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STATCOM Control for Power System Voltage Control Applications

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Abstract—**A Static Compensator (StatCom) is a device that can provide reactive support to a bus. It consists of voltage sourced converters connected to an energy storage device on one side and to the power system on the other. In this paper, the conventional method of PI control is compared and contrasted with various feedback control strategies. A linear optimal control based on LQR control is shown to be superior in terms of response profile and control effort required. These methodologies are applied to an example power system.**

Index Terms—**Controller design, StatCom, voltage regulation.**

I. INTRODUCTION

THE USE OF FACTS (flexible AC transmission system) devices in a power system can potentially overcome limitations of the present mechanically controlled transmission systems. By facilitating bulk power transfers, these interconnected networks help minimize the need to enlarge power plants and enable neighboring utilities and regions to exchange power. The stature of FACTS devices within the bulk power system will continually increase as the industry moves toward a more competitive posture in which power is bought and sold as a commodity. As power wheeling becomes increasingly prevalent, power electronic devices will be utilized more frequently to insure system reliability and stability and to increase maximum power transmission along various transmission corridors.

The static synchronous compensator, or StatCom, is a shunt connected FACTS device. It generates a balanced set of three-phase sinusoidal voltages at the fundamental frequency, with rapidly controllable amplitude and phase angle. This type of controller can be implemented using various topologies. However, the voltage-sourced inverter, using GTO thyristors in appropriate multi-phase circuit configurations, is presently considered the most practical for high power utility applications [1]–[3]. A typical application of this type of controller is voltage support.

Fig. 1 shows the StatCom connection to a utility bus. The GTO inverter shown in the figure consists of several six step voltage sourced inverters. These inverters are connected by means of a multi-winding transformer to a bus. The use of several inverters reduces the harmonic distortion of the output
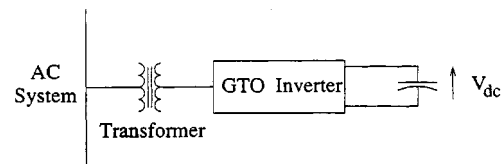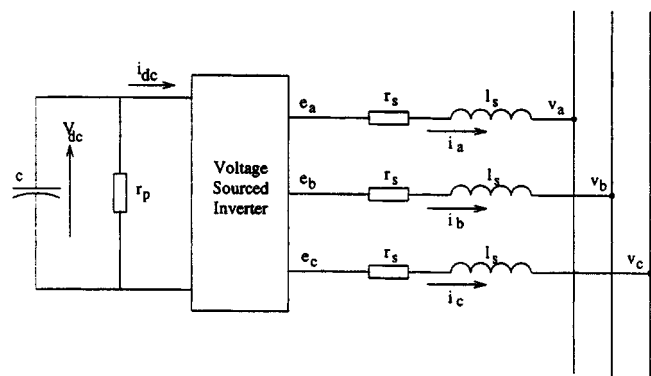
Fig. 1. Connection of a StatCom to a bus.

Fig. 2. Equivalent circuit of StatCom.

voltage. The inverters are connected to a capacitor which carries the DC voltage.

In practice, conventional proportional-integral (PI) control is typically used to achieve automatic voltage regulation. The standard response time is typically chosen to be on the order of a hundred microseconds ($\sim$0.1 s) [2]. In this paper, several state feedback control methods are developed and shown to have superior performance to the PI controller in terms of response dynamics and control effort required.

II. MODELING OF THE STATCOM

The equivalent circuit of the StatCom is shown in Fig. 2. In this circuit, the resistance r_s in series with the inverter represents the sum of the transformer winding resistance losses and the inverter conduction losses. The inductance l_s represents the leakage inductance of the transformer. The resistance r_p in shunt with the capacitor c represents the sum of the switching losses of the inverter and the power loss in the capacitor. The inverter block represents a lossless transformer. The voltages e_a, e_b and e_c are the inverter AC side phase voltages suitably stepped up. The loop equations for the circuit may be written in vector form as [4]:

$$\frac{d}{dt} i_{abc} = -\frac{r_s}{l_s} i_{abc} + \frac{1}{l_s} \left(e_{abc} - v_{abc} \right) \tag{1}$$

The a phase bus voltage is given by

$$v_a = \sqrt{2} V_s \cos(\omega_s t + \theta_s) \qquad (2)$$

where V_s is the rms value of the phase voltage at the bus and θ_s is the phase angle.

The output of the StatCom (neglecting harmonics) may be expressed as

$$e_a = k V_{dc} \cos(\omega_s t + \alpha) \qquad (3)$$

where

V_{dc} is the DC-side voltage,
α is the phase angle of the voltage and
k is a factor that relates the DC voltage to the peak voltage on the AC side.

Transforming the system to a synchronous reference frame and scaling the equations (where the primed quantities indicate per unit) results in the following model:

$$\frac{d}{dt} \begin{bmatrix} i'_d \\ i'_q \\ V'_{dc} \end{bmatrix} = A_s \begin{bmatrix} i'_d \\ i'_q \\ V'_{dc} \end{bmatrix} - \frac{\omega_s}{L'_s} \begin{bmatrix} V_s \cos \theta_s \\ V_s \sin \theta_s \\ 0 \end{bmatrix} \qquad (4)$$

where

$$A_s = \begin{bmatrix} \dfrac{-R'_s \omega_s}{L'_s} & \omega_s & \dfrac{\omega_s k}{L'_s} \cos(\alpha + \theta) \\[2ex] -\omega_s & \dfrac{-R'_s \omega_s}{L'_s} & \dfrac{\omega_s k}{L'_s} \sin(\alpha + \theta) \\[2ex] M_k \cos(\alpha + \theta) & M_k \sin(\alpha + \theta) & \dfrac{-C' \omega_s}{R'_p} \end{bmatrix} \qquad (5)$$

and

$$M_k = -\tfrac{3}{2} k \omega_s C' \qquad (6)$$

Note that although (4) appears to be linear, it is actually nonlinear. The nonlinearity of the StatCom is manifested by the inclusion of the state equation for the control angle α. Changes in the control angle α will results in nonlinear responses in the StatCom states i_d, i_q and V_{dc}.

The injected active and reactive power at the StatCom bus are given by

$$P = V_s(\cos \theta_s i_d + \sin \theta_s i_q) \qquad (7)$$

$$Q = V_s(\sin \theta_s i_d - \cos \theta_s i_q) \qquad (8)$$

The characteristic equation of the linearized system of (4) is given by

$$s^3 + s^2 \left\{ \frac{2R'_s \omega_s}{L'_s} + \frac{\omega_s C'}{R'_p} \right\}$$
$$+ s \left\{ \frac{R'_s \omega_s}{L'_s} \left(\frac{R'_s \omega_s}{L'_s} + \frac{2\omega_s C'}{R'_p} \right) + \omega_s^2 + \frac{3}{2} \frac{k^2 \omega_s^2 C'}{L'_s} \right\}$$
$$+ \left\{ \frac{\omega_s^3 C'}{R'_p} \left(1 + \frac{R'^2_s}{L'^2_s} \right) + \frac{3}{2} \frac{k^2 \omega_s^3 C' R'_s}{L'^2_s} \right\} = 0 \qquad (9)$$

Note that (9) indicates that the eigenvalues [roots of (9)] are independent of the control angle, α. This means that the sta-
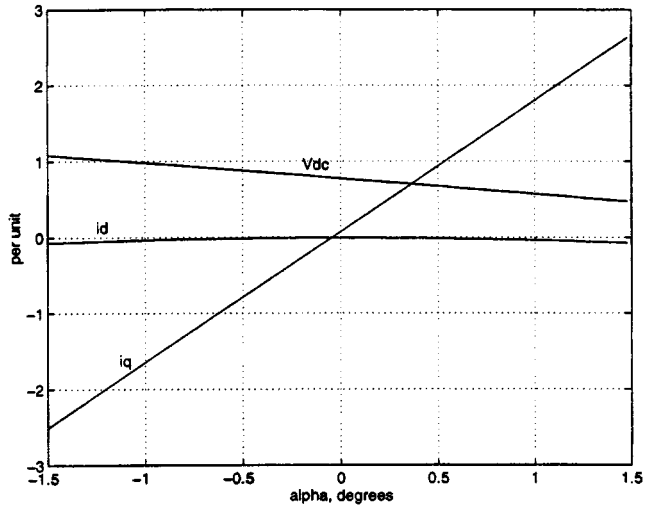

Fig. 3.   Steady-state operating points.

bility of the StatCom itself is independent of the control strategy applied.

The StatCom parameters (in pu) used in the following discussions are given in [4] and are repeated here:

$$L'_s = 0.15, \qquad C' = 0.88, \qquad k = 4/\pi$$
$$R'_s = 0.01, \qquad R'_p = 100/k, \qquad \omega_b = 377.0$$

These StatCom parameters yield the following eigenvalues of the linearized system:

$$s = -23.8, \qquad -15.4 \pm j1473.0$$

Note that these eigenvalues indicate that the StatCom is a highly damped and stable system at this operating point. The set of complex eigenvalues indicate the existence of a high frequency oscillation in the dynamic response of the StatCom.

A plot of the steady-state operating condition of the StatCom as a function of the control angle α using the parameters given previously is shown in Fig. 3. Note that although this is a plot of the nonlinear state relationships, the current component i'_q varies almost linearly with α. Since the reactive power supplied by the StatCom depends almost solely on i'_q (i'_d simply supplies the active power losses), the reactive power supplied by the StatCom also varies nearly linearly with α. This leads to the voltage at the StatCom bus being nearly linearly dependent on the control angle α as well. Thus, linear methods of control will yield satisfactory results for a wide range of disturbances, even though the StatCom and power system are inherently nonlinear.

The nonlinear power system may be modeled as:

$$\dot{x} = f(x, z) \qquad (10)$$

$$\dot{y} = g(y, z) \qquad (11)$$

$$0 = h(x, y, z) \qquad (12)$$

where x represents the generator states of the system (such as generator rotor angle and speed, dq axis voltages, excitation system states, etc.) and (10) represents the n sets of dynamic models corresponding to the generators. The states y represent the StatCom states and (11) represents the StatCom dynamic model given in (4). The states z represent the bus voltage mag-

nitude and angle of the system buses. Equation (12) represents the network constraints governing the active and reactive power flow through the system. Note that the generator and StatCom states are coupled only through the algebraic network equations.

This system can be linearized about an operating point to yield:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ 0 \end{bmatrix} = \begin{bmatrix} A_{gen} & 0 & B_{xz} \\ 0 & A_{stat} & B_{yz} \\ C_{zx} & C_{zy} & D \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ E \\ 0 \end{bmatrix} \alpha \qquad (13)$$

where E for each StatCom is given by:

$$E_i = \begin{bmatrix} -\dfrac{\omega_s k_i}{L_{s_i}} \sin(\alpha_i + \theta_i) V_{dc_i} \\ \dfrac{\omega_s k_i}{L_{s_i}} \cos(\alpha_i + \theta_i) V_{dc_i} \\ \frac{3}{2} k_i \omega_s C_i' (\sin(\alpha_i + \theta_i) i_{d_i} - \cos(\alpha_i + \theta_i) i_{q_i}) \end{bmatrix} \qquad (14)$$

Assuming that the network equations are solvable and invertible, then a reduced order linear system can be found:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} A_{xx} & A_{xy} \\ A_{yx} & A_{yy} \end{bmatrix} + \begin{bmatrix} 0 \\ E \end{bmatrix} \alpha \qquad (15)$$

where

$$A_{xx} = A_{gen} - B_{xz} D^{-1} C_{zx} \qquad A_{xy} = -B_{xz} D^{-1} C_{zy}$$
$$A_{yy} = A_{stat} - B_{yz} D^{-1} C_{zy} \qquad A_{yx} = -B_{yz} D^{-1} C_{zx}$$

III. STATCOM CONTROL

The linearization given in (15) can be used to design and compare various state feedback controllers of the form:

$$y_i = -K_{FB} \alpha_i \qquad (16)$$

which yields the following controlled system:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} A_{xx} & A_{xy} \\ A_{yx} & A_{yy} - E K_{FB} \end{bmatrix} \qquad (17)$$

State feedback control is used infrequently in power system control since many of the power system dynamics states are unavailable as control inputs due to the dispersed geographic nature of the power system. In this case however, state feedback control is possible, since the only states that are required for feedback are the local StatCom states, which are available. Thus, due to the near-linear nature of the StatCom response and the requirement that only locally measurable states are necessary, numerous controllers based on state feedback control may be developed for StatCom voltage response control.

A. Proportional–Integral (PI) Control

In practice, conventional proportional–integral (PI) control is typically used to control the voltage response of the StatCom [5], [2]. The first controller presented in this paper involves a PI control scheme. The block diagram of the control system is shown in Fig. 4.

The gains K_p and K_i are the proportional and integral parts of the controller respectively. The angles $\alpha_{\min}$ and $\alpha_{\max}$ are limits imposed on the value of the control angle by consideration of the maximum reactive power generation capability of the StatCom as well as the maximum value of capacitor voltage allowable.
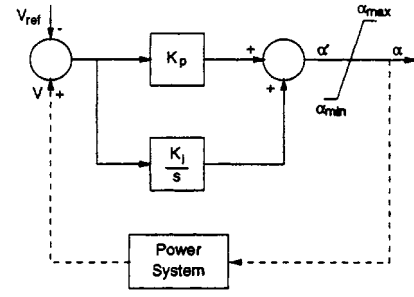

Fig. 4. PI control scheme.

PI control may be represented in state feedback form. For example, define the integral of the error signal as the state w:

$$\dot{w} = (V_s - V_{ref}) \qquad (18)$$

then the control α becomes:

$$\alpha = K_p(V_s - V_{ref}) + K_i w + \alpha_0 \qquad (19)$$

From (13), the StatCom bus voltage V is

$$V_s = C_{sys} D^{-1} \begin{bmatrix} C_{zx} & C_{zy} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \qquad (20)$$

where C_{sys} is a vector of zeros, except for a 1 at the position corresponding to the StatCom bus voltage magnitude. By appending the StatCom state vector y with w, the state feedback control vector for PI control becomes

$$K_{FB} = (K_i Q_{sys} - K_p C_{sys} D^{-1} C_{zy}) \qquad (21)$$

where Q_{sys} is a vector of zeros, with a 1 at the position of the integrator state w.

B. Pole-Placement Methods

The dynamic response of a linear system is governed by the magnitude and location of its eigenvalues, or poles. The response of a system may be affected by relocating some, or all, of the system poles. Common usages of pole placement control is to stabilize an unstable system by moving one or more poles from the right-half complex plane to the left, increasing damping of system response by increasing the magnitude of certain poles, or changing oscillatory frequency by manipulating the complex part of poles. In many cases, it is not desirable, or feasible, to adjust the placement of the majority of the system poles. Therefore, it is desirable to move only those eigenvalues which significantly impact the dynamics of interest.

Since the linearized system matrix given in (15) may be quite large, depending on the number of generators in the system, it is preferable to work with just the StatCom portion of the system, A_{yy}. The generator and StatCom portions of the system are coupled only through the network variables, thus the eigenvalues of A_{yy} are close representations of the eigenvalues of the full system associated with the StatCom. One exception to this is if there is tight coupling between a StatCom and generator. In practice this would seldom occur since a StatCom is not likely to be placed in close proximaty with a generator, because of the voltage regulation capabilities of the generator.

For each StatCom, this will involve choosing the placement of four eigenvalues, one each for i_q, i_d, V_{dc}, and the integrator

state w. The uncontrolled StatCom system eigenvalues will typically contain one real negative eigenvalue, corresponding most closely to i_q and the reactive power output of the StatCom, a complex pair of eigenvalues with negative real parts and large imaginary parts (reflecting the high frequency response of the StatCom), and a zero eigenvalue corresponding to the integrator state w. The eigenvalues may be chosen to shape the dynamic response of the StatCom. The choice of large negative eigenvalues will result in a highly-damped response. The magnitude of the eigenvalues will govern the settling time of the bus voltage magnitude. After the placement of the eigenvalues have been chosen, the state feedback gain matrix K_{FB} may be found using any of the methods given in [6], such as Ackermann's method.

C. Linear Quadratic Regulator (LQR)

The linear quadratic regulator, or LQR, approach is also a pole-placement method. However, in this method, the poles of the system are placed indirectly by minimizing a given performance index. A quadratic performance index may be defined as:

$$J = \tfrac{1}{2} \int_0^\infty (X^T Q X + U_c^T R U_c)\, dt \tag{22}$$

where Q is a positive semi-definite matrix and R is a positive definite matrix [7]. The given performance index can be optimized to yield the state feedback gain matrix K_{FB}. In the LQR control problem, the effectiveness of the control depends on the choice of the Q and R matrices, thus these must be chosen with care. The proposed method for the appropriate selection of Q is detailed below. In this work the Q matrix is chosen to be a diagonal matrix, thus the elements of Q act as a "weighting" of the StatCom states in the performance index.

The elements corresponding to the StatCom states i_d, i_q, and V_{dc} are chosen according to their respective impact on the StatCom voltage magnitude. From (20), one measure of this is the vector

$$C_{sys} D^{-1} [C_{zx} \quad C_{zy}]$$

which relates the StatCom bus voltage magnitude to the system state variables. An appropriate choice is to choose the diagonal elements of Q as the absolute values of the elements of the vector corresponding to the StatCom states.

The element corresponding to w must be chosen to be much larger than any other element in the matrix. The aim is to constrain the integral of the error in voltage at the StatCom bus ($\int(V_s - V_{ref})$) thereby, indirectly, constraining the error ($V_s - V_{ref}$).

The R matrix is also chosen to be a diagonal matrix. The elements of R must be chosen carefully. Very small values result in excessive control force demand while very large values result in sluggish system performance. The best range of values to be used varies depending on the system under consideration and the operating point.

The pole-placement and LQR controller design strategies are based on consideration of the A_{yy} matrix decoupled from the full system. This procedure yields a small discrepancy between the behavior of the full system design and the decoupled system design. In most practical instances however, this discrepancy is small and may be safely neglected.

IV. STATCOM PLACEMENT

The primary function of a StatCom is to provide voltage regulation within the power system. To provide the best performance, a StatCom should be placed at those buses which provide high voltage response for incremental changes in reactive power injection. One means of quantifying the sensitivity in voltage magnitude to changes in reactive power is to use a voltage security indicator such as the singular value decomposition of the system Jacobian:

$$J = U \Sigma V^T = \sum_{i=1}^n \sigma_i u_i v_i^T \tag{23}$$

where u_i and v_i are the columns of the $n \times n$ orthonormal matrices U and V, and Σ is a diagonal matrix of positive real singular values σ_i, such that $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_n \geq 0$ [8]. The smallest singular value σ_n is an indicator of the proximity to the static voltage limit, the right singular vector v_n corresponding to σ_n indicates the sensitive voltage magnitudes and angles, and the left singular vector u_n corresponding to σ_n indicates the most sensitive direction for changes of active and reactive power injections.

This approach can be used to determine the placement of a StatCom by increasing the system loading pattern until the minimum singular value is sufficiently small. This may be accomplished using the continuation power flow or similar method [9]. At this critical loading point, the relatively large elements of left singular vector will correspond to those system buses which are highly sensitive to reactive power injections. StatComs placed at these buses will provide the most effective voltage regulation.

V. POWER SYSTEM STUDIES

The controllers described previously were applied to the IEEE 118 bus system shown in Fig. 5. Using the placement method described in Section IV, three voltage-weak areas were identified. The most severe area was concentrated about bus 86. From Fig. 5, this area is heavily loaded and remote, thus making it susceptible to voltage problems. The two other identified areas were concentrated around buses 20 and 33, which are similarly remote from generation and voltage support. For the initial control studies, a StatCom was placed at bus 86, which was then subjected to sudden increases in load.

Each generator in the system is fully modeled with excitation system, voltage-regulator, and turbine/governor dynamics. All simulation results are based on full nonlinear modeling and simulation of the IEEE 118 bus test system.

Fig. 6 shows the voltage response at bus 86 to a 0.175 pu (0.9 power factor lagging) step increase in load. The steady-state voltage magnitude drops by almost 3%.

A. PI Control

The PI control system was designed for a desired settling time of 0.1 seconds. The desired voltage setpoint is the initial voltage value of 0.9445 pu. The PI controller gains were chosen to be $K_p = 0.5$ and $K_i = 32.34$. The voltage response of this controller is shown as the solid line in Fig. 7. The response of the bus voltage magnitude shows the highly damped, oscillatory contribution of the StatCom, with a settling time of 0.1 seconds to
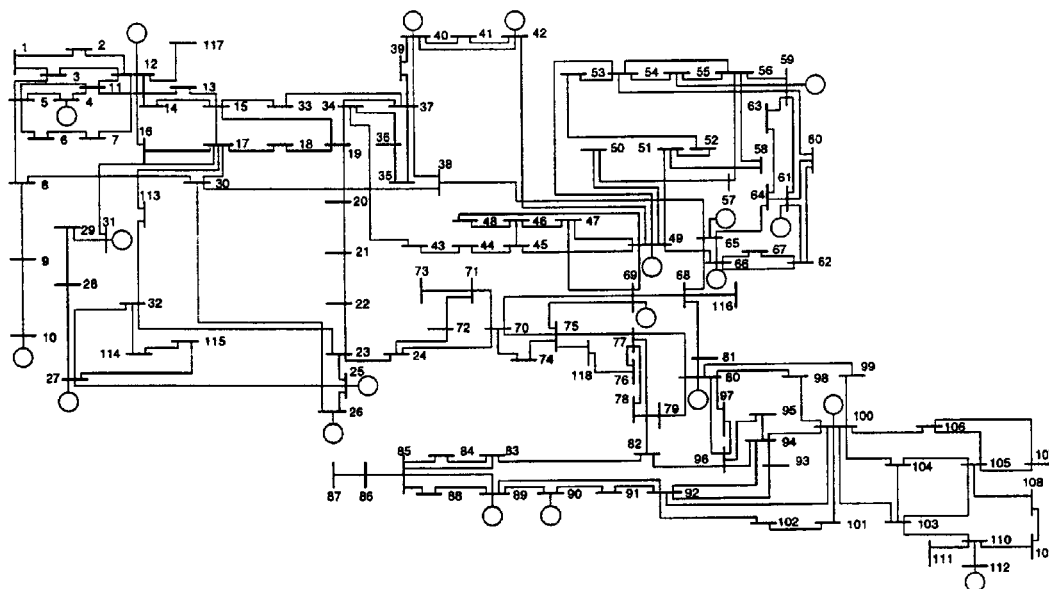
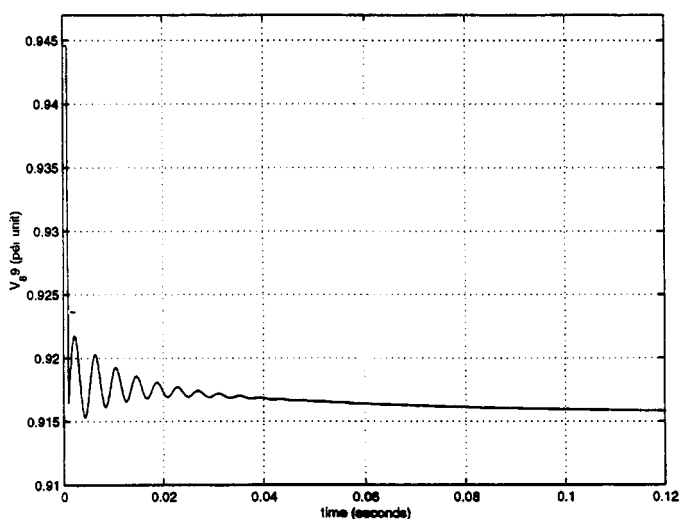Fig. 5.   IEEE 118 bus test system.

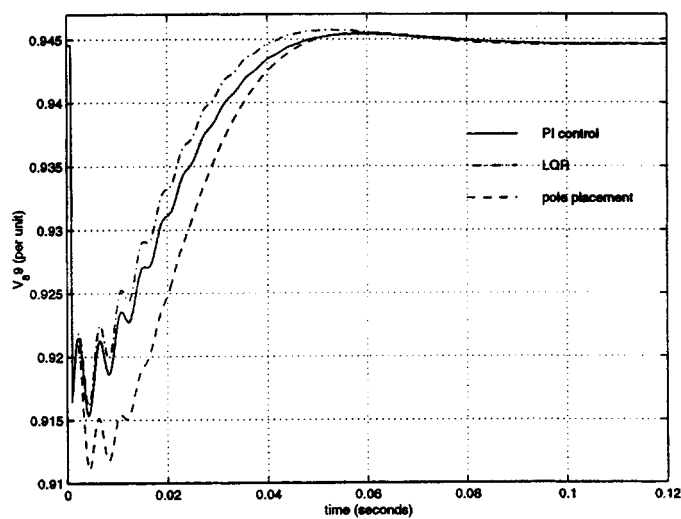Fig. 6.   Uncontrolled voltage response at bus 86.

Fig. 7.   Controlled voltage response at bus 86.

the voltage reference setpoint. The PI control angle is shown in Fig. 8. Note that the control angle requires a change of approximately 0.5 degrees. From Fig. 3, this corresponds to a change of nearly 0.14 pu in i_q and a proportional change in reactive power injection.

B. Pole Placement Control

The pole placement control system was also designed for a desired settling time of 0.1 seconds, with a reference voltage setpoint of 0.9445 pu. In this control scheme the StatCom eigenvalues were moved from their original placements to new positions which yielded the desired response. These eigenvalues are summarized in Table I. The zero initial eigenvalue corresponds to the integrator state w.

The response of bus 86 voltage magnitude is shown as the dashed line in Fig. 7. Note that to obtain the same settling time response as with PI control, a greater voltage excursion occurred. However, the high-frequency oscillations in the voltage
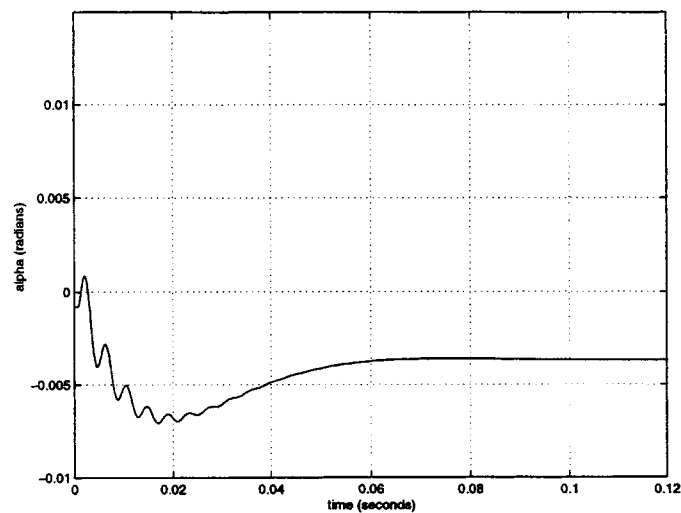

Fig. 8.   PI control—control angle α.

Initial	Placed
0	-130
-21.8	-125
-98.25 + j1535	-250 + j1535
-98.25 - j1535	-250 - j1535

Fig. 9.   Pole placement—control angle α.

Fig. 10.   LQR—control angle α.

Fig. 11.   PI vs. LQR control in an extreme loading case.
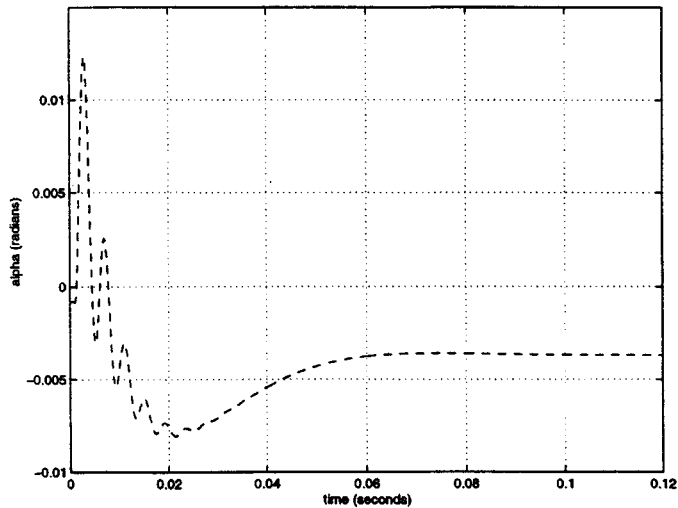
magnitude are more highly damped than with PI control. The pole placement control angle is shown in Fig. 9. To achieve the same basic voltage response as with PI control, a much larger control effort in α is required. This also results in a much greater excursion in i_q and subsequently, the injected reactive power.

C. LQR Control

The control system was designed as described in Section III-C, with the weighting element corresponding to w being chosen as 50 000 to insure a settling time of 0.1 seconds. A value of 40 for the weighting element R was found to yield satisfactory performance. The voltage magnitude response for the LQR control is shown as the dash-dot line in Fig. 7. The LQR control angle is shown in Fig. 10. Note that the required control effort with this controller is the least of the three controllers, and is a very smooth waveform.

A comparison of the various control efforts is informative. Firstly, recall that α is the *commanded* control angle. The StatCom power electronics will be programmed to fire in a manner to produce the commanded control angle trajectory. The smoother trajectory of the LQR controller will result in better performance from the power electronics, as opposed to the highly oscillatory response of the pole placement controller or PI controller. The reduction in the control angle and subsequent current i_q may also lead to lower rated devices which are less costly and have lower conduction losses.

D. Controller Robustness

The LQR and PI controller were subjected to a situation in which the bus 86 voltage magnitude exhibited instability
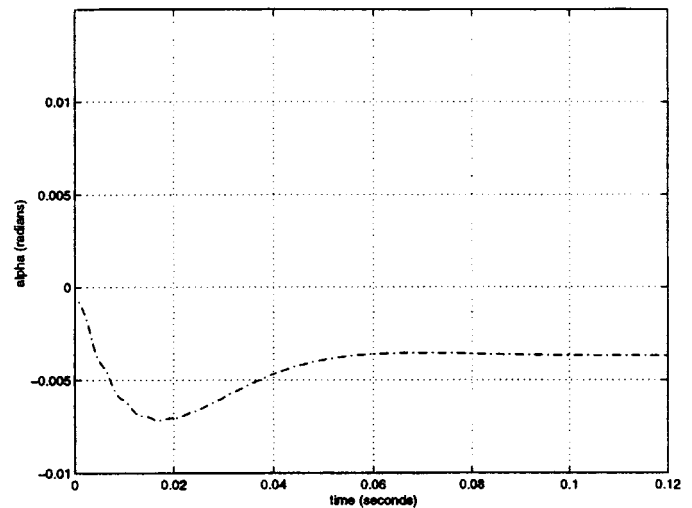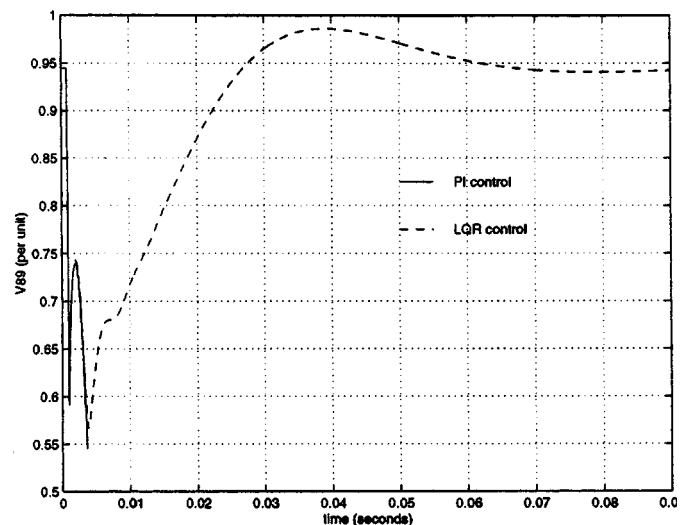
without a StatCom present (i.e. the voltage "collapsed"). Typically, the PI and LQR controller exhibited comparable responses. At extreme loading cases, however, the LQR controller had superior robustness. Fig. 11 shows such a case. In this example, an extreme loading was applied to the system. Without control, the bus voltage magnitude collapses. Similarly, with PI control of the StatCom (the solid line), the bus voltage magnitude also decreases to an unrecoverable level. The LQR controller (dashed line), however, is able to maintain the bus voltage magnitude and return it to the voltage reference setpoint by the desired settling time of 0.1 seconds.

E. StatCom Interaction

The control methodologies proposed in this paper can be extended to a system with more than one StatCom by considering each device individually. As noted previously, there is very little coupling between the StatCom dynamic states and the remaining power system states, thus control design is effected only negligibly by the presence of other StatComs in the system. The only deleterious interaction noted between StatComs in

a system is when two or more StatComs are placed in close proximity. In practice, this would seldom occur. If the placement scheme described in Section IV were used, then StatComs would not likely be placed in close proximity. It is far more cost effective to use a single StatCom per weak area with shunt capacitor banks. One difficulty with using capacitor banks alone in a weak voltage area, is that as the voltage profile decreases, the amount of injected reactive power from the capacitor bank will also decrease, thus exasercbating the voltage problem. However, if capacitor banks are used in conjunction with a StatCom, the StatCom will hold the local voltage profile at the reference voltage setpoint, thus maintaining the effectiveness of the capacitor banks. The interaction between StatComs and other power-electronic-based systems, such as HVDC lines, was not studied.

Unless outfitted with a DC power source, such as a battery or SMES across the DC capacitor, StatComs may only inject reactive power into the power system. Reactive power injection has only localized effects on power flow and power system dynamics, thus StatComs are not effective in control of active power problems, such as transient stability, interarea oscillations, and subsynchronous resonance.

VI. CONCLUSION

The modeling of a StatCom for power system applications is presented in this paper. Three types of state feedback controllers are developed and compared. One of the main advantages of these approaches is that the nearly linear behavior of the StatCom response enables linear control methods to be effectively employed. This allows a wide range of design strategies to be explored with linear tools, while a nonlinear simulation need only be applied during the final stages of design to substantiate the control.

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