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Damping Inter-Area Oscillations by UPFCs Based on Selected Global Measurements

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***Abstract*—This paper introduces a method of using a selected set of the global data for controlling inter-area oscillations of the power network using Unified Power Flow Controllers. This novel algorithm utilizes reduced order observers for estimating the missing data the purpose of control when all the data is unavailable through frequency measurements in a Wide Area Control approach. The paper will also address the problem of time-delay in data acquisition through examples.**

***Index Terms*—UPFC, Inter-Area Oscillation, PMU, Wide Area Control, Reduced Order Observer**

I. Introduction

DAMPING inter-area oscillations in a power network is one of the important applications of a Unified Power Flow Controller (UPFC) [1]-[6]. These oscillations can occur in a system as a result of a number of contingencies including sudden load changes or power system faults. Fig. 1 shows a schematic diagram of a UPFC, which is a series-shunt flexible AC Transmission (FACTS) device. Controlling power oscillations can be done by rapidly changing the power flow via the series part of the UPFC, which in turn can impact the power flow through the entire network. The needed electrical power to do this action is channeled from the UPFC's shunt part. Namely, the series part is controlled by the modulation gain and phase angle (k_2, α_2) of the injected voltage vector and the shunt part is controlled by modulation gain and phase angle (k_1, α_1) of the internal voltage of the shunt converter.

As explained earlier, damping inter-area oscillations demands the dynamic change of active power flow through the network. To rapidly damp interarea oscillations requires knowledge of the many modes that are inherent in oscillatory behavior. Frequency responses are rich in modal content and have been shown to be excellent inputs in controller design. In previous work [7], the authors have shown that using global frequency data as input to the UPFC controller provides effective inter-area oscillation damping. Although the introduction of frequency measurements (such as from FNET [12]) has made wide area control of the power networks feasible, it is still not reasonable to expect that the full set of frequency measurements is available for controller use. Therefore most UPFC controls still rely heavily on locally available measurements. This is why decentralized control schemes for damping inter-area oscillations must be developed. However, decentralized control schemes that depend primarily on local information can be significantly improved by the addition of select global frequency information.

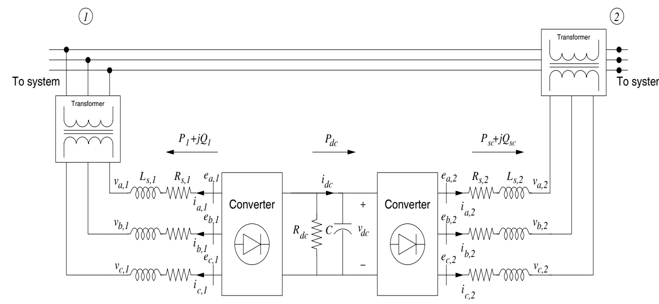

Fig. 1. Unified Power Flow Controller Diagram

This paper introduces an approach which shows that it is possible to damp out power network oscillations using only a limited set of measurements. The work is based on the two stage type of controller introduced in [7], which instead of using a whole set of global state feedback data, uses a subset of the global data plus an estimated set of data to run the controller. Simulations on the IEEE 14 bus test system show that the scheme is satisfactory.

In the following sections, the two stage controller will be reviewed. Then the method for designing a reduced order estimator based on the available feedback data will be provided. Next, simulation examples and their comparison with full state feedback controllers will be provided. In the end, the effect of time delay in global data acquisition will be shown and further work will be proposed.

II. Two Stage Controller

In this section, the two stage controller design is summarized from [7].

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Using a power injection model for the UPFC schematically shown in Fig. 2, the dynamical equations for the UPFC can be written as [8]:

$$\frac{1}{\omega_S}\dot{i}_{d_{1j}} = -\frac{R_{1j}}{L_{1j}} i_{d_{1j}} + i_{q_{1j}} + \frac{k_{1j}}{L_{1j}}\cos(\theta_{1j}+\alpha_{1j}) v_{dc_j} - \frac{V_{1d_j}}{L_{1j}} \quad (1)$$

$$\frac{1}{\omega_S}\dot{i}_{q_{1j}} = -\frac{R_{1j}}{L_{1j}} i_{q_{1j}} - i_{d_{1j}} + \frac{k_{1j}}{L_{1j}}\sin(\theta_{1j}+\alpha_{1j}) v_{dc_j} - \frac{V_{1q_j}}{L_{1j}} \quad (2)$$

$$\frac{1}{\omega_S}\dot{i}_{d_{2j}} = -\frac{R_{2j}}{L_{2j}} i_{d_{2j}} + i_{q_{2j}} + \frac{k_{2j}}{L_{2j}}\cos(\theta_{1j}+\alpha_{2j}) v_{dc_j} - \frac{V_{2d_j}}{L_{2j}} + \frac{V_{1d_j}}{L_{2j}} \quad (3)$$

$$\frac{1}{\omega_S}\dot{i}_{q_{2j}} = -\frac{R_{2j}}{L_{2j}} i_{q_{2j}} - i_{d_{2j}} + \frac{k_{2j}}{L_{2j}}\sin(\theta_{1j}+\alpha_{2j}) v_{dc_j} - \frac{V_{2q_j}}{L_{2j}} + \frac{V_{1q_j}}{L_{2j}} \quad (4)$$

$$\frac{C}{\omega_S}\dot{v}_{dc_j} = -k_{1j}\cos(\theta_{1j}+\alpha_{1j}) i_{d_{1j}} - k_{1j}\sin(\theta_{1j}+\alpha_{1j}) i_{q_{1j}}$$
$$-k_{2j}\cos(\theta_{1j}+\alpha_{2j}) i_{d_{2j}} - k_{2j}\sin(\theta_{1j}+\alpha_{2j}) i_{q_{2j}} - \frac{v_{dc_j}}{R_{p_j}} \quad (5)$$

$$V_{1d_j} = V_{1j}\cos\theta_{1j} \quad (6)$$

$$V_{1q_j} = V_{1j}\sin\theta_{1j} \quad (7)$$

$$V_{2d_j} = V_{2j}\cos\theta_{2j} \quad (8)$$

$$V_{2q_j} = V_{2j}\sin\theta_{2j} \quad (9)$$

where:

n_u : Number of UPFCs

$j = 1, ..., n_u$

$i_{1,j} = i_{d_{1j}} + j i_{q_{1j}}$: Shunt injection current in UPFC *j* (pu)

$i_{2,j} = i_{d_{2j}} + j i_{q_{2j}}$: Series injection current in UPFC *j* (pu)

R_{1j} : Equivalent shunt resistance of UPFC *j* (pu)

L_{1j} : Equivalent shunt inductance of UPFC *j* (pu)

R_{2j} : Equivalent series resistance of UPFC *j* (pu)

L_{2j} : Equivalent series inductance of UPFC *j* (pu)

v_{dc_j} : DC bus voltage in UPFC *j* (pu)

C_j : Equivalent capacitance of UPFC *j* (pu)

R_{p_j} : Equivalent dc resistance in UPFC j (pu)

k_{1j}, α_{1j} : Modulation amplitude and angle of the shunt part of UPFC *j*

k_{2j}, α_{2j} : Modulation amplitude and angle of the series part of UPFC *j*

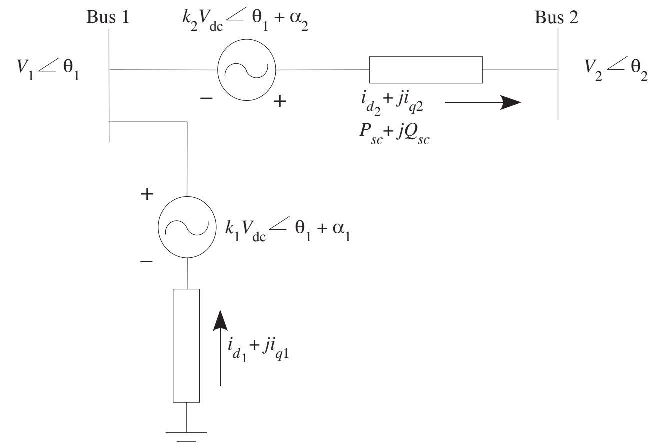

Fig. 2. Power Injection Model for UPFC

To derive the two stage controller, the loads in the system are initially converted to constant impedance. Further, the generators are modeled as the classical "transient reactance behind constant voltage." Note that these assumptions are made only to develop the UPFC control – the proposed two stage control will be validated on a system with constant power loads and with fully modeled (10th order) generators with voltage regulators. With the constant impedance assumption, it is possible to find a reduced admittance matrix of order $n_g + 2n_u$, where n_g internal machine buses are connected to $2n_u$ UPFC buses as shown in Fig. 3. The dashed lines in the figure show that in the reduced order network, virtually all the buses are connected to each other.

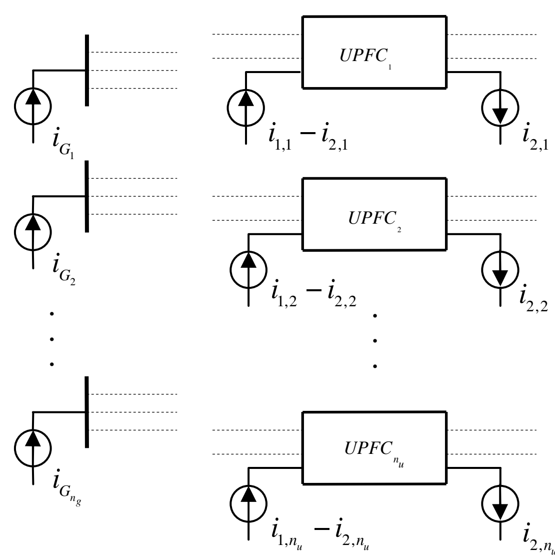

Fig. 3. Equivalent Power System from the Controller's View

Using the above, the resulted state space system for the power network would be of the following format:

$$\dot{\delta}_j = \omega_j - \omega_1 \qquad j = 2, \ldots, n_g \quad (10)$$

and for

$$j = 1, \ldots, n_g$$

$$\dot{\omega}_j = (1/M_j)[P_{M_j} - E_j \sum_{k=1}^{n_g} E_k Y_{jk} \cos(\delta_j - \delta_k - \Phi_{jk})$$
$$- E_j \sum_{k=n_g+1}^{n_g+n_u} (E_k Y_{jk} \cos(\delta_j - \Phi_{jk}) r_{2(k-n_g-1)+1} + E_k Y_{jk} \sin(\delta_j - \Phi_{jk}) r_{2(k-n_g-1)+2}) \quad (11)$$
$$- E_j \sum_{k=n_g+n_u+1}^{n_g+2n_u} (E_k Y_{jk} \cos(\delta_j - \Phi_{jk}) r_{2(k-n_g-1)+1} + E_k Y_{jk} \sin(\delta_j - \Phi_{jk}) r_{2(k-n_g-1)+2})]$$

where:

ω_j : Speed of machine j (rad/s) $\quad j = 1, \ldots, n_g$

M_j : Inertia at machine j (pu) $\quad j = 1, \ldots, n_g$

P_{M_j} : Mechanical input at machine j (pu) $\quad j = 1, \ldots, n_g$

δ_i : Angle at bus i (Radians) $\quad i = 1, \ldots, n_g + 2n_u$

E_i : Bus magnitude at bus i (pu) $\quad i = 1, \ldots, n_g + 2n_u$

$Y_{jk} \angle \Phi_{jk}$: Admittance matrix of the equivalent reduced system for $j, k = 1, \ldots, n_g + 2n_u$

This nonlinear state space system is of the order $2n_g - 1$ with intermediate control inputs r_i defined as:

$$\begin{aligned} r_{2(j-1)+1} &= V_{1dj} \quad j = 1, \ldots, n_u \\ r_{2(j-1)+2} &= V_{1qj} \\ r_{2(j-1)+2n_u+1} &= V_{2dj} \\ r_{2(j-1)+2n_u+2} &= V_{2qj} \end{aligned} \quad (12)$$

The above state space set, describes the first stage of the control process. Note that this first stage is independent of the UPFC dynamics of equations (1)-(5). Linearizing this system results in a linear time invariant system of the form:

$$\dot{X} = AX + BR \quad (13)$$

Linear feedback control is given by:

$$R = -KX \quad (14)$$

where K is chosen using optimal LQR control processes to minimize speed and angle deviations in the generators.

If the system were linear, the above control results in optimal values of voltages at both sending and receiving buses of the UPFC. The second control stage is to convert the control inputs R into modulation gain and phase angles k and α for each UPFC. The first step in the second control stage is to find the values of $i_{d_1}, i_{q_1}, i_{d_2}, i_{q_2}$ from the following four active and reactive power balance equations at the sending and receiving buses of the UPFC:

$$P_{1j} = V_{1dj}(i_{d1j} - i_{d2j}) + V_{1qj}(i_{q1j} - i_{q2j}) \quad (15)$$

$$Q_{1j} = V_{1qj}(i_{d1j} - i_{d2j}) - V_{1dj}(i_{q1j} - i_{q2j}) \quad (16)$$

$$P_{2j} = V_{2dj} i_{d2j} + V_{2qj} i_{q2j} \quad (17)$$

$$Q_{2j} = V_{2qj} i_{d2j} - V_{2dj} i_{q2j} \quad (18)$$

where:

P_{1j} : Active Power from j^{th} UPFCs sending end bus

Q_{1j} : Reactive Power from j^{th} UPFCs sending end bus

P_{2j} : Active Power from j^{th} UPFCs receiving end bus

Q_{2j} : Reactive Power from j^{th} UPFCs receiving end bus

The calculation made above is based on the fact that the UPFC's dynamics are much faster than the machines' dynamics so that $i_{d_1}, i_{q_1}, i_{d_2}, i_{q_2}$ can be taken to be algebraic quantities. With this assumption, converting (1) to (4) to algebraic equations results in:

$$0 = -\frac{R_{1j}}{L_{1j}} i_{d_{1j}} + i_{q_{1j}} + \frac{k_{1j}}{L_{1j}} \cos(\theta_{1j} + \alpha_{1j}) v_{dc_j} - \frac{V_{1d_j}}{L_{1j}} \quad (19)$$

$$0 = -\frac{R_{1j}}{L_{1j}} i_{q_{1j}} - i_{d_{1j}} + \frac{k_{1j}}{L_{1j}} \sin(\theta_{1j} + \alpha_{1j}) v_{dc_j} - \frac{V_{1q_j}}{L_{1j}} \quad (20)$$

$$0 = -\frac{R_{2j}}{L_{2j}} i_{d_{2j}} + i_{q_{2j}} + \frac{k_{2j}}{L_{2j}} \cos(\theta_{1j} + \alpha_{2j}) v_{dc_j} - \frac{V_{2d_j}}{L_{2j}} + \frac{V_{1d_j}}{L_{2j}} \quad (21)$$

$$0 = -\frac{R_{2j}}{L_{2j}} i_{q_{2j}} - i_{d_{2j}} + \frac{k_{2j}}{L_{2j}} \sin(\theta_{1j} + \alpha_{2j}) v_{dc_j} - \frac{V_{2q_j}}{L_{2j}} + \frac{V_{1q_j}}{L_{2j}} \quad (22)$$

Solving the differential-algebraic equation set of (19) to (22) and (5) will result in the values of $k_1, \alpha_1, k_2, \alpha_2, v_{dc}$. It should be noted that in a network having multiple UPFCs, this procedure can be done independently for each of them, since the first stage control provides the network coupling in the determination of the input R which are the dq sending and receiving voltages. Fig. 4 shows a flowchart which describes the two stage control.

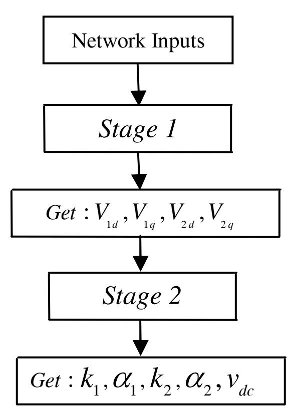

Fig. 4. Two Stage Control Design

III. Reduced Order Estimator Design

The control scheme explained in section II is best implemented if all the feedback data (machines' speeds and angles) are available to the controller. However, in a wide-area network spread over long distances this assumption may not be feasible. The idea behind a reduced order observer is to estimate the unavailable feedback data using the available data. Dividing the states into a directly available set of X_1 and observable set of X_2, equation (3) can be rewritten [9]:

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} U \tag{23}$$

The directly available set of X_1 can be verified through:

$$Y = C_1 X_1 \tag{24}$$

where C_1 is a square and nonsingular matrix. The estimated set of $\hat{X}_2$ is of the form:

$$\hat{X}_2 = LY + Z \tag{25}$$

Considering the dynamics of Z to be of the form:

$$\dot{Z} = FZ + GY + HU \tag{26}$$

the estimation error can be written as:

$$e_2 = X_2 - \hat{X}_2 \tag{27}$$

Therefore, the dynamics of the error is given by:

$$\begin{aligned} \dot{e}_2 &= (A_{21} - LC_1A_{11} + FLC_1 - GC_1)X_1 + \\ &(A_{22} - LC_1A_{12} - F)X_2 + (B_2 - LC_1B_1 - H)U + Fe_2 \end{aligned} \tag{28}$$

Setting the coefficients of X_1, X_2 and U to be zero in equation (28) results in:

$$\dot{e}_2 = Fe_2 \tag{29}$$

$$H = B_2 - LC_1B_1 \tag{30}$$

$$F = A_{22} - LC_1A_{12} \tag{31}$$

$$G = A_{21}C_1^{-1} - LC_1A_{11}C_1^{-1} + FL \tag{32}$$

Defining L to be the observer gain, L can be chosen such that F is negative definite. This guarantees the estimator to be stable and accurate.

Based on the designed estimator, the applied control takes the form of:

$$U = -[K_1, K_2] \begin{bmatrix} X_1 \\ \hat{X}_2 \end{bmatrix} \tag{33}$$

IV. Example and Discussion

The IEEE 14 bus test system [10] has been used to validate the reduced order estimator and the proposed two stage control process in the control of inter-area oscillations. This system has 5 machines and can be roughly considered to have two areas, where machines 1, 2 and 3 form one area and machines 4 and 5 form the other area. The generators are modeled with 10th order models containing the two-axis generator model, Type I Exciter/AVR model and turbine and governor models. The diagram of the network is shown in Fig. 5.

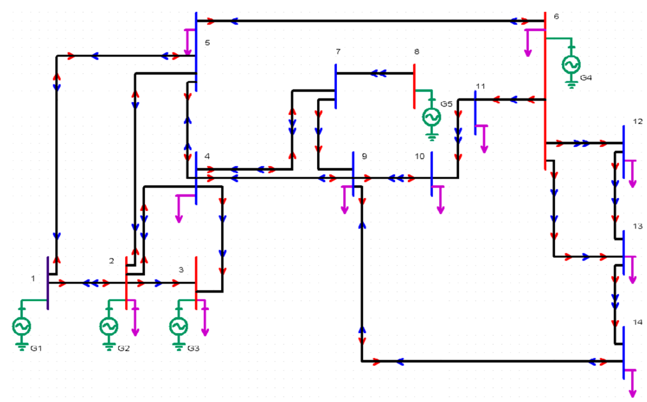

Fig. 5. IEEE 14 Bus Test System

The full differential-algebraic system has been simulated using MATLAB with a high impedance fault of 1 pu applied on bus 10 at 0.2 s and removed at 0.4 s. One UPFC has been installed in the system on the line between buses 5 and 6 with the shunt (sending) part of the UPFC on bus 5. The operating point of the UPFC has been initialized using the method discussed in [11]. The characteristics of the UPFC along with its pre-fault steady state operating points are as follows:

$$R_1 = 0.01pu, L_1 = 0.15pu$$

$$R_2 = 0.001pu, L_2 = 0.015pu$$

$$R_p = 200pu$$

$$C = 5pu$$

$$PLoss = 0.2pu$$

$$k_1 = 0.1654, \alpha_1 = 358.3661^\circ$$

$$k_2 = 0.0015, \alpha_2 = 98.4359^\circ$$

$$v_{dc} = 6.3147\text{pu}$$

$$i_{d_1} = -0.2036\text{pu}, i_{q_1} = -0.0401\text{pu}$$

$$i_{d_2} = 0.4369\text{pu}, i_{q_2} = 0.0442\text{pu}$$

TABLE I
SIMULATION CASES ACCORDING TO THE FEEDBACK AND ESTIMATED STATES

	Feedback States	Feedback States' Delay (ms)	Estimated States
Case I (No Control)	---	0	---
Case II (All Feedback Available)	$\omega_1, \omega_2, \omega_3, \omega_4, \omega_5$ $\delta_2, \delta_3, \delta_4, \delta_5$	0	---
Case III	ω_2, ω_5	0	$\omega_1, \omega_3, \omega_4$ $\delta_2, \delta_3, \delta_4, \delta_5$
Case IV	ω_2, ω_4	0	$\omega_1, \omega_3, \omega_5$ $\delta_2, \delta_3, \delta_4, \delta_5$
Case V	ω_2, ω_4	5<t<15	$\omega_1, \omega_3, \omega_5$ $\delta_2, \delta_3, \delta_4, \delta_5$
Case VI	ω_2, ω_4	5<t<20	$\omega_1, \omega_3, \omega_5$ $\delta_2, \delta_3, \delta_4, \delta_5$

Table I provides the detail of six different cases that were simulated. The first case is the base case in which there is no UPFC control. In Case II, the proposed two-stage control is utilized with full state feedback. Cases III and IV provide a comparison between scenarios in which different global signals are available (frequencies only according to FNET) and the remaining frequencies and all angles are estimated. The final two cases (V and VI) illustrate the impact of time delay on the global signal feedback.

Fig. 6 shows the simulation results for the speeds of machines 2, 4 and 5 (from top to bottom, respectively), for case I (thin) and II (bold). The figure shows that the proposed two stage control using the entire machine's data as feedback to the controller can effectively damp out inter-area oscillations uniformly in a short time.

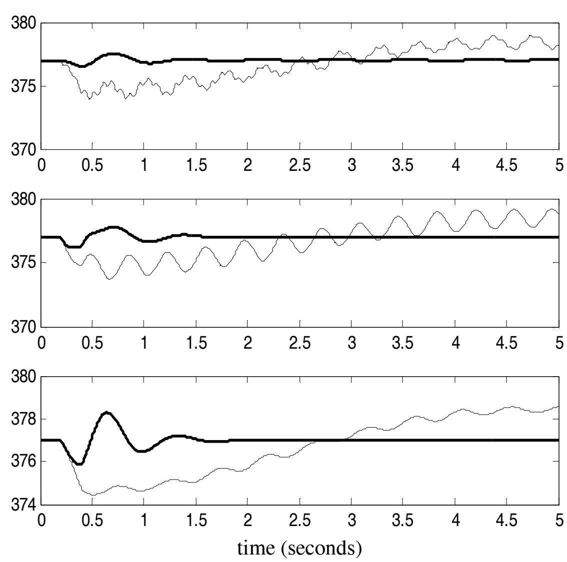

Fig. 6. Machine speeds $\omega_2, \omega_4, \omega_5$ (rad/s) (thin: case I, bold: case II)

Fig. 7 shows a comparison between cases II (bold), III (thin) and IV (boldest). Note that although in cases III and IV the same number of feedback states is available, the overall performance of case IV shows better performance. This shows that selecting the proper feedback states can play an important role in the performance of the controller. On the other hand, although in case IV, we get good speed deviations' damping for machines 2 and 4, but for machine 5 this is not happening.

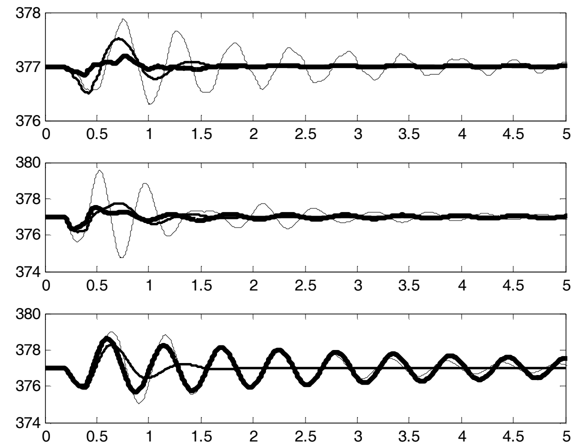

Fig. 7. Machine speeds $\omega_2, \omega_4, \omega_5$ (rad/s) (bold: Case II, thin: Case III, boldest: Case IV)

To verify how time delays from the available states in a wide-area controlled system impact the performance of a controller, cases V and VI introduce a randomly varying time delay into each of the globally available signals. In Case V, the time delay varies randomly between 5 and 15 ms. In Case VI, the time delay varies randomly between 5 and 20 ms.

Fig. 8 compares Cases IV and V, which both have the same observer design. The only difference is in the amount of time delay in the global signal feedback. The comparison shows that there is a slight degradation in performance caused by the signal delay, the controller and estimator still perform well and suitably control the system.

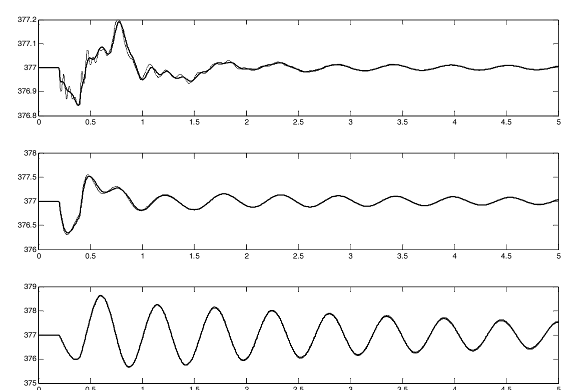

Fig. 8. Machine speeds $\omega_2, \omega_4, \omega_5$ (rad/s) (thin: Case V, bold: Case IV)

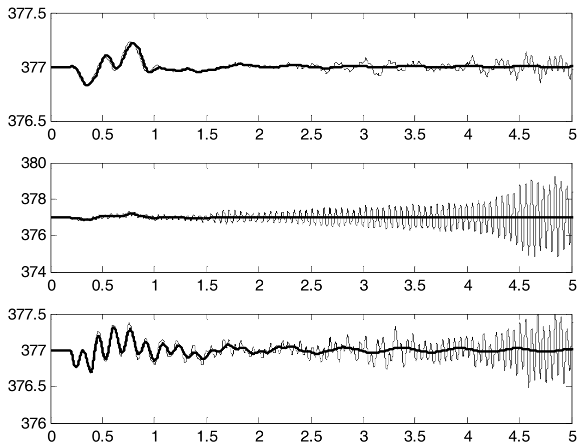

Fig. 9. Machine speeds $\omega_1, \omega_2, \omega_3$ (rad/s) (thin: case VI, bold: case IV)

Fig. 9 shows the comparison of Case IV and Case VI in which the random delays can be up to 20ms. This difference has a significant impact on the ability of the UPFC to damp the system oscillations. In fact, the UPFC actually destabilizes the system by causing undamped high frequency oscillations. At this time, the authors are investigating whether the estimator or the control is affected by the time delay causing the instability.

V. Conclusions and Further Work

This work shows that using a selected group of measurements in a wide-area controlled network can provide suitable inter-area oscillation damping performance provided the remaining states are estimated through properly designed observers. In a multi-area system, the selected measurements must be chosen from all the major areas of the system to guarantee the controller's successful performance. However, the choice of measurements within an area and the optimal number and type of measurements is still an open question.

Further work is needed to verify the number and type of the optimized measurements in a power network in a more organized scheme based on the control theory. Validity of the proposed approach should be tested in larger networks with more areas and oscillatory modes. Moreover, it is possible that local measurements used as auxiliary outputs of controller's linearized state space would contribute to the control process and reduce the number of needed global measurements.

VI. References

[1] HaiFeng Wang, "A Unified Model for the Analysis of FACTS Devices in Damping Power System Oscillations---Part III: Unified Power Flow Controller," *IEEE Trans. Power Delivery*, vol. 15, no. 3, pp. 978-983, July 2000.

[2] Mehrdad Ghandhari, G. Andersson and Ian A. Hiskens, "Control Lyapunov Functions for Controllable Series Devices," *IEEE Trans. Power Systems*, vol. 16, no. 4, pp. 689-694, Nov. 2001.

[3] B.C. Pal, "Robust damping of interarea oscillations with unified power-flow controller," *IEE Proceedings- Generation, Transmission and Distribution*, vol. 149, pp. 733-738, Nov. 2002.

[4] S. Robak, M. Januszewski, D.D. Rasolomampionona, "Power system stability enhancement using PSS and Lyapunov-based controllers: A comparative study," *IEEE Power Tech Conference Proceedings* 2003 Bologna, vol. 3, pp. 6.

[5] N. Tambey and M.L. Kothari, "Damping of power system oscillations with unified power flow controller (UPFC)," *IEE Proceedings- Generation, Transmission and Distribution*, vol. 150, pp. 129-140, March 2003.

[6] M. Januszewski, J. Machowski and J.W. Bialek, "Application of the direct Lyapunov method to improve damping of power swings by control of UPFC," *IEE Proceedings- Generation, Transmission and Distribution*, vol. 151, pp. 252-260, March 2004.

[7] M. Zarghami, M. Crow, "Discussion on Effective Control of Inter-Area Oscillations by UPFCs," *39'th North American Power Symposium, New Mexico State University,* Sep.-Oct. 2007.

[8] L. Dong, M.L. Crow, Z. Yang, C. Shen, L. Zhang, S. Atcitty, "A Reconfigurable FACTS System for University Laboratories," *IEEE Transactions on Power Systems*, Vol. 19, no. 1, pp. 120-128, Feb. 2004.

[9] Notes of the course ME 4811 by Prof. Fotis A. Papoulias, http://web.nps.navy.mil/~me/blackboard/me4811/docs/notes3.pdf

[10] http://www.ee.washington.edu/research/pstca/pf14/pg_tca14bus.htm

[11] M. Zarghami, M.L. Crow, "The Existence of Multiple Equilibria in the UPFC Power Injection Model," *IEEE Transactions on Power Systems*, vol. 22, no. 4, pp. 2280-2282, Nov. 2007.

[12] Zhian Zhong; Chunchun Xu; B. J. Billian, Li Zhang; S. J. Tsai, R. W. Conners, V. A. Centeno, A. G. Phadke, and Yilu Liu, "Power system frequency monitoring network (FNET) implementation," *IEEE Transactions on Power Systems,* November 2005, vol. 20, no. 4, pp. 1914-1921.