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Space-Time Fading Correlation Functions of a 3-D MIMO Channel Model

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Abstract— **Space-time correlation functions between the links of MIMO Rayleigh fading channels are derived using a new three-dimensional (3-D) cylinder scattering model. Closed form, mathematically tractable formulas are obtained for the space-time correlation functions for general MIMO systems where the base station and mobile station antennas may be arranged in 3-D space. It is shown that the correlation functions computed by the 3-D cylinder model are of significant difference than those of the conventional 2-D Clarke's isotropic scattering model for vertically placed antennas. The general formulas of the correlation functions includes the 2-D Clarke's model and the 3-D SIMO, MISO models as special cases.**

I. Introduction

The Multiple-input multiple-output (MIMO) communication technique has recently emerged as a new paradigm for high data rate wireless communications in rich multipath fading environments. By effectively exploiting the multipath fadings instead of mitigating them, the MIMO communication system shows greatly improved channel capacity potential far beyond that of traditional methods. It has been reported [1], [2] that the MIMO capacity scales linearly with the number of antennas under some spatially uncorrelated, time quasi-static, and frequency flat Rayleigh fading channels. However, in practice, the optimum relative antenna separation and placement may not be feasible due to space limitations and other practical constraints. Consequently, subchannels of a MIMO system are usually correlated in both space and time. The correlation between MIMO subchannels can substantially affect the performances of the MIMO systems[3], [4], [5]. Furthermore, the correlation functions also provide a critical tool and guidelines for system design and performance analysis, such as the design of space-time coding [6], the design of antenna arrays [5], and the analysis of optimum combining and equalization, etc. Therefore, further researches in modeling the physical MIMO channels are essential on providing accurate and in-depth understanding and estimation of the MIMO channels.

There have been many studies on the MIMO channel modeling and the space-time correlation of the MIMO channels, see [4]–[18] and the references therein. The most commonly used model is the two-dimensional (2-D) Clarke's isotropic scattering model. This model assumes that all random scatterers are uniformly reflected via a ring surrounding the mobile station (MS) and no line-of-sight (LOS) component is present between the MS and the base station (BS). In the literature, there also exist a large number of simulation models [9], [11], [12] based on this 2-D isotropic scattering model.

Despite its wide acceptance in the area of wireless communications, the 2-D isotropic scattering model is argued by some 3-Dimensional models. A 3-D cylinder model was first proposed by Aulin [14] based on the fact that, in highly urbanized areas, the locations of the random scatterers may be better described by a cylinder rather than a ring. This cylinder model has been improved and analyzed for SISO, SIMO (single input multiple output), and MISO (multiple input single output) channels in [15], [16]. It was shown that the correlation functions of the 3-D cylinder model is of significant difference than those of the 2-D isotropic model for vertically placed antennas. Experimental measurements in [19] and [20] also shows good agreement with the 3-D cylinder model. The cylinder model includes the 2-D scattering model as a special case by setting the maximum elevation angle (or (the height) of the cylinder to zero.

In this paper, we extend the 3-D cylinder model to frequency non-selective Rayleigh fading MIMO channels. Closed form, mathematically tractable formulas are derived for the space-time correlation functions between subchannels of the MIMO channel where the BS and MS antennas may be arranged in 3-D space. The effect of the mobility (the Doppler) of the MS is also considered in our derivation. As will be seen in Section III, the general formulas includes the 2-D model and the SIMO, MISO channels as special cases.

II. THE 3-D MIMO MODEL

Consider a downlink MIMO channel that employs n_{BS} BS and n_{MS} MS antennas. All antennas are omni-directional without beamforming. The antenna elements are numbered as $1 \le p \le q \le n_{BS}$ and $1 \le l \le m \le n_{MS}$. The pth BS antenna transmits a signal $s_p(t)$ and is received by the lth MS antenna as $r_l(t)$ through a 3-D "cylinder" scattering environment, as shown in Fig. 1.

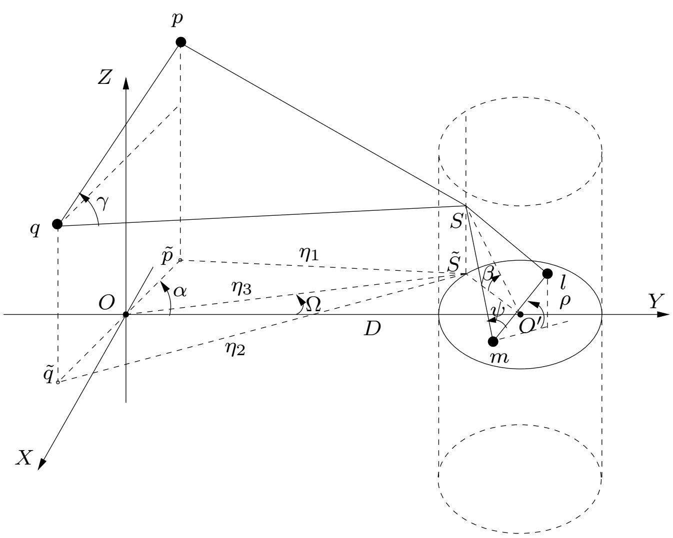

Fig. 1. The Three-Dimensional MIMO Model

To aid our analysis, we define a Cartesian coordinate system as follow: first define the X-Y plane to contain the center circle of the cylinder, then project the BS antennas p and q to the X-Y plane as $\tilde{p}$ and $\tilde{q}$. Choose the center between $\tilde{p}$ and $\tilde{q}$ as the coordinate origin O and the Y axis as the line connecting O and the center of the cylinder O'. Denote $D_{a,b}$ as the distance between two points a and b. Let the radius of the cylinder be R and the distance between O and O' be D. The elevation of the BS antenna q is $H = D_{q,\tilde{q}}$ and the displacement between the two BS antennas is $V = D_{p,\tilde{p}} - D_{q,\tilde{q}}$. Suppose there are N effective scatterers impinging on the MS antennas from a random position S on the cylinder. The elevation angle of S relative to the cylinder center O' is β and the azimuth angle is θ. Other parameters are better presented in Fig. 2 which is the projection on the X-Y plane.

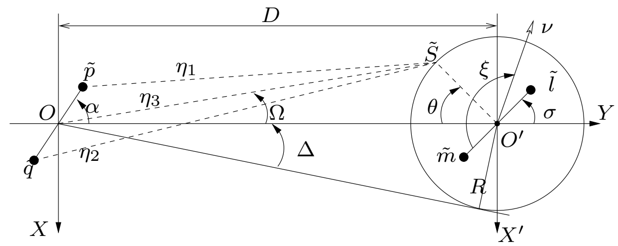

Fig. 2. Projection of the 3-D MIMO model on the X-Y plane, where ν is the direction of motion of the MS. The narrow angle of spread is $\Delta \approx \arcsin(R/D) \approx R/D$ when $D \gg R \gg max(D_{p,q}, D_{l,m})$.

The baseband input-output relationship of the MIMO system can be written as the following matrix notation:

$$\mathbf{r}(t) = \mathbf{H}(t)\mathbf{s}(t) + \mathbf{u}(t), \tag{1}$$

where the input vector $\mathbf{s}(t) = [s_1(t), \cdots, s_{n_{BS}}(t)]'$, the output vector $\mathbf{r}(t) = [r_1(t), \cdots, r_{n_{MS}}(t)]'$, and the additive white Gaussian noise (AWGN) $\mathbf{u}(t) = [u_1(t), \cdots, u_{n_{MS}}(t)]'$, with the superscript $(\cdot)'$ being the transpose. Assume that the MIMO channel is frequency nonselective, then the channel matrix $\mathbf{H}(t)$ is an $n_{MS} \times n_{BS}$ matrix whose (l,p)th element is the subchannel fading coefficient connecting the antenna elements p and l, that is $[\mathbf{H}(t)]_{lp} = h_{lp}(t)$.

Without the line-of-sight component, the subchannel impulse response $h_{lp}(t)$ can be expressed as

$$h_{lp}(t) = \lim_{N\to\infty} \frac{1}{\sqrt{N}} \sum_{k=1}^{N} g_k \cdot \exp\left\{-j\frac{2\pi}{\lambda}(D_{p,S} + D_{S,l}) + j2\pi f_d t \cos(\xi - (\theta + \sigma)) + j\phi_k\right\} \tag{2}$$

where g_k and ϕ_k are, respectively, the amplitude and random phase of the kth scatterer, f_d is the maximum Doppler frequency, and λ is the wavelength of the carrier frequency. When the gain of $h_{lp}(t)$ is normalized, we have $N^{-1}\sum_{k=1}^{N} E\left[g_k^2\right] = 1$ as $N \to \infty$, where $E[\cdot]$ is the expectation operator. According to the central limit theorem, when $N \to \infty$, the impulse response $h_{lp}(t)$ in (2) can be modeled as a low pass zero mean complex Gaussian process [21]. Thus its envelope $|h_{lp}(t)|$ is Rayleigh distributed.

III. The New Space-Time Correlation Functions

The space-time cross-correlation $\rho_{lp,mq}(\tau)$ between two subchannels $h_{lp}(t)$ and $h_{mq}(t)$ can be written as

$$\rho_{lp,mq}(\tau) = E[h_{lp}(t)h^*_{mq}(t+\tau)]$$

$$= \int_{-\beta_m}^{+\beta_m}\int_0^{2\pi} p_\theta(\theta)p_\beta(\beta)\exp\Big\{-\frac{2\pi j}{\lambda}[D_{p,S} - D_{q,S} + D_{l,S} - D_{m,S}] - j2\pi f_d\tau\cos(\xi-(\theta+\sigma))\Big\}d\theta d\beta \quad (3)$$

where $p_\theta(\theta)$ and $p_\beta(\beta)$ are the probability density functions (PDF) of the random angles of arrival θ and β, and β_m is the maximum elevation angle. We adopt the uniform distribution for $p_\theta(\theta)$ and the PDF of $p_\beta(\beta)$ used in [15], [16]. That is

$$p_\theta(\theta) = \frac{1}{2\pi} \qquad 0 \le \theta \le 2\pi. \quad (4)$$

$$p_\beta(\beta) = \frac{\pi}{4|\beta_m|}\cos\left(\frac{\pi}{2}\frac{\beta}{\beta_m}\right) \qquad |\beta| \le |\beta_m| \le \frac{\pi}{2} \quad (5)$$

The PDF defined in (4) is a flexible function of the degree of the urbanization. The parameter β_m is in the range of 10° to 20°, according to the experimental results reported in [14], [20]. When β_m is zero and $p_\beta(\beta) = 1$, the cylinder model becomes the commomly used 2-D one-ring scattering model.

Generally, the far field propagation assumption is held in practical wireless MIMO communication systems such that $D \gg R \gg max(D_{p,q}, D_{l,m})$. Therefore we can approximate the distances in (3) as

$$D_{p,S} - D_{q,S} \approx D_{p,q}\cos\gamma, \quad (6)$$

$$D_{l,S} - D_{m,S} \approx D_{l,m}\cos\psi. \quad (7)$$

Apparently, it is necessary to express $\cos\gamma$ and $\cos\psi$ in terms of the random variables θ or β and other measurable parameters. Following the procedures in [15], [16], we can obtain

$$\cos\psi = (\sin\beta\sin\rho - \cos\beta\cos\rho\cos(\theta+\sigma)) \quad (8)$$

Similarly, using the law of cosines and the small angle approximations $\sin x \approx x$ and $\cos x \approx 1$, we can obtain

$$D_{p,q}\cos\gamma = \left[\frac{\eta_2 D_{\tilde{p},\tilde{q}}\cos(\alpha-\Omega) - (H - R\tan\beta)V}{\sqrt{\eta_2^2 + (H-R\tan\beta)^2}}\right] \quad (9)$$

When V is small comparing to η_2 or D, the eq. (9) can be further simplified to

$$D_{p,q}\cos\gamma \approx D_{\tilde{p},\tilde{q}}\cos(\alpha-\Omega) \approx D_{\tilde{p},\tilde{q}}(\cos\alpha + \Delta\sin\alpha\sin\theta) \quad (10)$$

Substitute eqns. (4)-(10) into (3) and use the integral of exponential function [22]

$$\int_{-\pi}^{\pi}\exp(x\sin z + y\cos z)dz = 2\pi I_0\left(\sqrt{x^2+y^2}\right) \quad (11)$$

where $I_0(jx) = J_0(x)$, and $I_0(\cdot)$ is the zero-th order modified Bessel function of the first kind and $J_0(\cdot)$ the zero-th order Bessel function of the first kind, we derive the cross-correlation function as (12) and (13) at the top of next page, where the simplified notations are $a = 2\pi D_{\tilde{p},\tilde{q}}/\lambda$, $b = 2\pi D_{l,m}/\lambda$, $c = 2\pi f_d\tau$, and $d = \pi/(2\beta_m)$, $e = 2\pi V\Delta/\lambda$. The second approximations in (12) and (13) are obtained under the assumption that Δ is very small.

The general formulas of the MIMO channel correlation functions (12) and (13) can be further simplified for the special cases of SIMO and MISO channels.

CASE I: In MISO channels with multiple BS antennas and one MS antenna, if $V \ll D$, $H \ll D$, and $f_d = 0$, then the correlation function between the subchannels $h_{lp}(t)$ and $h_{lq}(t)$ is simplified from (12) as

$$\rho_{lp,lq}(\tau) \approx \exp\left\{\frac{-j2\pi}{\lambda}D_{\tilde{p}\tilde{q}}\cos\alpha\right\}J_0\left(\frac{2\pi}{\lambda}D_{\tilde{p}\tilde{q}}\Delta\sin\alpha\right) \quad (14)$$

The closed-form equation (14) agrees with the results in [4].

CASE II: In SIMO channels with one BS antenna and multiple MS antennas, if the MS antennas are on the same horizontal plane meaning that $\rho = 0^\circ$, and $f_d = 0$, then the correlation function between the subchannels $h_{lp}(t)$ and $h_{mp}(t)$ is simplified as

$$\rho_{lp,mp}(\tau) \approx \int_{-\beta_m}^{\beta_m} p_\beta(\beta)J_0\left(\frac{2\pi}{\lambda}D_{l,m}\cos\beta\right)d\beta \approx J_0\left(\frac{2\pi}{\lambda}D_{l,m}\right) \quad (15)$$

The second approximation is derived under the condition that β_m is in the range of 10° to 20° and $\cos\beta \approx 1$. This result is in agreement with the formulas in [15] and [4] derived from the 2-D scattering model. It is apparent that if the MS antennas are in the same horizontal plane and $\beta_m < 20^\circ$, the 3-D fading model does not differ significantly from the commonly used 2-D model.

CASE III: In SIMO channels with one BS antenna and multiple MS antennas, if the MS antennas are

When $V \ll D$,

$$\begin{aligned}\rho_{lp,mq}(\tau) &\approx \frac{1}{2\pi}\int_{-\beta_m}^{\beta_m}\int_0^{2\pi} p_\beta(\beta)\cdot\exp\left\{-\frac{2\pi j}{\lambda}[D_{\tilde{p},\tilde{q}}(\cos\alpha+\Delta\sin\Omega\sin\theta)\right.\\ &\quad \left.+D_{l,m}(\sin\beta\sin\rho-\cos\beta\cos\rho\cos(\theta+\sigma)] \;\; -2\pi f_d\tau\cos(\xi-(\theta+\sigma))\right\}d\theta d\beta\\ &\approx \exp\{-ja\cos\alpha\}\int_{-\beta_m}^{\beta_m} p_\beta(\beta)\exp\{-jb\sin\beta\sin\rho\}\cdot J_0(\{(a\Delta\sin\alpha)^2+(b\cos\beta\cos\rho)^2+c^2\\ &\quad +2a\Delta\sin\alpha[b\cos\beta\cos\rho\sin\sigma+c\sin(\xi-\sigma)]-2bc\cos\beta\cos\rho\cos\xi\}^{\frac{1}{2}})d\beta\\ &\approx \frac{\cos(\beta_m b\sin\rho)}{1-(\frac{b\sin\rho}{d})^2}\exp\{-ja\cos\alpha\}\\ &\quad \cdot J_0(\{(a\Delta\sin\alpha)^2+b\cos\rho)^2+c^2+2a\Delta\sin\alpha[b\cos\rho\sin\sigma+c\sin(\xi-\sigma)]-2bc\cos\rho\cos\xi\}^{\frac{1}{2}})\end{aligned} \tag{12}$$

When V is not very small comparing to D or R,

$$\begin{aligned}\rho_{lp,mq}(\tau) &\approx \frac{1}{2\pi}\int_{-\beta_m}^{\beta_m}\int_0^{2\pi} p_\beta(\beta)\cdot\exp\left\{-\frac{2\pi j}{\lambda}[D_{\tilde{p},\tilde{q}}(\cos\alpha+\Delta\sin\Omega\sin\theta)-\frac{V|H-R\tan\beta|}{\eta_2}\right.\\ &\quad +D_{l,m}(\sin\beta\sin\rho-\cos\beta\cos\rho\cos(\theta+\sigma)]-2\pi f_d\tau\cos(\xi-(\theta+\sigma))\}\,d\theta d\beta\\ &\approx \exp(-ja\cos\alpha)\left\{\frac{a^2}{2(a^2-(b+e)^2)}\exp\left(\frac{jeH}{R}\right)\right.\\ &\quad\times\left[\sin(a\Lambda)\exp\{-j(b+e)\Lambda\}-\frac{j(b+e)}{a}\cos(a\Lambda)\exp\{-j(b+e)\Lambda\}\right]_{\Lambda=-\beta_m}^{\Lambda=H/R}+\frac{a^2}{2(a^2-j(b-e)^2)}\exp\left(-\frac{jeH}{R}\right)\\ &\quad\left.\times\left[\sin(a\Lambda)\exp\{-j(b-e)\Lambda\}-\frac{j(b-e)}{a}\cos(a\Lambda)\exp\{-j(b-e)\Lambda\}\right]_{\Lambda=H/R}^{\Lambda=\beta_m}\right\}\\ &\quad\times J_0(\{(a\Delta\sin\alpha)^2+b\cos\rho)^2+c^2+2a\Delta\sin\alpha[b\cos\rho\sin\sigma+c\sin(\xi-\sigma)]-2bc\cos\rho\cos\xi\}^{\frac{1}{2}})\end{aligned} \tag{13}$$

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not placed on the same horizontal plane meaning that $\rho \neq 0°$, we have

$$\rho_{lp,mp}(\tau) \approx \frac{\cos(b\beta_m\sin\rho)}{1-(\frac{b\sin\rho}{d})^2} \tag{16}$$

The equation (16) shows that β_m and ρ have big impact on the cross-correlation function. The cross-correlation function from the 3-D model will differ significantly from those of the 2-D fading model. As will be shown in Fig. 3, the vertically placed MS antennas will not be completely correlated under the 3-D fading thus they can provide significant diversity gain.

IV. Simulation Results

Extensive simulations have been carried out and verified the derived formulas of the space-time correlation functions of the 3-D cylinder model. Several examples are presented here to show the effect of the new 3-D model and its difference from the conventional 2-D model. The effects of the antenna spacing and arrangement are also demonstrated. In all examples, we used the following parameters: the distance between the BS and MS was $D = 1200$ meters, the radius of the cylinder was $R = 100$ meters, the elevation of the lower BS antenna was $H = 30$ meters, and the angles $\alpha = 0$, and $\sigma = 0$. The angle spread was then $\Delta = \arcsin(R/D) \approx 5°$.

The first example is a SIMO channel (with one BS antenna and two MS antennas). The correlation functions of two vertically placed antennas are plotted in Fig. 3. As the maximum elevation angle β_m of the fading channel changes from 1° to 20°, the correlations between the two subchannels reduces dramatically. For instance, at the MS antenna separation $D_{MS} = D_{m,l} = 1.5\lambda$, the correlation $\rho_{mp,lp} = 0.3$ when $\beta_m = 20°$; while $\rho_{mp,lp} = 0.92$ when $\beta_m = 5°$. When $\beta = 0$, the 3-D cylinder model becomes the conventional 2-D fading model and we obtain $\rho_{mp,lp} = 1$ which suggests that the two vertically placed antenna are completely correlated and no diversity gain is available. However, the new 3-D fading model show that vertically placed BS antennas can have small correlations and are able to provide considerable diversity gain. This is in good agreement with field measurements.

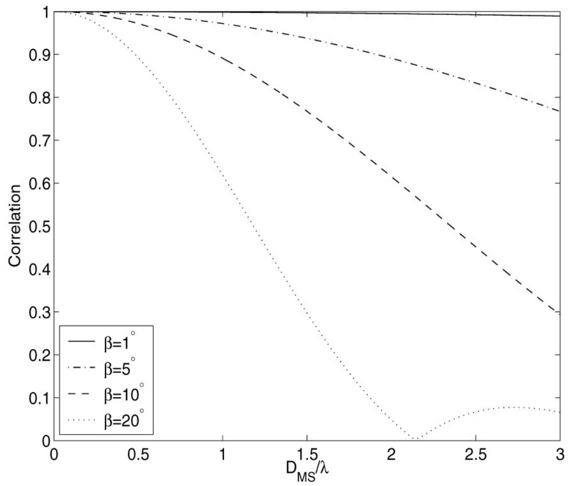

Fig. 3. The cross-correlation of a SIMO channel with one BS antennas and two MS antennas, vertically placed with $\rho = 90°$.

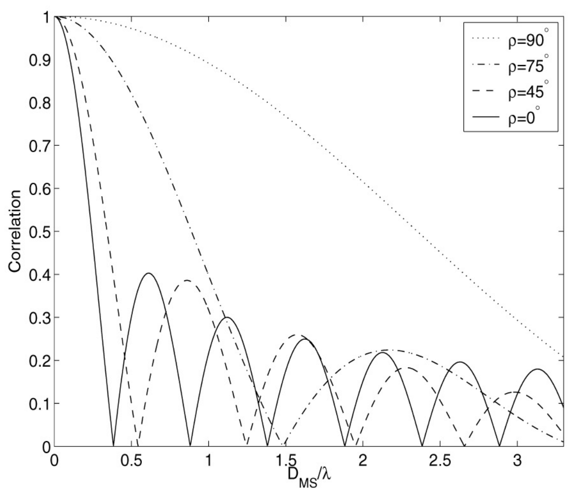

Fig. 4. The cross-correlation of a SIMO channel with one BS antennas and two MS antennas. The maximum elevation angle of the fading cylinder is $\beta_m = 10°$.

Figure 4 shows the effect of the MS antenna placement on the correlation of the SIMO channel. It is clear that the correlation of the horizontally placed MS antennas reduces much faster than that of the vertically placed MS antennas. It is also noted that the correlations of the horizontally placed antennas are almost the same with different β_m providing that $\beta_m < 20°$. Therefore, the 2-D Clark's fading model is sufficient and accurate for the SIMO channels with horizontally placed MS antennas.

The example of a MIMO channel is presented next. The correlations of the horizontally placed MS antennas are shown in Fig. 5 and those of the vertically placed MS antennas in Fig. 6. It is clear that the correlation functions of the MIMO channels are significantly affected by not only the spacing but also the arrangement of the antenna elements. Low correlations can be achieved by carefully arranging the MS and BS antennas such that their correlation falls in the "valleys" of the plots. Besides, lower correlations were generally observed with the 3-D fading model than those of the 2-D model with the same antenna configurations.

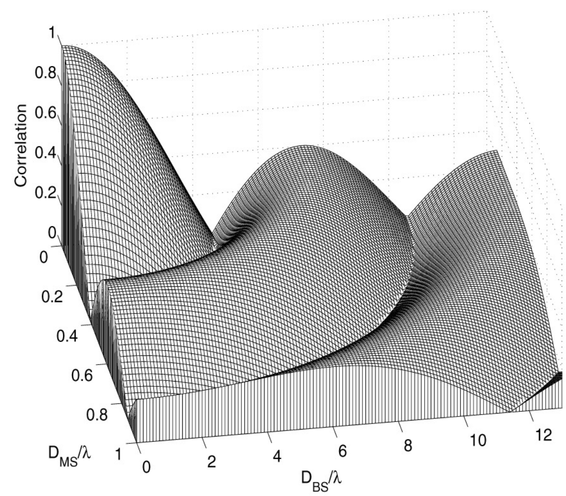

Fig. 5. Isometric view of the cross-correlation of a 2x2 channel with horizontally placed MS antennas ($\rho = 0°$) and $\beta_m = 10°$.

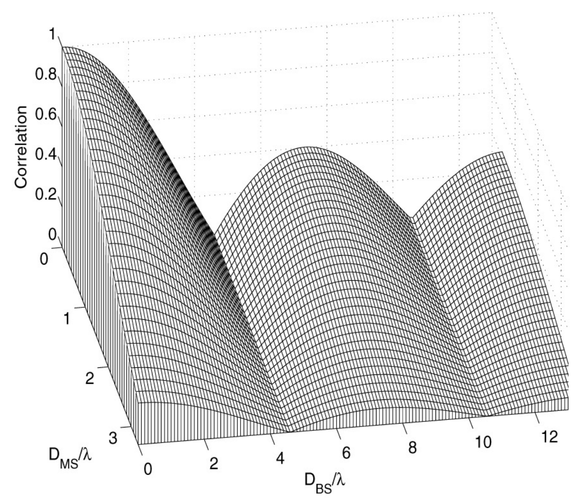

Fig. 6. Isometric view of the cross-correlation of a 2x2 channel with vertically placed MS antennas ($\rho = 90°$) and $\beta_m = 10°$.

V. Conclusion

The cross-correlation functions of MIMO fading channels have been analyzed under the new 3-D cylinder model. in contrast of the conventional 2-D Clarke's isotropic scattering model. Closed form formulas have been derived for the correlation between two arbitrary subchannels – no restrictions are imposed on the configurations of either BS or MS antennas. Computer simulations have verified our formula.

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