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On Discrete-Time Modeling of Time-Varying WSSUS Fading Channels

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***Abstract*— In this paper, we consider the serial concatenation of linear time-varying (LTV) systems and its impact on the discrete-time modeling of wide-sense stationary uncorrelated scattering (WSSUS) fading channels. By deriving an expression for the composite impulse response of the overall concatenated system, we find that unlike the time-invariant case, the concatenation of LTV systems is not commutative, i. e., the order of arrangement affects the overall impulse response. This has significant impact when a digital transmission over a time-varying fading channel, which is an LTV channel, is represented by an equivalent discrete-time model that incorporates both transmitter and receiver filters. We further show that if the maximum Doppler frequency is much smaller than the system bandwidth, the concatenation of LTV systems is approximately commutative, then a convenient and efficient representation in the discrete-time domain for WSSUS fading channels is obtainable.**

I. Introduction

Many mobile radio communication channels can be modeled as wide-sense stationary uncorrelated scattering (WSSUS) channels [1], [2], which are fully described by a two-dimensional Doppler-delay scattering function [3]. In rich scattering environments without line of sight, the uncorrelated paths are usually modeled as Rayleigh processes with a Doppler spectrum according to Clarke's two-dimensional isotropic scattering model [4]. For an efficient simulation of Clarke's reference model, various methods have been proposed in the literature, which are either based on filtered white noise or the sum-of-sinusoids principle (see [5] and the references therein).

Digital transmissions over linear time-varying WSSUS channels may be compactly represented and efficiently simulated using a fully discrete-time channel model that integrates transmitter and receiver filtering together with the LTV channel into an equivalent discrete-time channel impulse response [6], [7]. However, this discrete-time analog normally loses the uncorrelated scattering property [6] while preserving the wide-sense stationary (WSS) property. For accuracy reasons, the path correlations should be taken into account in a simulation model, or when designing a channel estimator that relies on proper second-order statistics. Moreover, the scattering function of an LTV channel is usually factorized into Doppler spectrum and power delay profile. Like the uncorrelated scattering (US) property, this separability property is also not preserved in the discrete-time model, as it is shown in this paper.

Therefor, we first consider the overall response of a serial concatenation of two LTV systems and then precisely analyze the discrete-time model of a digital transmission over an LTV channel. We further demonstrate that the separability property is approximately preserved if the channel is only slowly time-varying, as in most practical cases.

II. Serial Concatenation of Linear Time-Varying Systems

The serial concatenation of two LTV systems $a(t,\tau)$ and $b(t,\tau)$ is shown in Fig. 1, where $g(t,\tau)$ is the output of the system g at time t to a delta pulse at time $t-\tau$. Let $c(t,\tau)$ be the response of the concatenation of a and b, such that

$$z(t) = \int_{-\infty}^{\infty} c(t,\tau)x(t-\tau)\,d\tau. \tag{1}$$

We have

$$y(t) = \int_{-\infty}^{\infty} a(t,\alpha)x(t-\alpha)\,d\alpha, \tag{2}$$

$$z(t) = \int_{-\infty}^{\infty} b(t,\beta)y(t-\beta)\,d\beta. \tag{3}$$

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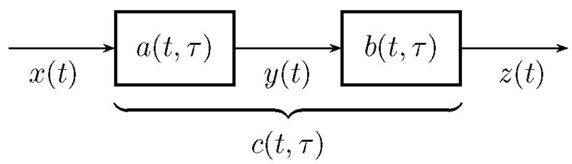

Fig. 1. Serial concatenation of two LTV systems.

Combining (2), (3) and substituting $\alpha + \beta \to \tau$ gives

$$z(t) = \iint b(t,\beta) a(t-\beta, \tau-\beta)\, d\beta\, x(t-\tau)\, d\tau. \quad (4)$$

By comparing (1) and (4), the response of the overall LTV system $c(t,\tau)$ comprising the two LTV systems $a(t,\tau)$ and $b(t,\tau)$, where $a(t,\tau)$ is followed by $b(t,\tau)$, is obtained as

$$\begin{aligned} c(t,\tau) &\triangleq a(t,\tau) \circledast b(t,\tau) \\ &= \int_{-\infty}^{\infty} b(t,\beta) a(t-\beta, \tau-\beta)\, d\beta. \end{aligned} \quad (5)$$

In (5), we have introduced the directional convolution operator "$\circledast$" which denotes the serial concatenation of two LTV systems. In contrast to time-invariant systems, the order of arrangement matters if at least one of the systems a and b is time-varying. In such a case, the convolution "$\circledast$" is not commutative anymore, i. e.,

$$a(t,\tau) \circledast b(t,\tau) \neq b(t,\tau) \circledast a(t,\tau) \quad (6)$$

in general. To verify (6), substitute $\tau - \beta \to \beta'$ in (5)

$$c(t,\tau) = \int_{-\infty}^{\infty} a(t-\tau+\beta', \beta') b(t, \tau-\beta')\, d\beta' \quad (7)$$

and compare (7) to the overall response of the reversed arrangement of a and b, which is $c'(t,\tau) = \int a(t,\beta') b(t-\beta', \tau-\beta')\, d\beta'$ and differs from (7) in general. However, if $a(t,\beta')$ is virtually time-invariant within the support of $b(\cdot, \tau-\beta')$ and vice versa, then the two systems may be interchanged without affecting the overall response. This is the case if the Doppler spread of any of the two systems is small compared to the system bandwidth of the respective other system. The latter conclusion becomes more apparent if we describe the two LTV systems a and b by their output Doppler-spread functions [3].

The output Doppler-spread function $G(\nu, f)$ of an LTV system g is the two-dimensional Fourier transform of its time-varying impulse response $g(t,\tau)$

$$G(\nu, f) = \iint g(t,\tau) e^{-j2\pi\nu t} e^{-j2\pi f\tau}\, dt\, d\tau. \quad (8)$$

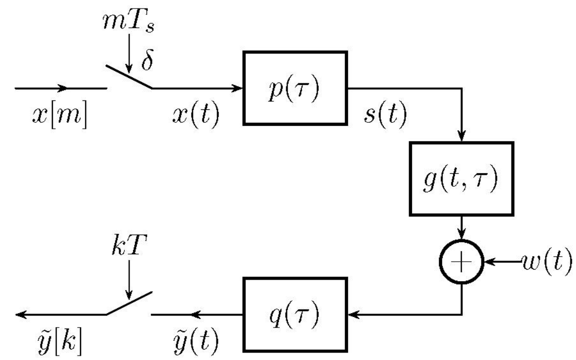

Fig. 2. Transmission model.

It allows to describe the input-output relationship of the two systems a and b in the frequency domain [cf. (2)–(3)]:

$$Y(f) = \int_{-\infty}^{\infty} A(\alpha, f-\alpha) X(f-\alpha)\, d\alpha, \quad (9)$$

$$Z(f) = \int_{-\infty}^{\infty} B(\beta, f-\beta) Y(f-\beta)\, d\beta. \quad (10)$$

Similar to (4)–(5), the overall output Doppler spread function $C(\nu, f)$ of the serial concatenation is found to be

$$C(\nu, f) = \int_{-\infty}^{\infty} B(\nu-\alpha, f+\alpha) A(\alpha, f)\, d\alpha. \quad (11)$$

(11) confirms the statement that if the ν-support (Doppler spread) of both systems is negligibly small compared to the f-support (system bandwidth), we may interchange the two systems (substitute $\nu - \alpha \to \alpha'$ and compare (11) to the result for a reversed arrangement). For fully time-invariant systems $A(\nu, f) = \delta(\nu) A(f)$ and $B(\nu, f) = \delta(\nu) B(f)$, the well-known result $C(\nu, f) = \delta(\nu) B(f) A(f)$ follows as a special case.

III. Discrete-Time Modeling of Linear Time-Varying WSSUS Channels

We now apply the previous findings to determine the correlation properties of an equivalent discrete-time channel model that complies with the transmission model shown in Fig. 2. Throughout the paper we make use of the complex baseband equivalent notation. After weighting the data sequence $x[m]$ with the transmit pulse $p(\tau)$ periodically at a symbol rate of $1/T_s$, the resultant signal $s(t)$ is transmitted over the LTV channel $g(t,\tau)$ and white noise $w(t)$ is added. $g(t,\tau)$ is assumed to be a WSSUS Rayleigh fading channel according to Clarke's

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isotropic scattering model [4]. The autocorrelation R_{gg} of $g(t,\tau)$ is separable and given by

$$R_{gg}(t, t-\Delta t; \tau, \tau') \triangleq \mathrm{E}\{g(t,\tau)g^*(t-\Delta t, \tau')\} \\ = \psi(\Delta t)\cdot\phi(\tau)\delta(\tau-\tau') \quad (12)$$

where $\psi(\Delta t) = J_0(2\pi B_d \Delta t)$ is the time correlation function for Clarke's model with B_d being the Doppler spread of the channel. $\phi(\tau)$ denotes the power delay profile with support T_g, i. e., $\phi(\tau) = 0$ for $\tau < 0$ and $\tau > T_g$.

After filtering with $q(\tau)$, the receive signal $\tilde{y}(t)$ is sampled at a rate of $1/T$ to yield the receive sequence $\tilde{y}[k]$. The sampling rate is either chosen equal to the symbol rate or as an integer multiple $\eta \geq 1$ of this ($T_s = \eta T$ with oversampling factor η) to account for the bandwidth spread that is introduced by the channel [8]. The receive filter $q(\tau)$ may either be a simple matched filter that is matched to the transmit pulse only, $q(\tau) = p^*(-\tau)$, or a more sophisticated pulse that maximizes the signal to noise ratio at the sampling instant for every channel realization or in a statistical sense.

In our model, the data bearing impulse signal

$$x(t) = \sum_{m=-\infty}^{\infty} x[m]\delta(t - mT_s) \quad (13)$$

is transmitted over an effective overall channel $h(t,\tau)$ that comprises the transmit filter $p(\tau)$, the LTV channel $g(t,\tau)$, and the receive filter $q(\tau)$

$$h(t,\tau) = p(\tau) \circledast g(t,\tau) \circledast q(\tau) \\ = \iint q(\beta)g(t-\beta,\alpha)p(\tau-\beta-\alpha)\,d\alpha\,d\beta \quad (14)$$

where (14) follows from applying (5) twice. As the channel $g(t,\tau)$ is a time-varying system, the order of arrangement is of importance and, therefore, affects the overall response $h(t,\tau)$ along with its correlation properties.

Based on $h(t,\tau)$, we can now describe the input-output relationship of our transmission model (see Fig. 2) in the discrete-time domain as follows:

$$\tilde{y}[k] = \sum_{l=-L_1}^{L_2} h[k,l]x[k-l] + n[k] \quad (15)$$

where $h[k,l] = h(kT, lT)$ is the discrete-time response at time k to an impulse with lag l. $n[k] = \int q(\tau)w(kT - \tau)\,d\tau$ is a colored noise sequence in general. In (15), the transmit sequence $x[l]$ is an oversampled version of $x[m]$ in order to allow for different rates at transmitter and receiver

$$x[l] = \begin{cases} x[m], & \text{if } l = \eta m, \\ 0, & \text{otherwise.} \end{cases} \quad (16)$$

Although the support of $g(t,\tau)$ was assumed to be finite and causal, $h(t,\tau)$ is of infinite duration in the strict sense, because $p(\tau)$ and $q(\tau)$ are usually band-limited filters. However, the effective support of $h(t,\tau)$ and, therefore, $h[k,l]$ is virtually finite, and denoted by the range $[-L_1, L_2]$, where L_1 and L_2 are non-negative integers.

To assure that the discrete-time model (15) is equivalent to its continuous-time counterpart, the statistical properties of the former should reflect those of the latter. Since the considered transmission model is linear and $g(t,\tau)$ is a Rayleigh process, also $h[k,l]$ will be a (now discrete-time) Rayleigh process. As already pointed out, $h[k,l]$ may however be lacking some of the specific correlation properties of $g(t,\tau)$, which was assumed to be WSSUS with separable scattering function. Using (14) and (12), the autocorrelation of the sampled overall response $h[k,l]$ is deduced as

$$R_{hh}[k_1,k_2;l_1,l_2] \triangleq \mathrm{E}\{h[k_1,l_1]h^*[k_2,l_2]\} \\ = \int_0^{T_g} \phi(\tau) \iint \psi([k_1-k_2]T - (\beta_1-\beta_2))\, q(\beta_1)q^*(\beta_2) \\ \cdot p(l_1T - \tau - \beta_1)p^*(l_2T-\tau-\beta_2)\,d\beta_2\,d\beta_1\,d\tau. \quad (17)$$

Since its autocorrelation depends on the difference $k_1 - k_2$ only, the discrete-time Rayleigh process $h[k,l]$ is still WSS. However, both the uncorrelated scattering and the separability property are lost and the above correlation function is an unpleasant expression mixing time and delay correlations. Especially, the lack of separability prohibits the use of efficient channel simulators for Clarke's reference model (like [5]) as the time correlation function is different from $\psi([k_1-k_2]T) = J_0(2\pi B_d[k_1 - k_2]T)$. Anyhow, the following approximation helps to handle this difficulty.

APPROXIMATION: If the Doppler spread B_d of the linear time-varying channel $g(t,\tau)$ is small compared to the bandwidth B_q of the receive filter $q(\tau)$, i. e., if $B_d/B_q \ll 1$, then the autocorrelation of the discrete-time response $h[k,l]$ is approximately separable:

$$R_{hh}[k_1,k_2;l_1,l_2] \approx \psi([k_1-k_2]T)\cdot\varphi[l_1,l_2] \quad (18)$$

where

$$\varphi[l_1,l_2] = \int_0^{T_g} \phi(\tau)r(l_1T-\tau)r^*(l_2T-\tau)\,d\tau \quad (19)$$

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with $r(\tau) = \int q(\alpha)p(\tau - \alpha)\, d\alpha$ being the convolution of $p(\tau)$ and $q(\tau)$.

The approximation is justified as follows. The right hand side (RHS) of (18) actually belongs to a notional transmission model that arranges $g(t, \tau)$ and the receive filter $q(\tau)$ in reversed order. The overall impulse response $\tilde{h}(t, \tau) \triangleq p(\tau) \circledast q(\tau) \circledast g(t, \tau)$ of such a configuration is given by

$$\tilde{h}(t, \tau) = \iint g(t, \alpha)q(\beta)p(\tau - \beta - \alpha)\, d\alpha\, d\beta. \quad (20)$$

In (20), the absolute time t in $g(t, \tau)$ is not being delayed [compare (14)], since the time-varying channel is now the last system in the sequence. Using (20), it is easily verified that the autocorrelation of the sampled response $\tilde{h}[k, l] = \tilde{h}(kT, lT)$ is given by (18). As stated in Section II, the condition $B_d/B_q \ll 1$ implies that the channel $g(t, \tau)$ remains virtually constant during receiver filtering with $q(\tau)$, which allows to change the order of the two systems.

As before, the validity of the approximation (18) is better seen in the frequency domain. By applying (11) twice, we obtain the dual of the overall response $h(t, \tau)$ [see (14)], the output Doppler-spread function $H(\nu, f)$, as

$$H(\nu, f) = Q(f + \nu)G(\nu, f)P(f). \quad (21)$$

Its autocorrelation involves the autocorrelation of $G(\nu, f)$, which follows from (12) and (8)

$$\begin{aligned} R_{GG}(\nu, \nu'; f, f') &\triangleq \mathrm{E}\{G(\nu, f)G^*(\nu', f')\} \\ &= \Psi(\nu)\delta(\nu - \nu') \cdot \Phi(f - f') \end{aligned} \quad (22)$$

where $\Psi(\nu)$ is the Doppler spectrum (Fourier transform of $\psi(\Delta t)$, the time correlation function) and $\Phi(f - f')$ is the Fourier transform of the power delay profile $\phi(\tau)$ at $f - f'$. Equation (22) shows the duality between R_{gg} and R_{GG}. As $g(t, \tau)$ is WSS in t and US in τ, $G(\nu, f)$ is US in ν and WSS in f.

From (21) and (22), we finally have

$$\begin{aligned} &R_{HH}(\nu, \nu'; f, f') \\ &= Q(f + \nu)Q^*(f' + \nu')R_{GG}(\nu, \nu'; f, f')P(f)P(f') \\ &\approx \Psi(\nu)\delta(\nu - \nu') \cdot \Phi(f - f')Q(f)Q(f')P(f)P(f'). \end{aligned} \quad (23)$$

In (23), the approximation holds for $B_d/B_q \ll 1$ and provides a frequency-domain representation of the impulse response $\tilde{h}(t, \tau)$ [see (20)] of the notional model that interchanges LTV channel and receive filter. If the condition $B_d/B_q \ll 1$ is fulfilled, then R_{HH} can be separated into Doppler spectrum and frequency correlation function. Although (23) applies to the continuous-time response $h(t, \tau)$, there is a corresponding correlation function for its sampled version $h[k, l]$. Let the two-dimensional discrete-time Fourier transform (DTFT) of $h[k, l]$

$$H\left(e^{j2\pi\nu T}, e^{j2\pi f T}\right) = \sum_k \sum_l h[k, l] e^{-j2\pi\nu T k} e^{-j2\pi f T l} \quad (24)$$

be the discrete-time counterpart of (8), the continuous-time output Doppler-spread function. Then, (23) applies to (24) as well and its inverse DTFT results in (18).

The requirement $B_d/B_q \ll 1$ is well fulfilled in most practical applications. Take the UMTS standard [9] as an example. The standard specifies a square-root raised cosine transmit filter with roll-off factor 0.22 and a symbol period of $T_s = 260.42\,\mathrm{ns}$. At a carrier frequency of 2 GHz, for a mobile velocity of 200 km/h, and assuming a receive filter that is equal to the transmit filter, we have $B_d/B_q < 10^{-4}$. In a multicarrier system like OFDM (orthogonal frequency division multiplexing) where the Doppler spread B_d is not always negligible with respect to the subcarrier bandwidth, the ratio B_d/B_q will be quite small, though, because of the usually large number of subcarriers.

Since the scattering function of the discrete-time channel $h[k, l]$ is approximately separable if the physical channel $g(t, \tau)$ is only slowly time-varying, every path $h[k, \cdot]$ of the discrete-time model (15) is a Rayleigh process with a Doppler spectrum according to Clarke's reference model. We may, therefore, use efficient sum-of-sinusoids simulators that have been designed for this model. If such a simulator is able to produce uncorrelated path processes $h[k, \cdot]$ (as the one proposed in [5]), then the path correlations according to (18)–(19) can be introduced by treating the impulse response $h[\cdot, l]$, $-L_1 \le l \le L_2$ as a vector and multiplying this vector with a correlation matrix whose entries are given by (19). More on this issue together with an extension to MIMO systems can be found in [7].

IV. Conclusions

In contrast to time-invariant systems, LTV systems are not commutative in general. This gains importance when a linear time-varying WSSUS channel with separable scattering function is modeled in the discrete-time domain, including transmitter and receiver filtering. In general, only the WSS property is preserved in a discrete-time model, but the uncorrelated scattering and separability property will be lost. For such a general

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discrete-time model, the autocorrelation of the LTV channel response is given by (17).

However, LTV systems are approximately commutative if the Doppler spreads are small with respect to the system bandwidths. In such a case, WSSUS channels have a convenient representation in the discrete-time domain, as the separability property of the scattering function is approximately preserved. If, in addition, the fading is Rayleigh or Rician and a Doppler spectrum due to Clarke applies, then efficient sum-of-sinusoids simulators can be used.

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