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Hamiltonian Theory Based Coordinated Nonlinear Control of Generator Excitation and STATCOMs

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Abstract—A coordinated controller for generator excitation and STATCOMs is studied based on the Hamiltonian function method. The Hamiltonian realization structure for multimachine power systems including STATCOMs is developed leading to a proposed coordinated scheme of excitation control and STAT-COM control. Simulation results illustrate the effectiveness of the proposed control strategy.

I. INTRODUCTION

Power oscillation damping is a critical issue for power system dynamic security. These oscillations can occur due to contingencies such as sudden faults or topology changes. Traditionally, generator field excitation control has been used to enhance transient stability. In recent years however, Flexible AC Transmission Systems (FACTS) that utilize power electronics devices have offered an alternative means to mitigate power system oscillations [1]. Typically, both generation excitation and FACTS control has employed linear control methods to improve transient performance. However, as power systems become increasingly stressed, nonlinear behavior begins to dominate their transient response. Therefore nonlinear controllers are becoming increasingly attractive to provide better oscillation damping.

In recent years, Hamiltonian system control has attracted considerable attention [2]-[11]. In [2]-[3], the generalized portcontrolled hamiltonian (PCH) system has been proposed to solve stabilization problems for general dynamics systems. This introduction led to considerable attention in applying generalized Hamiltonian theory into power system control and has yeilded several promising results [4] -[11]. In [4]-[8], the Hamiltonian system has been employed for generation excitation control. In [9], the passivation controller design for turbo-generators based on PCH has been proposed. In [10], the port-controlled Hamiltonian model is revised for application to a TCSC. In [11], a robust coordinated design is first proposed with for generation excitation and STATCOM control based on the generalized Hamiltonian theory, but without full consideration of the STATCOM model and the network interface between the STATCOM and the power system.

In this paper, we will extend the previous results and present the development of coordinated controllers in a multimachine power system with STATCOMs. First, the Hamiltonian realization for a multimachine power system with STATCOMs is developed. Secondly, the STATCOM model is extended to a higher order state-space model as opposed for the first order model used in [11]. Finally the excitation and STATCOM control is developed to stabilize the power system.

II. PORT-CONTROLLED HAMILTONIAN SYSTEM

Consider the Hamiltonian system [2] described as

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$$= [J(x) - R(x)]\frac{\partial H(x)}{\partial x} + g(x)u \tag{1}$$

$$y = g^{T}(x)\frac{\partial H(x)}{\partial x}$$
(2)

where $x^T = [x_1, x_2, \dots, x_n]^T \in \mathbb{R}^n$, $u, y \in \mathbb{R}^s$ denote control input and output. The matrix J(x) is a skew-symmetric, i.e. $J(x) = -J^T(x)$ and R(x) is a non-negative symmetric matrix. In general, the Hamiltonian function H(x) represents the total stored energy of the system. If the Lyapunov stability criterion is satisfied, then this function can also serve as a Lyapunov function.

Port-controlled Hamiltonian systems with dissipation satisfy the following the power balance equation:

$$\dot{H} = -\frac{\partial^T H}{\partial x}(x)R(x)\frac{\partial H}{\partial x} + u^T y$$
(3)

where $u^T y$ is the power externally supplied to the system and the first term on the right-hand side represents the energy dissipation due to the resistive elements in the system. If a system can be formulated in Hamiltonian form, then the Hamiltonian function can guarantee local stability. However, there is no general method to construct such a Hamiltonian function, and this is often the most difficult and important step in the controller development. References [5]-[11] have explored the Hamiltonian function and established conditions under which stability can satisfied in power systems. In this paper, we focus on extending the a Hamiltonian realization to power systems with STATCOMs.

III. HAMILTONIAN REALIZATION OF MULTI-MACHINE MULTI-STATCOM POWER SYSTEMS

A. Power Network Equations

Consider the power network which is modeled by n generators and m STATCOMs. Assume that the bus admittance matrix has been reduced to the generator internal buses with the STATCOM ac terminal buses explicitly retained as shown

in Fig.1. In this case, the reduced bus admittance matrix equation can be written as:



Fig. 1. Bus admittance matrix depiction

$$\begin{bmatrix} \mathbf{I}_{\mathbf{G}} \\ \mathbf{I}_{\mathbf{F}} \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{\mathbf{G}\mathbf{G}} & \mathbf{Y}_{\mathbf{G}\mathbf{F}} \\ \mathbf{Y}_{\mathbf{F}\mathbf{G}} & \mathbf{Y}_{\mathbf{F}\mathbf{F}} \end{bmatrix} \begin{bmatrix} \mathbf{E}_{\mathbf{G}} \\ \mathbf{V}_{\mathbf{F}} \end{bmatrix}$$
(4)

where $\mathbf{E}_{\mathbf{G}} \in C^n$ is the generator bus voltage vector and $\mathbf{V}_{\mathbf{F}} \in C^m$ is the STATCOM terminal bus voltage vector. The currents $\mathbf{I}_{\mathbf{G}} \in C^n$ and $\mathbf{I}_{\mathbf{F}} \in C^m$ are the generator injected current vector and the STATCOM injected current vector respectively. Note that $\mathbf{E}_{\mathbf{G}_i} = E'_{q_i} \angle \delta_i, i = 1 \cdots n$ and $\mathbf{I}_{\mathbf{F}_j} = i_{d_j} + ji_{q_j}, j = 1 \cdots m$. E'_q and δ are the state variables of the generator and i_d and i_q are the state variables of the STATCOM.

From the second equation of (4), the STATCOM terminal bus voltage equals

$$\mathbf{V}_{\mathbf{F}} = \mathbf{Y}_{\mathbf{FF}}^{-1}(\mathbf{I}_{\mathbf{F}} - \mathbf{Y}_{\mathbf{FG}}\mathbf{E}_{\mathbf{G}})$$

Substituting this expression into the first equation of (4) yields:

$$\mathbf{I}_{\mathbf{G}} = \mathbf{Y}_{\mathbf{G}} \mathbf{E}_{\mathbf{G}} + \mathbf{Y}_{\mathbf{F}} \mathbf{I}_{\mathbf{F}}$$
(5)

where

$$\begin{aligned} \mathbf{Y}_{\mathbf{G}} &= \mathbf{G}_{\mathbf{G}} + j\mathbf{B}_{\mathbf{G}} &= \mathbf{Y}_{\mathbf{G}\mathbf{G}} - \mathbf{Y}_{\mathbf{G}\mathbf{F}}\mathbf{Y}_{\mathbf{F}\mathbf{F}}^{-1}\mathbf{Y}_{\mathbf{F}\mathbf{G}} \\ \mathbf{Y}_{\mathbf{F}} &= \mathbf{G}_{\mathbf{F}} + j\mathbf{B}_{\mathbf{F}} &= \mathbf{Y}_{\mathbf{G}\mathbf{F}}\mathbf{Y}_{\mathbf{F}\mathbf{F}}^{-1} \end{aligned}$$

B. Single-Axis Generator Model

The third-order single-axis generator model is given by

$$\hat{\delta}_i = \omega_i - \omega_s \tag{6}$$

$$\frac{m_i}{\omega_s}\dot{\omega}_i = P_{m_i} - P_{e_i} - D_i \frac{\omega_i - \omega_s}{\omega_s}$$
(7)

$$\Gamma'_{d0_i} \dot{E'}_{q_i} = -E'_{q_i} - I_{d_i} (x_{d_i} - x'_{d_i}) + E^s_{fd_i} + u_{fi} \quad (8)$$

where

2

$$P_{e_i} = Re\left(\mathbf{E}_{\mathbf{G}_i}\mathbf{I}_{\mathbf{G}}^*\right) = P_{eG_i} + P_{eF_i} \qquad (9)$$

$$I_{d_i} = Im \left(\mathbf{E}_{\mathbf{G}_i} \mathbf{I}_{\mathbf{G}}^{\star} \right) / E'_{q_i} = I_{dG_i} + I_{dF_i}$$
(10)

and δ is the rotor angle, ω is the rotor speed, and E'_q is the quadrature-axis voltage behind transient reactance. The subscript *i* denotes the *i*th generator. The constant $\omega_s = 2\pi f_s$ is the synchronous speed. The powers P_m and P_e are the mechanical and electrical powers respectively, and constants D and M are the damping coefficient and inertia constant. The reactances x_d and x'_d are the direct axis reactance and direct axis transient reactance respectively. E_{fd} and E_{fd}^s are the field voltage and the steady-state field voltage. The input u_f is the field voltage control input, where $E_{fd} = E_{fd}^s + u_f$.

From equations (4), (9), and (10), the active powers injected by the generators and STATCOMs are

$$P_{eG_i} = E'_{q_i} \sum_{j=1}^{n} E'_{q_j} \left(G_{G_{ij}} \cos \delta_{ij} + B_{G_{ij}} \sin \delta_{ij} \right)$$
$$P_{eF_i} = E'_{q_i} \sum_{j=1}^{m} \left\{ \left(G_{F_{ij}} \cos \delta_i + B_{F_{ij}} \sin \delta_i \right) i_{d_j} \right.$$
$$\left(G_{F_{ij}} \sin \delta_i - B_{F_{ij}} \cos \delta_i \right) i_{q_j} \right\}$$

where $\delta_{ij} = \delta_i - \delta_j$.

The direct-axis currents injected by the generators and STATCOMs are:

$$I_{dG_i} = \sum_{j=1}^{n} E'_{q_j} \left(G_{G_{ij}} \sin \delta_{ij} - B_{G_{ij}} \cos \delta_{ij} \right)$$
$$I_{dF_i} = \sum_{j=1}^{m} \left\{ \left(G_{F_{ij}} \sin \delta_i - B_{F_{ij}} \cos \delta_i \right) i_{d_j} - \left(G_{F_{ij}} \cos \delta_i + B_{F_{ij}} \sin \delta_i \right) i_{q_j} \right\}$$

C. The STATCOM model

The STATCOM equivalent circuit model is shown in Fig. 2. The model of the j-th STATCOM in the dq reference frames is given by:

$$\frac{d}{dt}i_{d_j} = -a_j i_{d_j} + b_j i_{q_j} + c_j (u_{d_j} - V_{d_j})$$
(11)

$$\frac{d}{dt}i_{q_j} = -a_j i_{q_j} - b_j i_{d_j} + c_j (u_{q_j} - V_{q_j})$$
(12)

where $a_j = \frac{\omega_s R_{s_j}}{L_{s_j}}$, $b_j = \omega$, $c_j = \frac{\omega_s}{L_{s_i}}$, $j = 1 \cdots m$, and i_{d_j} and i_{q_j} are the injected dq STATCOM currents. The parameters R_{s_j} and L_{s_j} are the coupling transformer resistance and inductance respectively. The STATCOM terminal bus voltage is $\mathbf{V_{F_j}} = V_{d_i} + jV_{q_j}$. The u_d and u_q are the dq components of the internal voltages $\mathbf{E_{F_j}}$ of the STATCOM voltage source converter (VSC).



Fig. 2. STATCOM Equivalent Circuit

D. The Hamiltonian function

A Hamiltonian function H(x) can be given by:

$$H(x) = \sum_{k=1}^{6} H_k(x)$$
(13)

where

$$H_{1}(x) = \frac{1}{2} \sum_{i=1}^{n} \frac{M_{i}}{\omega_{s}} (\omega_{i} - \omega_{s})^{2}$$

$$H_{2}(x) = -\sum_{i=1}^{n} P_{m_{i}}^{s'} (\delta_{i} - \delta_{i}^{s})$$

$$H_{3}(x) = -\frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} (E'_{q_{i}} E'_{q_{j}} B_{G_{ij}} \cos \delta_{ij})$$

$$H_{4}(x) = \frac{1}{2} \sum_{i=1}^{n} \frac{1}{x_{d_{i}} - x'_{d_{i}}} (E'_{q_{i}} - E_{fdi}^{s'})^{2}$$

$$H_{5}(x) = -\sum_{i=1}^{n} \sum_{j=1}^{m} \{E'_{q_{i}} B_{F_{ij}} (\cos \delta_{i} i_{d_{j}} + \sin \delta_{i} i_{q_{j}}) | S \}$$

$$H_{6}(x) = \frac{1}{2} \sum_{j=1}^{m} \{(i_{d_{j}} - i_{d_{j}}^{s})^{2} + (i_{q_{j}} - i_{q_{j}}^{s})^{2} \}$$

where H_1 denotes the rotating kinetic energy, $H_2 - H_6$ are terms of the potential energy. The superscript *s* denotes the value at the equilibrium point. Let the equilibrium point be expressed as $(\delta_i^s, \omega_i^s, E_{q_i}')$ for the generators and $(i_{d_i}^s, i_{q_i}^s)$ for the STATCOMs. Then,

$$P_{m_i}^s = P_{m_i}^{s\prime} + P_{m_i}^{s\prime\prime} = P_{e_i}^s \tag{14}$$

$$E_{fd_i}^s = E_{fd_i}^{s\prime} + E_{fd_i}^{s\prime\prime} = E_{q_i}^{\prime s} + I_{d_i}^s (x_{d_i} - x_{d_i}^\prime)$$
(15)

where

$$P_{m_{i}}^{s'} = E_{q_{i}}^{\prime s} \sum_{j=1}^{n} E_{q_{j}}^{\prime s} B_{G_{ij}} \sin \delta_{ij}^{s} + E_{q_{i}}^{\prime s} \sum_{j=1}^{n} B_{F_{ij}} (\sin \delta_{i}^{s} i_{d_{j}}^{s} - \cos \delta_{i}^{s} i_{q_{j}}^{s}) P_{m_{i}}^{s''} = E_{q_{i}}^{\prime s} \sum_{j=1}^{n} E_{q_{j}}^{\prime s} G_{G_{ij}} \cos \delta_{ij}^{s} + E_{q_{i}}^{\prime s} \sum_{j=1}^{n} G_{F_{ij}} (\cos \delta_{i}^{s} i_{d_{j}}^{s} + \sin \delta_{i}^{s} i_{q_{j}}^{s})$$

and

$$E_{fd_{i}}^{s\prime} = E_{q_{i}}^{\prime s} - (x_{d_{i}} - x_{d_{i}}^{\prime}) \sum_{j=1}^{n} E_{q_{j}}^{\prime s} B_{G_{ij}} \cos \delta_{ij}^{s}$$
$$-(x_{d_{i}} - x_{d_{i}}^{\prime}) \sum_{j=1}^{n} B_{F_{ij}} (\cos \delta_{i}^{s} i_{d_{j}}^{s} + \sin \delta_{i}^{s} i_{q_{j}}^{s})$$
$$E_{fd_{i}}^{s\prime\prime} = (x_{d_{i}} - x_{d_{i}}^{\prime}) \sum_{j=1}^{n} E_{q_{j}}^{\prime s} G_{G_{ij}} \sin \delta_{ij}^{s}$$
$$+(x_{d_{i}} - x_{d_{i}}^{\prime}) \sum_{j=1}^{n} B_{F_{ij}} (\sin \delta_{i}^{s} i_{d_{j}}^{s} - \cos \delta_{i}^{s} i_{q_{j}}^{s})$$

Differentiating the function H(x) results in

$$\begin{aligned} \frac{\partial H}{\partial \delta_i} &= -P_{m_i}^{s\prime} + E_{q_i}^{\prime} \sum_{j=1}^n E_{q_j}^{\prime} B_{G_{ij}} \sin \delta_{ij} \\ &+ E_{q_i}^{\prime} \sum_{j=1}^m B_{F_{ij}} (\sin \delta_i i_{d_j} - \cos \delta_i i_{q_j}) \\ \frac{\partial H}{\partial \omega_i} &= \frac{M_i}{\omega_s} (\omega_i - \omega_s) \\ \frac{\partial H}{\partial E_{q_i}^{\prime}} &= -\sum_{j=1}^n E_{q_i}^{\prime} B_{G_{ij}} \cos \delta_{ij} + \frac{1}{x_{d_i} - x_{d_i}^{\prime}} (E_{q_i}^{\prime} - E_{fd_i}^{s\prime}) \\ &- \sum_{j=1}^m B_{F_{ij}} (\cos \delta_i i_{d_j} + \sin \delta_i i_{q_j}) \\ \frac{\partial H}{\partial i_{d_j}} &= i_{d_j} - i_{d_j}^s - \sum_{i=1}^n E_{q_i}^{\prime} B_{F_{ij}} \cos \delta_i \\ \frac{\partial H}{\partial i_{q_j}} &= i_{q_j} - i_{q_j}^s - \sum_{i=1}^n E_{q_i}^{\prime} B_{F_{ij}} \sin \delta_i \end{aligned}$$

Therefore, the generator model can be written as

$$\dot{\delta}_{i} = \frac{\omega_{s}}{M_{i}} \frac{\partial H}{\partial \omega_{i}}$$

$$M_{i}\dot{\omega}_{i} = -\omega_{s} \frac{\partial H}{\partial \delta_{i}} - \frac{D_{i}\omega_{s}}{M_{i}} \frac{\partial H}{\partial \omega_{i}}$$

$$+\omega_{s} \left(P_{mi}^{s\prime\prime} - E_{qi}^{\prime} \sum_{j=1}^{n} G_{G_{ij}} E_{qj}^{\prime} \cos \delta_{ij}$$

$$-E_{qi}^{\prime} \sum_{j=1}^{m} G_{F_{ij}} (\cos \delta_{i} i_{dj} + \sin \delta_{i} i_{qj}) \right)$$

$$T_{d0}^{\prime} \dot{E}_{qi}^{\prime} = -(x_{di} - x_{di}^{\prime}) \frac{\partial H}{\partial E_{qi}^{\prime}} + u_{fi}$$

$$(16)$$

$$d \partial L' q_{i} = (x_{d_{i}} - x_{d_{i}}') \frac{\partial E'_{q_{i}}}{\partial E'_{q_{i}}} + a_{f_{i}} + C (x_{d_{i}} - x'_{d_{i}}) \sum_{j=1}^{m} G_{F_{ij}} (\sin \delta_{i} i_{d_{j}} - \cos \delta_{i} i_{q_{j}}) + E_{fd_{i}}^{s''} - (x_{d_{i}} - x'_{d_{i}}) \sum_{j=1}^{n} E'_{q_{i}} G_{G_{ij}} \sin \delta_{ij}$$
(18)

and the STATCOM model is

$$\frac{d}{dt}i_{d_{j}} = -a_{j}\frac{\partial H}{\partial i_{d_{j}}} + b_{j}\frac{\partial H}{\partial i_{q_{j}}} + c_{j}(u_{d_{j}} - V_{d_{j}})$$

$$-a_{j}(i_{d_{j}}^{s} + \sum_{i=1}^{n} E'_{q_{i}}B_{F_{ij}}\cos\delta_{i})$$

$$+b_{j}(i_{q_{j}}^{s} + \sum_{i=1}^{n} E'_{q_{i}}B_{F_{ij}}\sin\delta_{i})$$

$$\frac{d}{dt}i_{q_{j}} = -a_{j}\frac{\partial H}{\partial i_{q_{j}}} - b_{j}\frac{\partial H}{\partial i_{d_{j}}} + c_{j}(u_{q_{j}} - V_{q_{j}})$$

$$-a_{j}(i_{q_{j}}^{s} + \sum_{i=1}^{n} E'_{q_{i}}B_{F_{ij}}\sin\delta_{i})$$
(19)

$$-b_j(i_{d_j}^s + \sum_{i=1}^n E'_{q_i} B_{F_{ij}} \cos \delta_i)$$
(20)

Let the virtual control input be defined as $v_j = [v_{d_j} \ v_{q_j}]^T$ and let $u_j = [u_{d_j} \ u_{q_j}]^T$, then:

$$v_j = u_j + w_j \tag{21}$$

where

$$w_{j} = \begin{bmatrix} -V_{d_{j}} - a_{j}/c_{j}(i_{d_{j}}^{s} + \sum_{i=1}^{n} E_{q_{i}}^{\prime}B_{F_{ij}}\cos\delta_{i}) \\ +b_{j}/c_{j}(i_{q_{j}}^{s} + \sum_{i=1}^{n} E_{q_{i}}^{\prime}B_{F_{ij}}\sin\delta_{i}) \\ -V_{q_{j}} - a_{j}/c_{j}(i_{q_{j}}^{s} + \sum_{i=1}^{n} E_{q_{i}}^{\prime}B_{F_{ij}}\sin\delta_{i}) \\ +b_{j}/c_{j}(i_{d_{j}}^{s} + \sum_{i=1}^{n} E_{q_{i}}^{\prime}B_{F_{ij}}\cos\delta_{i}) \end{bmatrix}$$

If equations (19)-(20) are rewritten in Hamiltonian formulation using vector form, then:

$$\dot{x}_i = [J_i - R_i] \frac{\partial H}{\partial x_i} + g_{1_i} w_{1_i} + g_{2_i} u_{f_i}$$
 (22)

$$\dot{x}_j = [J_j - R_j] \frac{\partial H}{\partial x_j} + g_{3_j} v_j$$
(23)

where $x_i = \begin{bmatrix} \delta_i \ \omega_i \ E'_{q_i} \end{bmatrix}^T$, $\frac{\partial H}{\partial x_i} = \begin{bmatrix} \frac{\partial H}{\partial \delta_i} \ \frac{\partial H}{\partial \omega_i} \ \frac{\partial H}{\partial E'_{q_i}} \end{bmatrix}^T$, $i = 1 \dots n$ and $x_j = \begin{bmatrix} i_{d_j} \ i_{q_j} \end{bmatrix}^T$, $\frac{\partial H}{\partial x_j} = \begin{bmatrix} \frac{\partial H}{\partial i_{d_j}} \ \frac{\partial H}{\partial i_{q_j}} \end{bmatrix}^T$, $j = 1 \dots m$, and

$$J_{i} = \begin{bmatrix} 0 & \frac{\omega_{s}}{M_{i}} & 0 \\ -\frac{\omega_{s}}{M_{i}} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, R_{i} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{D_{i}\omega_{s}}{M_{i}^{2}} & 0 \\ 0 & 0 & \frac{x_{d_{i}}-x_{d_{i}}'}{T_{d0_{i}}'} \end{bmatrix}$$
$$g_{1_{i}} = \begin{bmatrix} 0 & 0 \\ \frac{\omega_{s}}{M_{i}} & 0 \\ 0 & \frac{1}{T_{d0_{i}}'} \end{bmatrix}, g_{2_{i}} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T_{d0_{i}}} \end{bmatrix}$$
$$w_{1_{i}} = \begin{bmatrix} \left\{ P_{m_{i}}^{s\prime\prime} - E_{q_{i}}^{\prime} \sum_{j=1}^{n} E_{q_{j}} G_{G_{ij}} \cos \delta_{ij} \\ -E_{q_{i}}^{\prime} \sum_{j=1}^{n} G_{F_{ij}} (\cos \delta_{i} i_{d_{j}} + \sin \delta_{i} i_{q_{j}}) \right\} \\ \left\{ E_{fd_{i}}^{s\prime\prime} - (x_{d_{i}} - x_{d_{i}}') \sum_{j=1}^{n} E_{q_{j}} G_{G_{ij}} \sin \delta_{ij} \\ -(x_{d_{i}} - x_{d_{i}}') \sum_{j=1}^{m} G_{F_{ij}} (\sin \delta_{i} i_{d_{j}} - \cos \delta_{i} i_{q_{j}}) \right\}$$

$$J_j = \begin{bmatrix} 0 & b_j \\ -b_j & 0 \end{bmatrix}, R_j = \begin{bmatrix} -a_j & 0 \\ 0 & -a_j \end{bmatrix}, g_{3j} = \begin{bmatrix} c_j & 0 \\ 0 & c_j \end{bmatrix}$$

E. Control Law

If the power network is lossless, then $w_{1_i} = [0, 0]^T$. The output signal of the *i*th generator is given by

$$y_{i} = g_{2_{i}}^{T} \frac{\partial H}{\partial x_{i}}$$

$$= \frac{1}{T_{d0_{i}}^{\prime}} \left(\frac{1}{x_{d_{i}} - x_{d_{i}}^{\prime}} \left(E_{q_{i}}^{\prime} - E_{fd_{i}}^{s\prime} \right) - \sum_{j=1}^{n} E_{q_{j}}^{\prime} B_{G_{ij}} \cos \delta_{ij} - \sum_{j=1}^{n} B_{F_{ij}} (\cos \delta_{i} i_{d_{j}} + \sin \delta_{i} i_{q_{j}}) \right)$$
(24)

then the control law for excitation is

$$u_{f_i} = -k_i y_i \tag{25}$$

where k_i is the feedback gain.

Similarly, the *j*th STATCOM has the output signal given by

$$y_{j} = g_{3_{j}}^{T} \frac{\partial H}{\partial x_{j}} = c_{j} \begin{bmatrix} i_{d_{j}} - i_{d_{j}}^{s} - \sum_{i=1}^{n} E_{q_{i}}^{\prime} B_{F_{ij}} \cos \delta_{i} \\ i_{q_{j}} - i_{q_{j}}^{s} - \sum_{i=1}^{n} E_{q_{i}}^{\prime} B_{F_{ij}} \sin \delta_{i} \end{bmatrix}$$
(26)

Then the virtual control input of STATCOM is given by

$$v_j = - \begin{bmatrix} k_j & 0\\ 0 & k_j \end{bmatrix} y_j \tag{27}$$

where k_j is feedback gain.

From (21), the control input for STATCOM is as follows

$$u_j = v_j - w_j \tag{28}$$

IV. SIMULATION

A simple two-area system has been used to validate the proposed controller. The diagram of the network is shown in Fig. 3. This system has four generators located in two areas. The generator parameters, network parameters and operating condition can be found in [12]. The STATCOM has been installed in the system on bus 8. The parameters of the STATCOM is shown in Table I. The feedback gain for the Hamiltonian control are chosen as $k_i = 0.01$ and $k_j = 0.02$. A solid symmetrical fault has been applied on bus 9 of at 0.1 seconds and has been cleared at 0.2 seconds. Transient simulations have been carried out using single-axis generator models and the STATCOM model described in Section III. Two cases have been considered for comparison. Case I is uncontrolled and case II is the proposed control. Fig. 4 shows the response of the rotor angles $(\delta_{i1} = \delta_i - \delta_1, (i = 2 \cdots 4)).$ Fig. 5 shows the voltage response of STATCOM terminal bus. As the figures show, the proposed controllers can not only mitigate the oscillation of the rotor angles between area 1 and area 2, but also exhibit good performance in maintaining the bus voltage magnitude.



Fig. 3. two-area system

TABLE I

STATCOM PARAMETERS

Symbol	R_s	L_s	ω_s
Unit	p.u.	p.u.	rad/sec
Value	0.01	0.15	377



Fig. 4. Generator Angle (uncontrolled: thin, proposed: bold)



Fig. 5. STATCOM terminal bus voltage (uncontrolled: thin, proposed: bold)

V. CONCLUSION

A coordinated control scheme of generation excitation and STATCOM has been proposed to stabilize the power system based on the Hamiltonian function method. The Hamiltonian realization structure for a power system with multiple machines and STATCOMs is developed. The illustrative example shows the effectiveness of the proposed control. Further investigation will consider the effect of uncertain parameters and unmodeled dynamics using robust control.

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