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A Novel Morphological Operator to Calculate Euler Number

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Abstract

This paper will introduce a novel morphological operator to calculate the Euler number for binary images. The operator is based on the condition of eight-connectedness for foreground and four-connectedness for background. It is significantly faster than the previous operators.

1. Introduction

In this paper, the morphological operations used in border detection will be discussed. Borders found by computers are usually represented as binary images, so common binary morphological operations such as dilation, erosion, thinning, edge extraction, opening and closing are very useful for border post-processing. Here one advanced operation, Euler number counting, will be discussed. This operator is the base for more advanced operations such as flood-filling, hole filling, and island deleting [1].

2. The EULER number operator

In a binary image, the Euler number is defined as the number of objects minus the number of holes inside the objects.

$$Euler\_Number = \#Objects - \#Holes$$

For example, for Figure 1, the Euler number is 1, while for Figure 2, the Euler number is 0.

When dealing with a square tessellation binary image, the Euler number is different depending on the definition of connectedness. There are three possibilities:

- Four-connectedness: only edge-adjacent cells are considered neighbors.

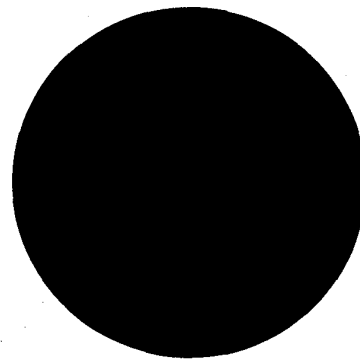- Six-connectedness: two corner-adjacent cells are considered neighbors, also.

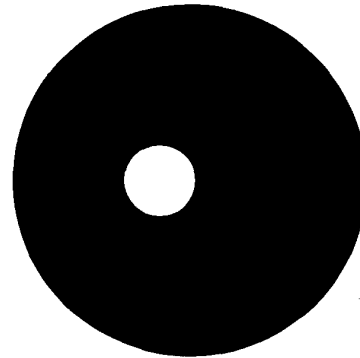**Figure 1. An example of Euler number equals one (One object, zero holes)**

Figure 2. An example of Euler number equals zero (One object, one hole)

- Eight-connectedness: all four corner-adjacent cells are considered neighbors, as well.

For six-connectedness, the two corner cells must be on the same diagonal to ensure symmetry in the relationship.

In this research, four-connectedness for the background, and eight-connectedness for the object is used. This satisfies human intuition about connected components in continuous binary images. For example, a simple closed curve should separate the image into two simply-connected regions. This is called the Jordan curve theorem [2].

The Euler number also satisfies the additive set property. This means that for two images X and Y, with the CONNECTED AND and CONNECTED OR of the two images denoted as $X \wedge Y$ and $X \vee Y$, respectively, then

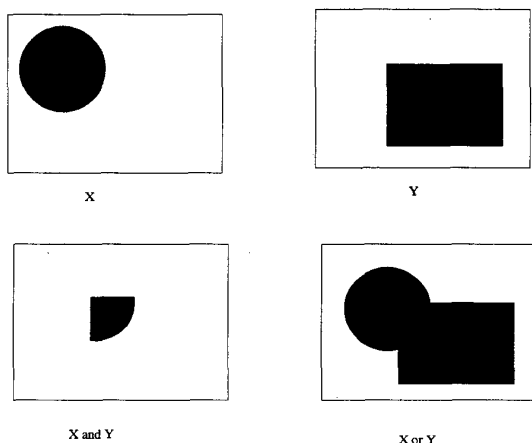$$E(X) + E(Y) = E(X \wedge Y) + E(X \vee Y)$$

as shown in Figure 3. [2]

Figure 3. Additive set property (from [2])

Any measurement that satisfies the additive set property can be calculated by a local counting technique. Consider a 64x64 discrete binary image. The image can be divided into 64 sub-images, counting each scan line as a sub-image. Each scan line is denoted as I_n, where the subscript n is the scan line number. Denote the Euler number of I_n as $E(I_n)$, and the Euler number of the whole image as $E(image)$. Then from the additive set property,

$$\sum_{n=1}^{64} E(I_n) = \sum_{n=1}^{63} E(I_n \wedge I_{n+1}) + E(image)$$

so,

$$E(image) = \sum_{n=1}^{64} E(I_n) - \sum_{n=1}^{63} E(I_n \wedge I_{n+1})$$

$E(I_n)$ is the Euler number of the segment of each sub-image, and it is easy to calculate. $I_n \wedge I_{n+1}$ is the CONNNECTED AND of the two neighboring scan lines. It has different results for different definitions of connectivity. Since the object (foreground) in this study uses eight-connectedness, the CONNECTED AND is different from the ordinary LOGICAL AND. For example, for the two neighboring scan lines shown as Table 1, the result of an ordinary LOGICAL AND between the two pixels of each vertical column is shown as Table 2, and its Euler number is two. The result is correct for four-connectedness CONNNECTED AND, but for eight-connectedness CONNECTED AND, the previous result missed the connectedness between the fourth pixel of row one and the third pixel of row two, and between the fourth pixel of row one and the fifth pixel of row two. The correct result should be as shown in Table 3. There are two additional pixels in the result, the fourth pixel represents the first missed connectedness, and the sixth pixel represents the second missed connectedness. The Euler number is four, and it is the correct result for eight-connectedness. This method of calculating the Euler number is used in this research.

Table 1. A binary image with two rows

1	1	0	1	0	1	0
1	1	1	0	1	1	1

Table 2. The result of logical AND

1	1	0	0	0	1	0

Table 3. The result of eight-connectedness AND

1	1	0	1	0	1	0	1	0

Gray [3] has devised a systematic method of computing the Euler number by matching the logical state of regions of an image to binary patterns. He first defined a set of 2x2 pixel patterns called bit quads as shown in Tables 4- 6.

The Euler number of an image for eight-connectedness can be expressed in terms of the number of bit quad counts in the image as

$$E = \frac{1}{4}[n\{Q1\} - n\{Q3\} - 2n\{QD\}]$$

where $n\{.\}$ means the number of bit quads counted. There are ten different bit quad patterns that need to be counted in Gray's method.

[3] S. B. Gray, "Local Properties of Binary Images in Two Dimensions," *IEEE Trans. Computers*, **Vol. C-20** (1971), pp. 551-561.

Table 4. Q1

0	0	0	1	1	0	0	0
0	1	0	0	0	0	1	0

Table 5. Q3

0	1	1	1	1	1	1	0
1	1	0	1	1	0	1	1

Table 6. QD

0	1	1	0
1	0	0	1

Based on the previous discussion, the Euler number for the previous 64x64 binary image can also be calculated by a local counting technique using the following equation:

$$E(image) = \sum_{n=1}^{64} E(I_n) - \sum_{n=1}^{63} E(I_n \ LOGICAL{-}AND \ I_{n+1})$$

$$-n\{QD\}$$

Where $n\{QD\}$ is the number of bit quad patterns as shown in Table 6 which are missed by the LOGICAL AND operation. This method only needs to count two bit quad patterns, so it is much faster than Gray's method.

3. Test results

This novel algorithm is part of the automatic skin tumor segmentation package the authors have developed [1]. It was tested on thousands of borders with no errors. The source code, written in the C programming language, is available upon request.

4. Discussion and conclusions

The Euler number has many applications in the areas of machine vision, automatic border detection and image understanding. A fast, easy and reliable morphological operator for Euler number counting will improve the performance and reliability for these systems.

References

[1] Z. Zhang, W. V. Stoecker and R. H. Moss, "Border Detection on Digitized Skin Tumor Images," *IEEE Trans. on Medical Imaging*, **Vol. 19** (2000) pp. 1128-1143.

[2] B. K. P. Horn, *Robot Vision* (McGraw-Hill, New York, 1986), pp. 90-102, 161-184.