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**Abstract:** A composite media containing particles with a high internal field of magnetic anisotropy (hexagonal ferrites) useful for numerous EMC applications in a wide frequency band is considered. Effective constitutive parameters of a high-loss composite gyromagnetic media are represented in the Lorentzian form. It is convenient for the numerical analysis using the finite-difference time-domain (FDTD) algorithm with a recursive convolution procedure. The equations for the electric and magnetic field updating in such media are represented.

## INTRODUCTION

A gyromagnetic medium (GM) is a magneto-dielectric medium composed of particles of microwave ferrites. Such media find multiple applications for EMC purposes:

-for design of anechoic chambers for electronic equipment testing;
-in radio electronics for spurious radiation suppression in transmitting devices, and for improved immunity in receiving devices;
-for protecting people in everyday life from harmful radiation (in industry using microwaves, in microwave ovens, medical equipment, mobile telephones, etc.);
-for protecting biological objects in outer space conditions, and
-for design of computer processors operating at GHz frequencies.

A gyromagnetic medium due to the spin mechanism of interaction with the electromagnetic field has the following properties [1]:

• conductivity losses are negligible;
• magnetic losses are very high;
• frequency-selective absorption of energy takes place at ferromagnetic (or antiferromagnetic) resonance;
• natural ferromagnetic resonance (NFMR) phenomenon takes place in ferrites with high internal field of crystallographic anisotropy;
• permeability is a tensor (in general case).

An individual GM particle absorbs electromagnetic energy instantaneously and is insensitive to the phase and polarization of the electromagnetic field. "Non-current", "non-inertial", and "non-phase" mechanisms of GM and electromagnetic field interaction determine the effect of all-directional and all-wave matching of the GM impedance with that of free space [2].

Wideband composite GM using a mixture of hexagonal ferrite (HF) powders of various chemical content have been designed for application in the frequency range from 2.5 to 170 GHz [3]. Due to the phenomenon of natural ferromagnetic resonance (NFMR) in HF particles, coatings and devices (waveguide filters for harmonics, isolators, matched loads) operate without external magnets. "All-mode" absorbers have good electrophysical parameters, simple design, and their production is comparatively easy and low-cost [4].

## GYROMAGNETIC COMPOSITE MEDIA FDTD ANALYSIS

Effective medium theory (EMT), based on introducing constitutive parameters averaged in space and time, allows using methods of computational electrodynamics for design of devices on the basis of GM. The robust finite-difference time-domain (FDTD) technique can be efficient for analysis of such structures.

Two alternative FDTD approaches are known to be applicable for dispersive media (either dielectric or magnetic). The first uses recursive convolution of constitutive parameters in the time domain and corresponding field component, and the second is based on an auxiliary differential equation for the corresponding field vectors [5,6]. In the case of magnetic media, either the Landau-Lifshitz magnetization vector equation of motion with a damping term in one of the convenient forms is used [7,8], or the known Polder's tensor and recursive convolution is applied [9].

The effective electrodynamic parameters of composite media containing particles with high a internal field of magnetic anisotropy (hexagonal ferrites) can be represented in a form convenient for further FDTD analysis using recursive convolution computations.

For a dispersive magneto-dielectric medium, magnetic flux and displacement vectors in the frequency domain have time-domain forms as convolutions

$$\vec{D}(\omega) = \varepsilon(\omega)\vec{E}(\omega) \Leftrightarrow \vec{D}(t) = \varepsilon(t) * \vec{E}(t), \qquad (1)$$

$$\vec{B}(\omega) = \mu(\omega)\vec{H}(\omega) \Leftrightarrow \vec{B}(t) = \mu(t) * \vec{H}(t). \qquad (2)$$

Frequency-dependent permittivity and permeability are related to the time-domain form via Fourier (or Laplace) transforms

$$\varepsilon(\omega) = \varepsilon_0(\varepsilon_\infty + \chi_\varepsilon(\omega)) \Leftrightarrow \varepsilon(t) = \varepsilon_0(\varepsilon_\infty \delta(t) + \chi_\varepsilon(t)), \qquad (3)$$

$$\mu(\omega) = \mu_0(1 + \chi_\mu(\omega)) \Leftrightarrow \mu(t) = \mu_0(\delta(t) + \chi_\mu(t)), \qquad (4)$$

where $\varepsilon_\infty$ is the "optic" relative dielectric constant, $\chi_{\varepsilon,\mu}(t)$ are the dielectric or magnetic susceptibility kernels, correspondingly. For the

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materials that satisfy Kramers-Kroenig causality relations [9], the complex magnetic and dielectric susceptibility can be represented as linear-fractional functions of frequency,

$$\chi(\omega)=\frac{P_m(\omega)}{Q_n(\omega)},\ n>m. \tag{5}$$

Then Heaviside's formula for the time-domain Laplace transform can be applied,

$$\chi(t)=\sum_i \frac{P_m(p_i)}{Q'_n(p_i)}e^{p_i t}\cdot u(t), \tag{6}$$

where $p_i$ are complex roots of the equation

$$Q_n(p_i)=0, \tag{7}$$

and $u(t)$ is a unit step-function.

Hexagonal ferrite mixture constitutive parameters can be represented in Lorentzian form. It can be a one-pole (one-resonance) curve if the material is a single-component (with one type of a hexagonal ferrite filler), or multi-pole (with several resonance peaks) if the material is multi-component (contains mixture of ferrites of different types). For a gyromagnetic composite media the magnetic susceptibility in frequency domain can be obtained using effective media theory approach (generalized J.C. Maxwell Garnett equation as in [1]),

$$\chi_\mu(\omega)=\frac{2}{3}\sum_{i=1}^{N_F}\frac{f_i\omega_{Mi}\overline{\omega}_{Ai}}{\overline{\omega}_{Ai}^2+2j\omega\omega_{Li}-\omega^2} \tag{8}$$

where

$$\omega_{Mi}=\gamma\mu_0 M_{Si};\ \overline{\omega}_{Ai}=\gamma\mu_0\overline{H}_{Ai};\ \omega_{Li}=\gamma\mu_0\Delta H_i/2.$$

In (8), $f_i$ is the volumetric fraction of ferrite powder of type $i$, and $M_{Si}$ is the saturation magnetization. The parameter $\overline{\omega}_{Ai}$ is an average value of angular frequency corresponding to the scatter of values of internal field of crystallographic anisotropy $\overline{H}_{Ai}$ for a ferrite powder of type $i$. This value determines the NFMR frequency for a conglomerate of ferrite particles of type $i$, and $\omega_{Li}$ is the loss parameter describing the width of the corresponding NFMR line. The parameters $\overline{\omega}_{Ai}$ and $\omega_{Li}$ are found using methods of mathematical statistics if the corresponding probability density function (for example, Cauchy distribution) is known [4].

Every Lorentzian peak can be considered as either "narrowband" or "wideband", depending on the relation between the resonance frequency and width of the corresponding resonance curve. It is shown below that recursive convolution algorithms in these two cases differ.

## "NARROWBAND" GYROMAGNETIC COMPOSITE MEDIA

If the material is "narrowband" ($\overline{\omega}_{Ai}>\omega_{Li}$), then poles of function (8) are complex,

$$p_{i1,2}=-\omega_{Li}\pm j\sqrt{\overline{\omega}_{Ai}^2-\omega_{Li}^2}, \tag{9}$$

and time-domain susceptibility kernel according to formula (6) is

$$\chi_\mu(t)=\sum_{i=1}^{N_F}G_i\exp(-\omega_{Li}t)\sin(\omega_i t)u(t), \tag{10}$$

where

$$G_i=\frac{2}{3}\cdot\frac{f_i\omega_{Mi}\overline{\omega}_{Ai}}{\omega_i}, \tag{11}$$

and

$$\omega_i=\sqrt{\overline{\omega}_A^2-\omega_{Li}^2}. \tag{12}$$

The susceptibility kernel can be represented by a complex exponential form as

$$\chi_\mu(t)=\sum_{i=1}^{N_F}\mathrm{Re}(\tilde{\chi}_{\mu i})=\sum_{i=1}^{N_F}\mathrm{Re}\{G_i\exp(\tilde{\gamma}_i t)\}, \tag{13}$$

where

$$\tilde{\gamma}_i=-\omega_{Li}+j\omega_i. \tag{14}$$

The time-dependent susceptibility kernel (13) can be substituted in (4) and (2), and the corresponding magnetic flux vector can be discretized as

$$\vec{B}^n(m)=\mu_0\vec{H}^n(m)+\mu_0\sum_{i=1}^{N_F}\sum_{k=0}^{n-1}\vec{H}^{n-k}(m)\cdot\chi_{\mu i}^k, \tag{15}$$

where $m=(x,y,z)$ corresponds to the discrete integer coordinates of a mesh, and $n$ is the discrete time-step. The second term in (15) is the discrete convolution function, where

$$\chi_{\mu i}^k=\int_{k\Delta t}^{(k+1)\Delta t}\chi_{\mu i}(\tau)d\tau. \tag{16}$$

Then, the Faraday's Law Maxwell equation discretized according to the FDTD algorithm can be written as

$$\vec{H}^{n+1}(m)=A_\mu\vec{H}^n(m)-B_\mu\nabla_d\times\vec{E}^n[m,m+1]]-\mu_0\sum_{i=1}^{N_F}\mathrm{Re}\left(\tilde{\vec{\Phi}}_i^n(m)\right)\cdot \tag{17}$$

In formula (17) the coefficients $A_\mu$ and $B_\mu$ are

$$A_\mu = \frac{2 - \sum_{i=1}^{N_F} \chi_{\mu i}^0 \Delta t}{2 + \sum_{i=1}^{N_F} \chi_{\mu i}^0 \Delta t} \quad (18)$$

and

$$B_\mu = \frac{2\Delta t}{2 + \sum_{i=1}^{N_F} \chi_{\mu i}^0 \Delta t}. \quad (19)$$

The symbol $\nabla_d \times$ is the central-difference discrete curl operator.

The complex discrete convolution function is

$$\tilde{\vec{\Phi}}_i^n(m) = \sum_{k=0}^{n-1} \vec{H}^{n-k}(m) \Delta\tilde{\chi}_{\mu i}^{k+1}, \quad (20)$$

where

$$\Delta\tilde{\chi}_{\mu i}^0 = \tilde{\chi}_{\mu i}^0 - \tilde{\chi}_{\mu i}^1 = \int_{0t}^{\Delta t} \tilde{\chi}_{\mu i}(\tau)d\tau - \int_{\Delta t}^{2\Delta t} \tilde{\chi}_{\mu i}(\tau)d\tau \quad (21)$$

and

$$\Delta\tilde{\chi}_{\mu i}^{k+1} = \tilde{\chi}_{\mu i}^{k+1} - \tilde{\chi}_{\mu i}^{k+2} = \int_{(k+1)\Delta t}^{(k+2)\Delta t} \tilde{\chi}_{\mu i}(\tau)d\tau - \int_{(k+2)\Delta t}^{(k+3)\Delta t} \tilde{\chi}_{\mu i}(\tau)d\tau. \quad (22)$$

Because of the exponential function, $\Delta\tilde{\chi}_{\mu i}^{k+1}$ can be calculated recursively,

$$\Delta\tilde{\chi}_{\mu i}^{k+1} = \exp(\tilde{\gamma}_i k\Delta t)\Delta\tilde{\chi}_{\mu i}^k \quad (23)$$

and (20 ) is also represented in recursive form

$$\tilde{\Phi}_i^{n+1}(m) = E^{n+1}(m)\Delta\tilde{\chi}_i^0 + \exp(\tilde{\gamma}_i \cdot \Delta t) \cdot \tilde{\Phi}_i^n(m) \quad (24)$$

If real and imaginary parts in (24) are separated, two related recursive equations are obtained

$$\mathrm{Re}\left(\tilde{\vec{\Phi}}_i^{n+1}(m)\right) = \vec{H}^{n+1} \cdot \mathrm{Re}(\tilde{\chi}_i^0) + e^{-\omega_{Li} \cdot t}\left[\mathrm{Re}\left(\tilde{\vec{\Phi}}_i^n(m)\right) \cdot \cos\omega_i t - \mathrm{Im}\left(\tilde{\vec{\Phi}}_i^n(m)\right) \cdot \sin\omega_i t\right] \quad (25)$$

and

$$\mathrm{Im}\left(\tilde{\vec{\Phi}}_i^n(m)\right) = \vec{H}^n \cdot \mathrm{Im}(\tilde{\chi}_i^0) + e^{-\omega_{Li} \cdot t}\left[\mathrm{Im}\left(\tilde{\vec{\Phi}}_i^{n-1}(m)\right) \cdot \cos\omega_i t + \mathrm{Re}\left(\tilde{\vec{\Phi}}_i^{n-1}(m)\right) \cdot \sin\omega_i t\right] \quad (26)$$

The complex susceptibility kernels in (21) can be represented as (13), and the result upon integration for a narrowband Lorentzian magnetic material is

$$\Delta\tilde{\chi}_{\mu i}^0 = -\frac{G_i}{\tilde{\gamma}_i}(1 - \exp(\tilde{\gamma}_i k\Delta t))^2. \quad (27)$$

The "static" susceptibility integral

$$\chi_i^0 = \mathrm{Re}\int_0^{\Delta t} \tilde{\chi}_i(t)dt \quad (28)$$

yields in

$$\chi_i^0 = \frac{G_i}{|\tilde{\gamma}_i|^2}\begin{pmatrix} \overline{\omega}_{Ai}(1 - \exp(-\omega_{Li}\Delta t))\cos\overline{\omega}_{Ai}\Delta t - \\ \omega_{Li}(\exp(-\omega_{Li}\Delta t)\sin\overline{\omega}_{Ai}\Delta t \end{pmatrix}. \quad (29)$$

The value $\Delta\tilde{\chi}_{\mu i}^0$ is substituted in (25) and (26) for calculating complex discrete convolution function, and $\chi_i^0$ is used for calculating coefficients $A_\mu$ and $B_\mu$ in FDTD updating equation (17). From (25) and (26), it follows that to apply a recursive convolution procedure to "narrowband" Lorentzian materials and calculate the function $\tilde{\Phi}_i^{n+1}(m)$, it is necessary to save real and imaginary parts of the function at time steps *n* and *(n-1)*.

### "WIDEBAND" GYROMAGNETIC COMPOSITE MEDIA

The case for $\overline{\omega}_{Ai} \le \omega_{Li}$, i.e., the material is essentially lossy, requires further work. In this case, the poles of function (8) are real,

$$p_{i1,2} = -\omega_{Li} \pm \sqrt{\omega_{Li}^2 - \overline{\omega}_{Ai}^2}. \quad (30)$$

Then, the susceptibility time-domain kernel for the frequency-domain function (8) found according to Heaviside's formula (6) is

$$\chi_\mu(t) = \sum_{i=1}^{N_F} G_i^W \cdot \exp(-\omega_{Li}t)\sinh(\omega_i^W \cdot t)u(t) \quad (31)$$

where

$$\omega_i^W = \sqrt{\omega_{Li}^2 - \overline{\omega}_A^2}, \quad (32)$$

and

$$G_i^W = \frac{2}{3} \cdot \frac{f_i \omega_{Mi} \overline{\omega}_{Ai}}{\omega_i^W}. \quad (33)$$

Equation (31) describes the susceptibility kernel as a non-oscillating damping function of time. In this case, according to (28), the "static" susceptibility is real

$$\chi_{\mu i}^0 = \frac{G_i^W}{2}\left(\frac{e^{h_i \Delta t} - 1}{h_i} + \frac{e^{-g_i \Delta t} - 1}{g_i}\right), \quad (34)$$

where the following notations:

$$h_i = \omega_i^W - \omega_{Li} \quad (35)$$

and

$$g_i = \omega_i^W + \omega_{Li}, \quad (36)$$

are used.

Calculating the analogous integral for $\chi^1_{\mu i}$ and substituting in (21), the following equation is obtained

$$\Delta\chi^0_{\mu i} = \Delta\chi^0_{\mu i(1)} + \Delta\chi^0_{\mu i(2)}, \quad (37)$$

where

$$\Delta\chi^0_{\mu i(1)} = -\frac{G_i^W}{2}\left(\frac{\left(e^{h_i\Delta t}-1\right)^2}{h_i}\right), \quad (38)$$

$$\Delta\chi^0_{\mu i(2)} = -\frac{G_i^W}{2}\left(\frac{\left(e^{-g_i\Delta t}-1\right)^2}{g_i}\right). \quad (39)$$

By taking several simple integrals for finding values $\Delta\chi^k_{\mu i}$ used in convolution procedure it can be shown that

$$\Delta\chi^{k+1}_{\mu i(1)} = \Delta\chi^k_{\mu i(1)} \cdot e^{h_i\Delta t} \quad (40)$$

$$\Delta\chi^{k+1}_{\mu i(2)} = \Delta\chi^k_{\mu i(2)} \cdot e^{-g_i\Delta t} \quad (41)$$

Then the summation in (19) can be represented in the form of two separate recursively calculated terms

$$\vec{\Phi}^n_i = \vec{\Phi}^n_{i(1)} + \vec{\Phi}^n_{i(2)}, \quad (42)$$

$$\vec{\Phi}^n_{i(1)} = \vec{H}^n \Delta\chi^0_{\mu i(1)} + e^{h_i\Delta t} \cdot \vec{\Phi}^{n-1}_{i(1)}, \quad (43)$$

$$\vec{\Phi}^n_{i(2)} = \vec{H}^n \Delta\chi^0_{\mu i(2)} + e^{-g_i\Delta t} \cdot \vec{\Phi}^{n-1}_{i(2)}. \quad (44)$$

Formulas (42)-(44) are used for updating H-field components using algorithm (17).

The "wideband" case also requires additional memory for saving convolution function terms at steps *n* and *n-1*.

The composite material can have complex frequency characteristics combining "narrowband" resonance peaks and "wideband" regions, and for such materials both algorithms will be used.

If the magneto-dielectric material under study exhibits frequency dependence of dielectric properties in the same frequency range as magnetic properties, then a combined algorithm for both electric field and magnetic field updating using a recursive convolution technique can be used. The updating equations and recursive terms are analogous to those presented for magnetic field

$$\vec{E}^{n+1}(m) = A_\varepsilon \vec{E}^n(m) - B_\varepsilon\left(\nabla_d \times \vec{H}^n[m,m+1] + \vec{J}^n(m)\right) - \varepsilon_0 \sum_{i=1}^{N_F} \mathrm{Re}\left(\vec{\Psi}^n_i(m)\right) \quad (45)$$

Coefficients $A_\varepsilon$ and $B_\varepsilon$ are found analogous to the coefficients in the magnetic case, but can contain conductivity losses.

## CONCLUSION

FDTD modeling of isotropic composite gyromagnetic medium can be accomplished using a recursive convolution procedure, if the magnetic susceptibility kernel of the medium is represented as a sum of complex exponents of time. The multipole Lorentzian model is used for the representation of magnetic susceptibility of the gyromagnetic composite multi-component media. Two types of Lorentzian models are considered – "narrowband" and "wideband" – depending on the ratio of the material bandwidth to the resonance frequency. Formulas using recursive convolution in the FDTD updating algorithm are obtained. Lorentzian models of material parameters representation are general and allow taking into account resonance effects in materials at microwave, mm-wave, IR, and optical frequency bands.

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