)C_q^b(\eta)\,d\xi\, d\eta \tag{14}$$

$$I_2(m,n,p,q)=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}V_2 S_m^a(-\xi)C_n^b(-\eta)S_p^a(\xi)C_q^b(\eta)\,d\xi\, d\eta \tag{15}$$

$$I_3(m,n,p,q)=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}V_3 C_m^a(-\xi)S_n^b(-\eta)C_p^a(\xi)S_q^b(\eta)\,d\xi\, d\eta \tag{16}$$

$$I_4(m,n,p,q)=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}V_1 S_m^a(-\xi)C_n^b(-\eta)C_p^a(\xi)S_q^b(\eta)\,d\xi\, d\eta \tag{17}$$

The variables $V_{1,2,3}$ contain information about the properties of the multilayered dielectric structure and defined as:

$$V_1=\frac{K_1^2\eta\xi-\zeta_1^2\eta\xi D_1^h D_1^e}{K_1\left(\xi^2+\eta^2\right)\zeta_1 D_1^e} \tag{18}$$

$$V_2=\frac{K_1^2\eta^2+\zeta_1^2\xi^2 D_1^h D_1^e}{K_1\left(\xi^2+\eta^2\right)\zeta_1 D_1^e} \tag{19}$$

$$V_3=\frac{K_1^2\xi^2+\zeta_1^2\eta^2 D_1^h D_1^e}{K_1\left(\xi^2+\eta^2\right)\zeta_1 D_1^e} \tag{20}$$

$$D_1^h=\frac{1-C_1^h e^{-2j\zeta_1 d_1}}{1+C_1^h e^{-2j\zeta_1 d_1}} \tag{21}$$

$$D_1^e=\frac{1-C_1^e e^{-2j\zeta_1 d_1}}{1+C_1^e e^{-2j\zeta_1 d_1}} \tag{22}$$

The integral in (14) to (17) may become singular, especially when the total loss in the multilayered dielectric structure is low. When numerically evaluating these integrals, it is advantageous to transform these integral to polar coordinates as reported in [5]. However unlike the cases investigated in

[5] and [6] where the structure is always an IHS and the singularity location is known and analytically extracted, in a multilayered structure, the singularity of the integrals depends on profile of the structure. Therefore, analytically extracting the singularities is not feasible and the integration is performed on a contour around singular points. Furthermore, a conversion test on these integrals is performed prior to obtaining a solution.

III. ANALYSIS

To verify the formulation, the complex reflection coefficient ($\Gamma = |\Gamma| e^{\phi_\Gamma}$) seen by an X-band (8.2 -12.4 GHz) waveguide radiating into free-space was compared to results obtained using HFSS™ which employs 3D electromagnetic simulations using finite element methods (FEM). The results are shown in Fig. 2. These results show that while there is a major advantage in including higher order-modes in the solution, their contribution becomes minimal beyond the first few modes. In fact with only 6 modes, it is possible to approximate the actual solution adequately as similarly reported in [6]. Consequently, from now on all solutions that include higher-order modes will only utilize 6 modes.

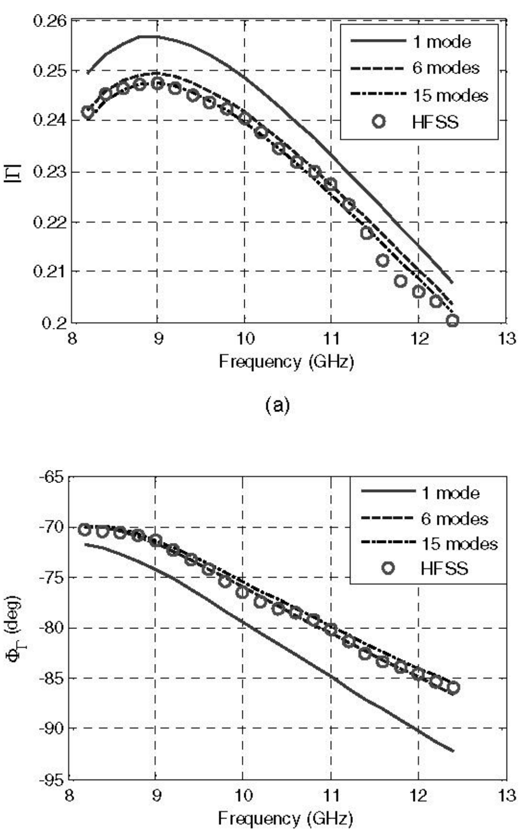

Fig. 2. Reflection coefficient seen by an X-band open-ended waveguide radiating into free space; the effect of higher order modes (a) magnitude, and (b) phase.

In [6], it was shown that the error in TE_{10} reflection coefficient due to the exclusion of higher-order modes decreases as permittivity and loss factor increase. However, that finding is only true when the waveguide is radiating into infinite half-space of dielectric. In the cases of a finite dielectric slab, or a multilayered case, the thickness of the dielectric layers has an effect on the contribution of higher-order modes as well. To investigate this phenomenon, simulations were performed for a slab of rubber with a relative dielectric constant of ($_r$ = 7.3 – j 0.3) and a slab of Plexiglas with a relative dielectric constant of ($_r$ = 2.6 – j 0.01). Fig. 3 shows the error in calculating TE_{10} mode reflection coefficient in terms of the slab thickness (normalized to the wavelength in the dielectric slab) due to the exclusion of higher-order modes. The results show that, as the thickness of the slab increases (moving toward infinite half-space), materials with higher permittivity and loss factor (i.e. rubber) produce less significant higher-order modes compared to material with low permittivity and loss factor (i.e. plexi glass) and it agrees with the results reported in [6]. Conversely, when the thickness of the slab becomes small, the opposite occurs and the contribution of higher-order modes becomes more significant for high permittivity materials.

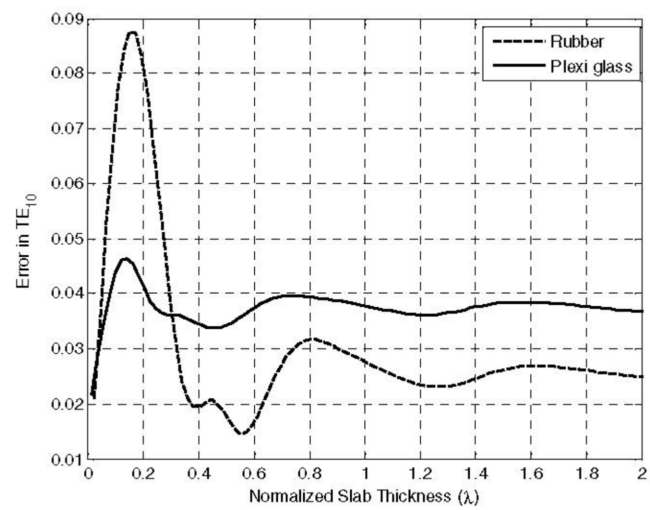

Fig. 3. Error in reflection coefficient calculation due to the exclusion of higher-order modes vs. normalized slab thickness.

IV. INVERSE PROBLEM

The main goal of this study is to extract the dielectric constant (or thickness, not specifically shown here) of a material within a multilayered structure. To illustrate this, measurements were performed with an Agilent 8510C vector network analyzer (VNA) using a sheet of 4.42 mm-thick rubber utilizing the setup shown in Fig. 4. The rubber sheet had an area of 15 x 15 mm^2, large enough to contain the radiations of the open-ended waveguide.

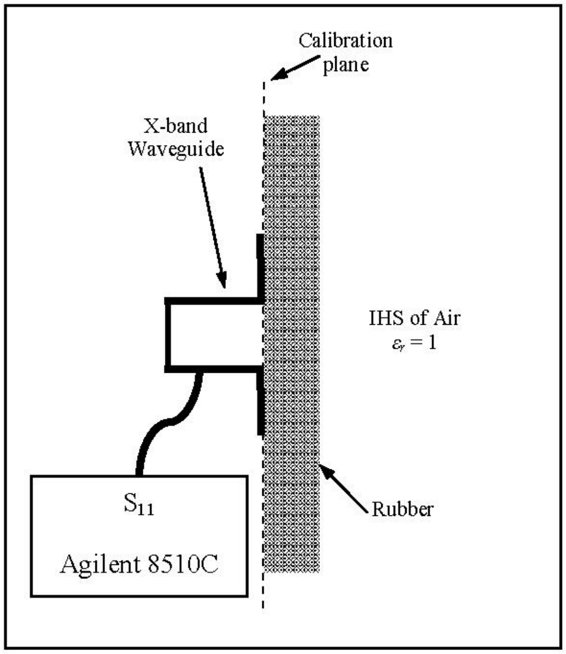

Fig. 4. Measurement setup.

The dielectric constant is found by minimizing the Euclidean distance, i.e. cost function (43), between the measured and simulated reflection coefficients, Γ_m (using VNA) and Γ_s (using the proposed formulation), respectively.

$$e = \sum_f \left|\Gamma_m - \Gamma_s\right|^2 \tag{43}$$

This Euclidean distance is iteratively minimized by varying relative dielectric constant $_r$ using the minimax optimization algorithm [7]. Table 1 shows a solution obtained for measuring dielectric constant of the rubber sheet, with and without taking into account the effect of the higher-order modes. These results are compared to measurements performed using loaded waveguide technique [8-9]. When taking the higher-order modes into consideration, the estimated $_r$ is within less than 1% of the actual $_r$ for the real part and within less than 10% for the imaginary part which is within the accuracy limits of the loaded waveguide technique [8]. These results also show that not taking higher-order modes into account, significant errors are encountered specially in estimating the loss factor.

Table 1. Measured dielectric constant of rubber.

Loaded waveguide technique	This Study	
Actual ε_r	Including HOM	Excluding HOM
7.28 – j 0.275	7.32 – j 0.25	7.45 – j 1.03

V. SUMMARY

Results of formulating reflection properties of an open-ended rectangular waveguide irradiating a stratified dielectric medium were presented. This formulation utilizes Fourier analysis which provides a complete non-approximate solution incorporating higher-order modes. It is shown that the contribution of higher-order modes is frequency and structure dependant. Most importantly, when measuring reflection coefficient from a dielectric slab, the effect of higher-order modes increases significantly when the slab is thin relative to the wavelength. This formulation is used to measure the dielectric constant of thin slab of rubber and significant improvement in measurement accuracy is obtained. Potentially, dielectric constant of a layer in a multilayered structure or the dielectric constants of multiple layers may be obtained using this formulation. Furthermore, if the thickness of dielectric layers is of interest, it may also be accurately estimated. The results of applying this technique to thickness evaluation will be presented in the full paper.

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