



Missouri University of Science and Technology
Scholars' Mine

Electrical and Computer Engineering Faculty
Research & Creative Works

Electrical and Computer Engineering

03 Oct 2007


Optimal Control of Class of Non-Linear Plants using Artificial Immune Systems: Application of the Clonal Selection Algorithm

S. A. Panimadai Ramaswamy

Ganesh K. Venayagamoorthy
Missouri University of Science and Technology

S. N. Balakrishnan
Missouri University of Science and Technology, bala@mst.edu

Follow this and additional works at: https://scholarsmine.mst.edu/ele_comeng_facwork

 Part of the [Aerospace Engineering Commons](#), [Electrical and Computer Engineering Commons](#), and the [Mechanical Engineering Commons](#)

Recommended Citation

S. A. Ramaswamy et al., "Optimal Control of Class of Non-Linear Plants using Artificial Immune Systems: Application of the Clonal Selection Algorithm," *Proceedings of the IEEE 22nd International Symposium on Intelligent Control (2017, San Jose, CA)*, Institute of Electrical and Electronics Engineers (IEEE), Oct 2007. The definitive version is available at <https://doi.org/10.1109/ISIC.2007.4450893>

This Article - Conference proceedings is brought to you for free and open access by Scholars' Mine. It has been accepted for inclusion in Electrical and Computer Engineering Faculty Research & Creative Works by an authorized administrator of Scholars' Mine. This work is protected by U. S. Copyright Law. Unauthorized use including reproduction for redistribution requires the permission of the copyright holder. For more information, please contact scholarsmine@mst.edu.

Optimal Control of Class of Non-linear Plants Using Artificial Immune Systems: Application of the Clonal Selection Algorithm

S.A. Panimadai Ramaswamy, G.K. Venayagamoorthy, *Senior Member, IEEE* and
S.N. Balakrishnan, *Member, IEEE*

Abstract—The function of natural immune system is to protect the living organisms against invaders/pathogens. Artificial Immune System (AIS) is a computational intelligence paradigm inspired by the natural immune system. Diverse engineering problems have been solved in the recent past using AIS. Clonal selection is one of the few algorithms that belong to the family of AIS techniques. Clonal selection algorithm is the computational implementation of the clonal selection principle. The process of affinity maturation of the immune system is explicitly incorporated in this algorithm. This paper presents the application of AIS for the optimal control of a class of non-linear plants which are affine in control. The clonal selection algorithm is adapted for optimal control. A new mutation operator that operates on real values and one that aids in fast convergence is developed in this paper. AIS is used to obtain constant coefficient Kalman gain matrices. The validation and evaluation of the results thus obtained are carried out by comparing with standard and the widely used State Dependent Algebraic Riccati Equation (SDARE) method for the non-linear plants. In case of non-linear systems with hard state constraints, the SDARE formulation requires the use of mathematically involved expressions to incorporate these state constraints. However, the modified clonal selection algorithm developed in this paper has been used with hardly any changes to incorporate the hard state constraints and obtain the Kalman gain matrix.

I. INTRODUCTION

Artificial Immune Systems (AIS) is a relatively new field and the interest in using AIS for solving engineering problems has been on the rise in the past few years [1]-[7]. AIS can be defined as computational systems that are inspired by theoretical immunology i.e. principles, mechanisms and the observed functions of the immune systems. Their development and domains of application are similar to those of other computational intelligence paradigms such as artificial neural networks, evolutionary algorithms, swarm intelligence and fuzzy logic based systems.

In this paper, the clonal selection algorithm that belongs to the family of AIS techniques has been used to solve the

optimal control problem. The clonal selection algorithm developed by Castro et al. [1] has been adapted to suit the application considered in this paper. The problem of optimal control involves finding the optimal gains for state feedback control by optimizing (maximizing or minimizing) a cost function which represents some performance index. AIS was considered when compared to other soft computing algorithms as it offers certain advantages over the others. Genetic algorithms cannot find a global optimum when the number of parameters involved increases and when they are epistatic in nature. Simulated annealing is very slow. AIS do not suffer from the above drawbacks.

The mutation operator encountered in clonal selection is generally applied to binary strings. In this paper, a mutation operator that performs goal directed mutation on real valued antibodies (control gains) is developed. The optimal gains have been found for both linear and non-linear plants using AIS. Non-linear plants considered are affine in control. The same algorithm without any modification has been used for scalar and vector plants. The work presented in this paper considers vector non-linear systems. In addition, the optimal control of plants with hard state constraints is performed. The results are compared with those obtained by using the State Dependent Algebraic Riccati Equation (SDARE) formulation, one of the most widely used methods for optimal control of non-linear systems [12]-[16]. There is plethora of literature available on the real time applications of SDARE technique. Note that the SDARE formulation requires the calculation of Kalman gain matrix by solving the Riccati equation at every time step. The new clonal selection algorithm returns a constant coefficient Kalman gain matrix calculated offline. Hence unlike the SDARE method the usage of clonal selection algorithm reduces the computational burden and saves time in real time applications.

The paper is organized as follows. Section II presents a brief background on the optimal control problem and an introduction to SDARE formulation, which is a widely used optimal control method for non-linear systems. Section III deals with the review of the clonal selection principle and the affinity maturation process. In Section IV, a description of the new adapted clonal selection algorithm for optimal control is presented. In Section V, the results are presented. Section VI involves a discussion of the results and comparisons with conventional approaches. Finally, the conclusion is given in Section VII.

Manuscript received January 9, 2007.

S.A. Panimadai Ramaswamy is with the Department of Electrical and Computer Engineering, University of Missouri-Rolla, MO 65409 USA (e-mail: sph58@umr.edu).

G.K. Venayagamoorthy is with the Real-Time Power and Intelligent Systems Laboratory, Department of Electrical and Computer Engineering, University of Missouri-Rolla, MO 65409 USA (e-mail: gkumar@ieee.org).

S.N. Balakrishnan is with the Department of Mechanical, Aerospace and Engineering Mechanics, University of Missouri-Rolla, MO 65409 USA (e-mail: bala@umr.edu).

II. OPTIMAL CONTROL PROBLEM

A common objective in many technical fields is to design control logic that commands a dynamic system to produce a desired output and augments the system's stability. When the control objective is expressed as a quantitative criterion, then optimization of this criterion results in a set of equations to be solved to obtain the controller. Optimal control theory governs strategies for maximizing a performance measure or minimizing a quantitative criterion as the state of a dynamic system evolves. The fundamentals of optimal control of continuous-time dynamic linear systems and non-linear systems are discussed below.

The process of design of optimal control for linear systems which have quadratic performance indices is called the linear quadratic (LQ) problem. The theory for optimal control of linear systems using linear quadratic regulator (LQR) can be found in [8]. Consider a non-linear dynamic system affine in control given by

$$\dot{\mathbf{x}} = f(\mathbf{x}) + g(\mathbf{x})\mathbf{u} \quad (1)$$

In the recent years, the SDARE method has been used to obtain the optimal control for non-linear systems. Considered below is the SDARE formulation. The problem considered here is the infinite-horizon regulation of general autonomous nonlinear systems which are affine in input [8]. Given the system equation in (1) and the performance index (PI)

$$\mathbf{J} = \frac{1}{2} \int_0^{\infty} (\mathbf{x}^T \mathbf{Q}(\mathbf{x})\mathbf{x} + \mathbf{u}^T \mathbf{R}(\mathbf{x})\mathbf{u}) dt \quad (2)$$

which allows for trading-off state error \mathbf{x} versus control input \mathbf{u} , via the weighting matrices $\mathbf{Q}(\mathbf{x}) \geq 0$, $\mathbf{R}(\mathbf{x}) > 0$, $\forall \mathbf{x}$, respectively. It is assumed that $f(0) = 0$ and $g(\mathbf{x}) \neq 0$, $\forall \mathbf{x}$. A feedback control law $\mathbf{u}(\mathbf{x})$ which regulates the system to the origin can be found by using the SDARE method [12]-[16] which approaches the problem by mimicking the LQR formulation for discussed for linear systems. Accordingly, the system equations have to be first written in the form given by:

$$\dot{\mathbf{x}} = \mathbf{A}(\mathbf{x})\mathbf{x} + \mathbf{B}(\mathbf{x})\mathbf{u} \quad (3)$$

where the state vector $\mathbf{x} \in \mathcal{R}^n$, control input $\mathbf{u} \in \mathcal{R}^m$, $\mathbf{A} \in \mathcal{R}^{n \times n}$, $\mathbf{B} \in \mathcal{R}^{n \times m}$, $f(\mathbf{x}) = \mathbf{A}(\mathbf{x})\mathbf{x}$ and $g(\mathbf{x}) = \mathbf{B}(\mathbf{x})$. The cost is denoted by \mathbf{J} . The objective here is to find the control that minimizes the quadratic PI in (2). The control weighting matrix $\mathbf{R} \in \mathcal{R}^{m \times m}$ and the state weighting matrix $\mathbf{Q} \in \mathcal{R}^{n \times n}$ are symmetric matrices. The former parameterization is possible if and only if $f(0) = 0$ and $f(\mathbf{x})$ is continuously differentiable. Then as in the linear time-invariant case [8], a state-feedback control law of the form of

$$\mathbf{u}(\mathbf{x}) = -\mathbf{K}(\mathbf{x})\mathbf{x} = -\mathbf{R}^{-1}(\mathbf{x})\mathbf{B}^T(\mathbf{x})\mathbf{S}(\mathbf{x})\mathbf{x} \quad (4)$$

can be found. The Kalman given by $\mathbf{K}(\mathbf{x})$ in non-linear systems is state dependent and changes at every time step.

$\mathbf{S}(\mathbf{x})$ is unique, symmetric, positive-definite solution of the state-dependent algebraic Riccati equation

$$\mathbf{A}^T(\mathbf{x})\mathbf{S}(\mathbf{x}) + \mathbf{S}(\mathbf{x})\mathbf{A}(\mathbf{x}) + \mathbf{Q}(\mathbf{x}) - \mathbf{S}(\mathbf{x})\mathbf{B}(\mathbf{x})\mathbf{R}^{-1}(\mathbf{x})\mathbf{B}^T(\mathbf{x})\mathbf{S}(\mathbf{x}) = 0 \quad (5)$$

The pair $(\mathbf{A}(\mathbf{x}), \mathbf{B}(\mathbf{x}))$ should be pointwise controllable in the linear sense so that the algebraic Riccati equation has a solution at that particular state \mathbf{x} [9], [12]-[16]. Due to the nonuniqueness of $\mathbf{A}(\mathbf{x})$, different $\mathbf{A}(\mathbf{x})$ choices yield different controllability matrices and thus different pointwise controllability characteristics. From the many choices for the parameterization $(\mathbf{A}(\mathbf{x}), \mathbf{B}(\mathbf{x}))$, a pointwise stabilizable pair is chosen. Parameterization is easy for lower order systems; it can be complicated for higher order systems. The solving of the state dependent algebraic Riccati equation is very cumbersome and hence numerical tools are used.

III. CLONAL SELECTION PRINCIPLE

An overview of the immune system fundamentals along with the clonal selection principle and the process of affinity maturation is presented here. A more detailed view on the subject can be obtained from [10]. The immune system is made up a vast array of cells, molecules and organs that work together to maintain life and protect us from foreign bodies and disease causing agents, called pathogens. It also helps in the elimination of malfunctioning self-cells in the body (cancerous cells). Since there are many pathogens, they have to be recognized before elimination. The pathogens cannot be directly recognized by the components of the immune system, but some small portions/molecules of the pathogens, called antigens, are recognized by the immune system. After recognizing a disease causing agent, the immune system eliminates it from the body in order to prevent or curb diseases. B-cells have antibody molecules on its surface which is released into the blood stream during an immune response. The main function of an antibody molecule is to bind with an antigen. Antigens and antibodies have complementary shapes so that they can bind together. The immune response is triggered by the binding together of the antibodies with the antigens. Adaptive immune response is the process that occurs once the immune cells are generated and the antigens have been recognized. One of the important processes in the immune mechanism of defense is the reproduction of the cells capable of recognizing and binding with specific antigens and is based on a process called cloning, i.e. the process of creation of offspring that are copies of their parent cells. These clones are subject to mutation at the next stage. Affinity maturation is the combined process of mutation that affect the portions of antibodies that bind with the antigen and the process of selection that guarantees the survival of the offspring (outcome of mutation) that match the antigen better. The clones that better recognize the antigen, which elicited the immune response, are selected to have long life spans; and these cells go on to become what are called as memory cells.

This enables the immune system to have a memory and gives it the capability of dealing with antigens it has dealt with in the past with greater effectiveness. The complete process of antigen recognition, cell proliferation and differentiation into memory cells is what is called as clonal selection [10]-[11].

IV. CLONAL SELECTION ALGORITHM AND ITS APPLICATION TO OPTIMAL CONTROL

Few of the immune aspects taken into account by Castro et al [1] in the development of the algorithm are namely - maintenance of a specific memory set, selection and cloning of the stimulated antibodies, affinity maturation and re-selection of the clones proportionally to their antigenic affinity, generation and maintenance of diversity. Finding optimal control and state histories for dynamic systems is an extension of the field of static optimization that involves finding control parameters that define maxima/minima of algebraic functions. Hence, the clonal selection algorithm for optimization described by Castro et al [1] is adapted in this paper for optimal control as follows:

- i. In the optimization process there is no explicit antigen population (Ag) to be recognized. So the objective function is to be optimized is taken as the Ag. In this case the PI given by (2) is the objective function/Ag to be minimized. So an antibody affinity corresponds to evaluation of the objective function for the given antibody.
- ii. A population N of antibodies (Ab) is randomly initialized. Each antibody represents a different constant coefficient Kalman gain matrix (refer to (1) and (4)).
- iii. Solve the system equation represented by the differential equation in (1).
- iv. In case of systems with state constraints, check if the constraints are satisfied and choose the N antibodies such that constraints are satisfied.
- v. Evaluate the cost or PI for each antibody in the population.
- vi. Sort the antibodies based on low to high cost.
- vii. All the antibodies are cloned/reproduced independently and proportionally to their affinity generating a set C of clones. The higher the affinity, the higher the number of clones generated. The number of clones generated is described by the equation

$$N_c = \sum_{i=1}^n \text{round} \left(\frac{B * N}{i} \right) \quad (6)$$

- where N = total number of antibodies
 B = multiplying factor and is taken to be 0.5
 i = 1 for highest affinity (i.e. least cost function) and 2 for second highest affinity, etc
 n = lowest rank which is usually equal to total number of antibodies.

- viii. The set C of clones is submitted to an affinity maturation process which is inversely proportional to the antigenic affinity (i.e. higher the cost more the mutation). The mutation rate is given by the equation below

$$\alpha = \exp(-\rho * f) \quad (7)$$

where α is the mutation rate

ρ controls the decay of the mutation rate and is taken to be 1

f represents the normalized antigenic affinity.

The process of mutation developed in this paper is given by

$$C^* = C + \alpha * \text{randn} * C + \alpha * \text{randn} * (C - Ab_{best}) \quad (8)$$

where C^* is the population of set C clones after the affinity maturation process and Ab_{best} is the antibody

with least cost in the set C of clones. Hence equation (8) results in a goal directed mutation process.

- ix. In case of systems with constraints, check if the constraints are satisfied for the set C^* of mutated clones and chose only those that satisfy. Denote new set by C^*_{new} .
- x. Determine the affinity of the mutated clones/affinity matured clones by evaluating the cost function and then sorting them based on cost. In the case of constrained systems, determine the affinity of the set C^*_{new} .
- xi. Then compare the affinity of the available population N of antibodies and the affinity matured clones C^* . Set C^*_{new} is used instead of C^* in the case of systems with hard state constraints.
- xii. Reselect N antibodies from the above set, i.e. select N highest affinity antibodies from the population of available antibodies and affinity matured clones.
- xiii. Repeat the process until stopping condition is reached. The stopping condition used in this paper is that if the cost of the highest affinity antibody does not change over a number of generations denoted by N_{gen} then the algorithm is stopped. N_{gen} is a user defined parameter. Thus the Kalman gain matrix that gives the least cost is found.

V. RESULTS

A new adapted clonal selection algorithm is used to obtain the Kalman gains directly without having to solve the SDARE. In all the cases discussed below, the control weighting matrix R and the state weighting matrix Q are taken to be identity matrices. The number of antibodies used is 15 and the stopping condition is negligible or no change in cost for 500 generations. Case 1 is given by the non-linear plant

$$\dot{x}_1 = x_1^2 + x_5^2 + x_3x_2 + x_4 + u_1 \quad (9)$$

$$\dot{x}_2 = x_3 + x_2x_5 + x_1^3 \quad (10)$$

$$\dot{x}_3 = x_2 + x_4 + x_1 + x_3^2 + u_2 \quad (11)$$

$$\dot{x}_4 = x_2^2 + x_1x_3 + x_5^2 + u_1 \quad (12)$$

$$\dot{x}_5 = x_1 + x_5 + x_2 + x_3 + x_4^2 + u_2 \quad (13)$$

Equations (9) through (13) can be parameterized and expressed in the form suitable for application of the SDARE method as follows

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \end{bmatrix} = \begin{bmatrix} x_1 & 0 & x_2 & 1 & x_5 \\ x_1^2 & 0 & 1 & 0 & x_2 \\ 1 & 1 & x_3 & 1 & 0 \\ 0 & x_2 & x_1 & 0 & x_5 \\ 1 & 1 & 1 & x_4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (14)$$

The above parameterization is one of the many possible parameterizations. The “best” Kalman gain matrix obtained by using clonal selection algorithm is given by $K_{BEST} = \begin{bmatrix} 0.8141 & 2.0399 & 3.1757 & 1.6606 & 1.9903 \\ 1.4795 & 1.6765 & 2.8558 & 1.4062 & 3.6266 \end{bmatrix}$. In the SDARE

method K has to be *calculated at every step*. The plots for the states (Figs. 1 to 5), cost (Fig. 6) and control (Figs. 7 and 8) are as given below.

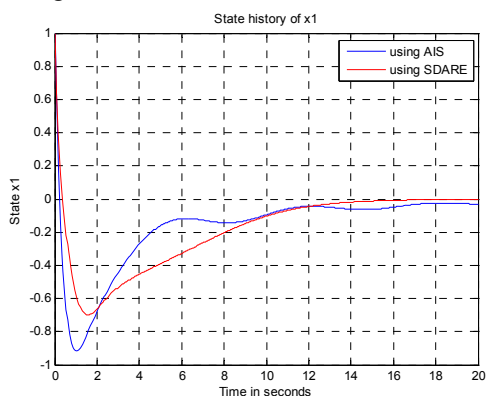


Fig. 1. State history of x_1 obtained using AIS and SDARE

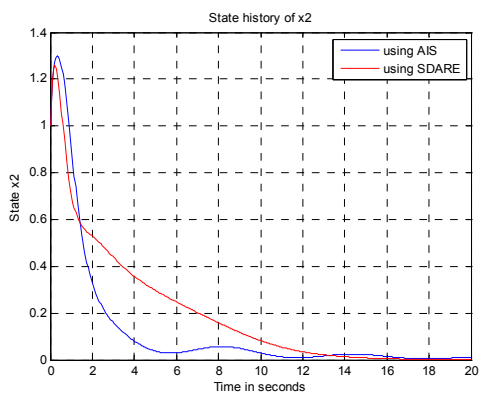


Fig. 2. State history of x_2 obtained using AIS and SDARE

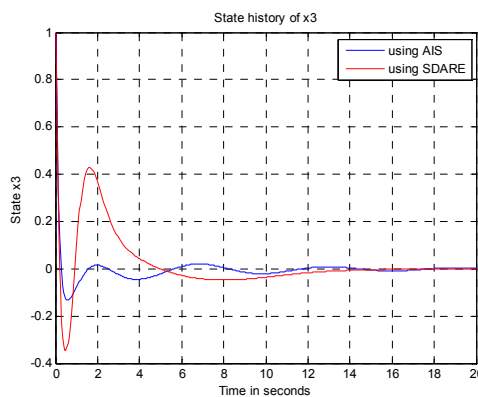


Fig. 3. State history of x_3 obtained using AIS and SDARE

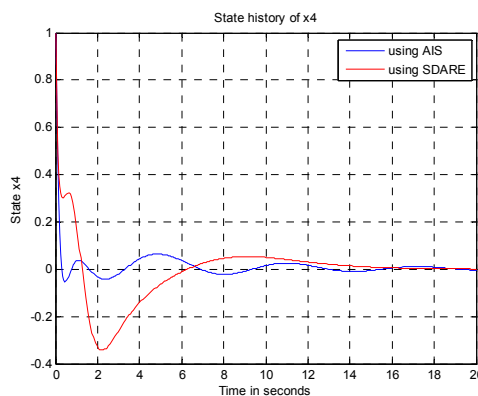


Fig. 4. State history of x_4 obtained using AIS and SDARE

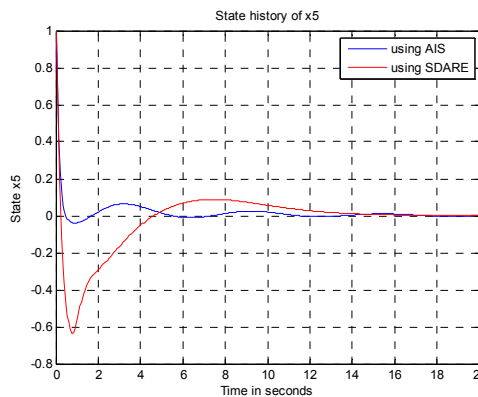


Fig. 5. State history of x_5 obtained using AIS and SDARE

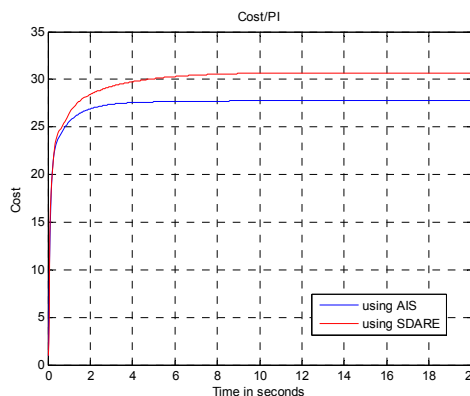


Fig. 6. Plot of cost/PI obtained using AIS and SDARE

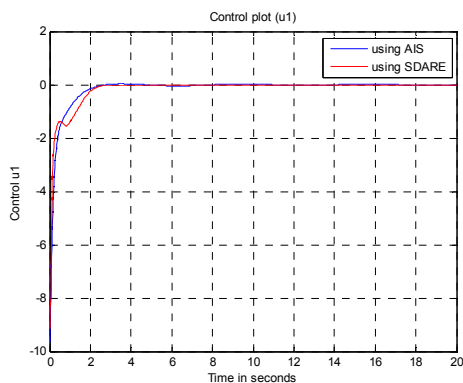
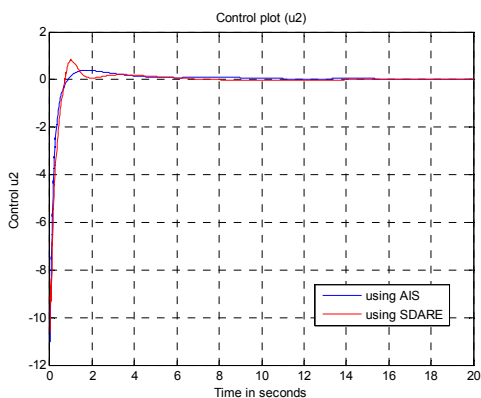
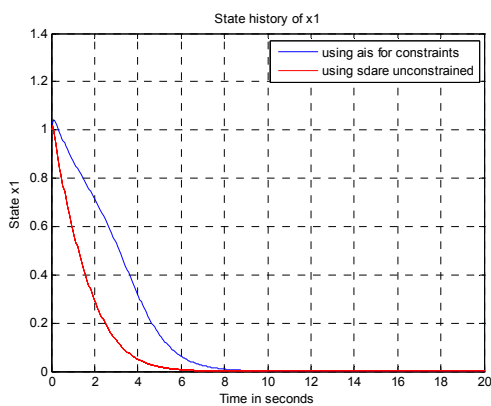
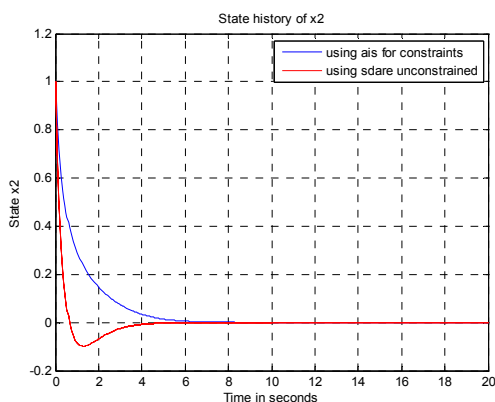
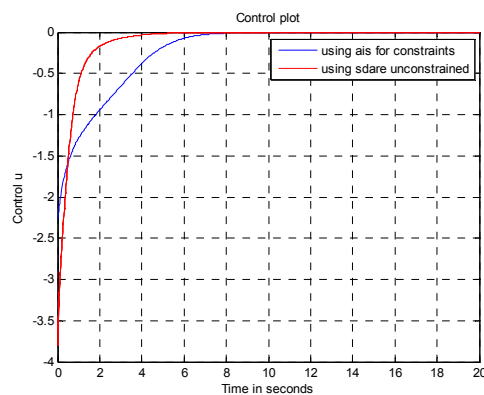
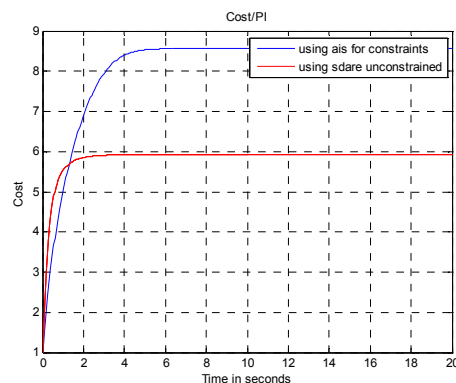
Fig. 7. Control input u_1 obtained using AIS and SDAREFig. 8. Control input u_2 obtained using AIS and SDAREFig. 9. State history of x_1 obtained using AIS and SDAREFig. 10. State history of x_2 obtained using AIS and SDAREFig. 11. Control input u obtained using AIS and SDARE

Fig. 12. Plot of cost/PI obtained using AIS and SDARE

VI. DISCUSSION

In case 1, a nonlinear plant has been considered and the optimal control obtained using SDARE and AIS methods are compared. It can be observed from the plots that using AIS method results in better regulation of the states and also results in the reduction of the cost and control effort needed. In case 2, there is a state constraint $x_2 > 0$ always. Note that these types of constraints are common in many fields. For example, in biosystems, there is a constraint on blood pressure, and in the case of vehicle dynamics, there is a constraint on the maximum and minimum velocity. From the plots, it can be seen that the state x_2 is constrained. Due to the constraint, the control effort needed to meet the constraint is more and this shows in the control plot for the constrained case. This in turn increases the overall cost. Highly involved mathematical expressions are required to incorporate hard state constraints in the SDARE formulation. AIS allows the constraints to be incorporated without any significant changes in the algorithm and gives out constant coefficient Kalman gain matrix for optimal control of a class of constrained non-linear systems. The simulations were also carried out for different Q and R matrices and it was found that AIS gave better results when compared to SDARE method. Different parameterizations of the A(x) matrix were also investigated and it was found that AIS gave better results for the problems considered. Note that in the algorithm the

upper bound on ρ (controls mutation rate) is 1. An increase in ρ defeats the purpose of equation (14). When $\rho > 1$ the diversity in mutated is very less and when $\rho < 1$ the diversity in mutated particles is more. Decay of mutation rate is faster when $\rho > 1$, which will not help in the convergence to global optimum. To have balance in diversity of the mutated particles ρ has been chosen equal to 1. B (multiplying factor) affects the number of clones generated and hence the computation time and the number of computations. Increase in B increases the search capability but at the same time also increases the computation time. It is to be noted that AIS works for all classes of non-linear systems. AIS can be employed for systems that are non-affine in control but not SDARE. Due to lack of comparison measure, results for such a case have not been presented.

VII. CONCLUSIONS

It can be concluded that artificial immune system based clonal selection algorithm is successfully applied to obtain the Kalman gains for optimal state feedback control. It is shown that the new adapted clonal selection algorithm performs at par with the standard and widely used technique for optimal control of non-linear systems namely, the SDARE method. It can also be seen from the results that the adapted clonal selection is able to perform optimal control of non-linear systems with ease, while avoiding the mathematically involved expressions required for the same in SDARE formulation. It gives results better than SDARE and this outcome can be associated with the difficulty of finding the right parameterization while using the SDARE method. The advantages of using AIS to obtain the optimal control gains over the conventional methods are: 1) it is a compact algorithm which can be used for both linear and non-linear cases; 2) also takes into account the constraints imposed on state with ease, unlike its conventional counterpart; 3) performs better than the SDARE and this is because most often it is very difficult to obtain the right parameterizations for the SDARE method; 4) the optimal gains are calculated offline unlike the SDARE method and hence saves computational time in real time; and 5) it gives a constant gain for non-linear plants unlike the SDARE method. This makes it a compact method for performing optimal control satisfactorily. Hence, AIS allows constrained optimization, in turn constrained sub-optimal control is made possible. The constrained SDARE case is very mathematically involved the reference for one of the ways for using the SDARE method when state constraints are used is given in [17].

REFERENCES

[1] L. N. de Castro and F.J.Von Zuben, "Learning and optimization using the clonal selection principle", *IEEE Trans. on Evolutionary Computing*, vol. 6, no. 3, Jun. 2002, pp. 239-251.
 [2] Y. Ishida, "The Immune System as a Self-Identification Process: A Survey and a Proposal", *ICMAS International Workshop on Immunity-Based Systems (IMBS96)*, Kyoto, Dec. 10-13, pp. 2-12.

[3] J. E. Hunt and D. E. Cooke, "Learning Using an Artificial Immune System", *Journal of Network and Computer Applications*, vol. 19, issue 2, Apr. 1996, pp. 189-212.
 [4] D. Dasgupta, *Artificial Immune Systems and Their Applications*, Ed., Springer-Verlag, 1999.
 [5] S. A. Hofmeyr and S. Forrest, "Immunity by Design: An Artificial Immune System", in *Proc. Of GECCO'99*, 1999, pp. 1289-1296.
 [6] L. N. de Castro and F. J. Von Zuben, "Artificial Immune Systems: Part II – A Survey of Applications", Tech. Rep. – RT DCA 02/00, p.65. [Online]. Available: <http://www.dca.fee.unicamp.br/~lnunes/immune.html>.
 [7] "Artificial Immune Systems: Part I – Basic Theory and Applications", FEEC/Unicamp, Campinas, SP, Tech. Rep. – RT DCA01/99, p.95. [Online]. Available: <http://www.dca.fee.unicamp.br/~lnunes/immune.html>.
 [8] F. L. Lewis "Applied optimal control and estimation", Prentice Hall 1992, ch.3.
 [9] Z. Qu and J.R. Cloutier, "A New Suboptimal Control Design for Cascaded Non-linear Systems" *Proc. Of American Control Conference*, Chicago, Illinois, Jun. 2000, pp. 3903-3907.
 [10] L. N. de Castro, J. Timmis, "Artificial immune systems: A new computational intelligence approach" (Eds) Springer, London, 2002.
 [11] C. Berek and M. Zeigner, "The maturation of the immune response", *Imm Today*, 1993, vol. 14(8), pp. 400-402.
 [12] J.R. Cloutier and D.T.Stansbery, "The Capabilities and Art of State-Dependent Riccati Equation-Based Design", *Proc. Of American Control Conference*, Anchorage, AK, May 2002, pp. 86-91.
 [13] R.A. Hull, J.R. Cloutier, C.P. Mracek and D.T.Stansbery, "State-Dependent Riccati Equation Solution of the Toy Nonlinear Optimal Control Problem", *Proc. Of American Control Conference*, Philadelphia, Pennsylvania, Jun. 1998, pp. 1658-1662.
 [14] J.R. Cloutier, "State-dependent Riccati equation techniques: an overview" *Proc. of the American Control Conference*, Albuquerque, New Mexico, Jun. 1997, pp. 932-936.
 [15] E.B. Erdem and A.G. Alleyne, "Design of a class of nonlinear controllers via state dependent Riccati equations" *IEEE Trans. On Control Systems Technology*, vol. 12, no. 1, Jan. 2004, pp.133-137
 [16] C. P. Mracek and J. R. Cloutier, "Control designs for the nonlinear benchmark problem via the State-Dependent Riccati Equation method," *Int. J. Robust Nonlinear Control*, vol. 8, no. 4/5, pp. 401-433, 1998.
 [17] J.R.Cloutier and J.C.Cockburn, "The state-dependent nonlinear regulator with state constraints", *American Control Conference*, 2001. Proceedings of the 2001, Vol.1, Iss., 2001 Pages:390-395 vol.1.