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Direction of Arrival Estimation Based on Support Vector Regression: Experimental Validation and Comparison With MUSIC

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Abstract—In this letter, the problem of estimating the directions of arrival (DOAs) of coherent electromagnetic waves impinging upon a uniform linear array (ULA) is considered. In particular, an efficient DOA estimation approach based on the support vector regression is assessed using experimental measurements. Moreover, the obtained results are compared with those yielded by the multiple signal classification (MUSIC) algorithm.

Index Terms—Directions of arrival (DOAs), support vector machines (SVMs)/regression, uniform linear array (ULA).

I. INTRODUCTION

IN the past few years, there has been a growing interest in the development of smart antennas for various applications including wireless communications and radar systems. One of the critical issues associated with the implementation of smart antennas is the estimation of directions of arrival (DOAs) of incoming electromagnetic waves [1]–[3].

A number of methods have been devised for DOA estimation. One of the most popular methods is the multiple signal classification (MUSIC) algorithm [4]. Other methods, such as the maximum likelihood (ML) method and the estimation of signal parameters via rotational invariance technique (ESPRIT) [5] have been successfully applied as well. Recently, very accurate DOA estimation results have been also reported using stochastic methods based on neural networks [6], [7]. In [8], the authors proposed an alternative method based on a support vector regression approach [9]. This method is built upon the support vector machines (SVMs) developed following the learning theory [10].

The key features of the SVMs pertain to their rigorous mathematical formulation and their excellent robustness, i.e., after a training phase in which several known input/output mappings are used to determine the parameters of the SVMs, they perform well in response to input signals that have not been initially included in the training set.

In [8], the performance of the SVM approach has been assessed using synthetic signals with the main objective of evaluating its robustness. Furthermore, the computational aspects of the SVM have been discussed and compared with other approaches, i.e., the MUSIC algorithm and neural networks.

In this letter, the performance of the SVM-based DOA estimation is substantiated by experimental signal measurements. The SVM performance analysis is extended beyond the preliminary study reported in [11]. To this end, the SVM estimation accuracy is investigated using uniform linear array (ULA) of varying number of elements and plane waves arriving from different directions. Furthermore, the experimental results are compared to the simulation results produced by emulating the measured signal structure. Subsequently, the SVM estimation results are compared with those obtained by applying MUSIC algorithm on the same measurements.

The remaining of this letter is organized as follows. In Section II, an outline of the mathematical formulation of the SVM approach is presented. In Section III, the experimental results are reported together with a comparison with the MUSIC algorithm. Finally, Section IV concludes the letter.

II. OUTLINE OF THE MATHEMATICAL FORMULATION

A ULA composed by N elements with inter-element spacing d is used to intercept M narrowband plane waves transmitted by sources located at angles θ_m, $m = 1, \ldots, M$. Assuming isotropic array elements, the response of the ULA to the incident waves can be written as [7]

$$\mathbf{x} = \mathbf{As} + \mathbf{n} \tag{1}$$

where $\mathbf{x} = [x_1, x_2, \ldots, x_N]^T$ is the vector of the baseband signals seen at the output of each element, $\mathbf{s} = [s_1, s_2, \ldots, s_M]^T$ is the vector containing the complex amplitudes of the transmitted signals, and $\mathbf{n} = [n_1, n_2, \ldots, n_N]^T$ is the receiver's noise vector. The noise term n_i is assumed to be a sample from complex Gaussian white noise process with zero mean value and variance $\sigma^2/2$ per dimension. Finally, $\mathbf{A}$ is the steering matrix whose generic element is given by $a_{im} = e^{-j(i-1)kd\sin\theta_m}$, $i = 1, \ldots, N$, $m = 1, \ldots, M$, where k is the wavenumber.

After estimating the autocovariance matrix of the received signal, defined as $\mathbf{R} = E\{\mathbf{x}\mathbf{x}^H\} = \mathbf{A}\mathbf{S}\mathbf{A}^H + \sigma^2\mathbf{I}$ (where $\mathbf{S}$ is the $M \times M$ source autocovariance matrix given by $\mathbf{S} = E\{\boldsymbol{ss}^H\}$, and $\mathbf{I}$ is the identity matrix), a mapping $\mathbf{G} : \Theta \rightarrow \Sigma$ is defined between the DOA values $\boldsymbol{\theta} = [\theta_1, \ldots, \theta_M] \in \Theta \subset R^M$

and the elements of matrix $\mathbf{R}$. For post processing, the elements of the matrix $\mathbf{R}$ are rearranged in a normalized array $\mathbf{z} \in \Sigma \subset C^{N(N+1)/2}$ [8].

The retrieval of the array $\boldsymbol{\theta}$ starting from the knowledge of the array $\mathbf{z}$ is obtained by approximating the inverse mapping $\mathbf{F} : \Sigma \to \Theta$ using the SVMs. Specifically, the mth component of $\mathbf{F}$ is approximated as

$$\tilde{F}_m(\mathbf{z}) = \sum_{i=0}^{L-1} (\alpha_i - \alpha_i')\, \psi(\mathbf{z}_i, \mathbf{z}) + b \qquad (2)$$

in which $\psi(\mathbf{z}_i, \mathbf{z})$ is a radial kernel function given by $\psi(\mathbf{z}_i, \mathbf{z}) = \exp\left(-\gamma \|\mathbf{z} - \mathbf{z}_i\|^2\right)$, α_i', α_i, and b are variables that minimize the regression risk defined as [9]

$$R_{\text{reg}} = C \sum_{i=0}^{L-1} c\left(\mathbf{z}^i, \hat{\theta}_1^i\right) + \frac{1}{2} \|\mathbf{w}\|^2 \qquad (3)$$

In (3), C is a constant, and $c\left(\mathbf{z}^i, \hat{\theta}_1^i\right)$ is the ϵ-insensitive loss function [9] given by $c\left(\mathbf{z}, \hat{\theta}_1\right) = 0$, if $\left|\hat{\theta}_1 - \tilde{F}_1(\mathbf{z})\right| \le \varepsilon$ and $c\left(\mathbf{z}, \hat{\theta}_1\right) = \left|\hat{\theta}_1 - \tilde{F}_1(\mathbf{z})\right| - \varepsilon$, otherwise.

Moreover, L is the number of samples used to train the SVM and $\mathbf{w} = \sum_{i=0}^{L-1} (\alpha_i - \alpha_i')\, \boldsymbol{\Phi}\left(\mathbf{z}^i\right)$, being $\boldsymbol{\Phi}$ a nonlinear vector-valued function related to the kernel function by $\psi\left(\mathbf{z}^i, \mathbf{z}\right) = \langle \boldsymbol{\Phi}\left(\mathbf{z}^i\right), \boldsymbol{\Phi}(\mathbf{z}) \rangle$. The details of this approach are provided in [8] and they are not repeated here.

III. EXPERIMENTAL RESULTS

In this letter, the SVM-based DOA estimation approach has been tested by using experimental data. An antenna array, operating at 10 GHz, was synthesized by scanning an X-band open-ended rectangular waveguide linearly along the x axis. The waveguide was connected to port 1 of a VNA, i.e., HP8510C. Another open-ended rectangular waveguide was used as a transmitter while connected to port 2 of the VNA. The distance between the source and the array elements was such that they were well in the far-field of each other. Thereafter, calibrated transmission measurements were taken at each scan point. To randomize the positioning errors, five snapshots were taken (each time by repositioning the transmitter).

The transmitting source was positioned sequentially at three different angles; $\theta_1 = 90°$, $\theta_1 = 100°$, and $\theta_1 = 110°$. For each position, the transmitted signal was received using a 27-element ULA with inter-element spacing $d = \lambda/4$ and the measured signals were recorded for further processing.

ULAs of number of elements N varying in the range $[3, 27]$ were constructed as subarrays from the original 27 measurement points. The signal received by each subarray was subsequently presented to the SVM to estimate the DOA.

Fig. 1 shows the absolute error in estimating the three DOAs versus the number of elements in the ULA. The SVM training set was composed by $L = 361$ samples, whose DOAs, $\hat{\theta}_1^l$, $l = 1, \ldots, L$, were uniformly distributed in the range $[-90°, +90°]$, i.e., $\hat{\theta}_1^l = -90 + 0.5(l - 1)$, $l = 1, \ldots, L$. The SVMs were trained by using one sample per direction. However, in order
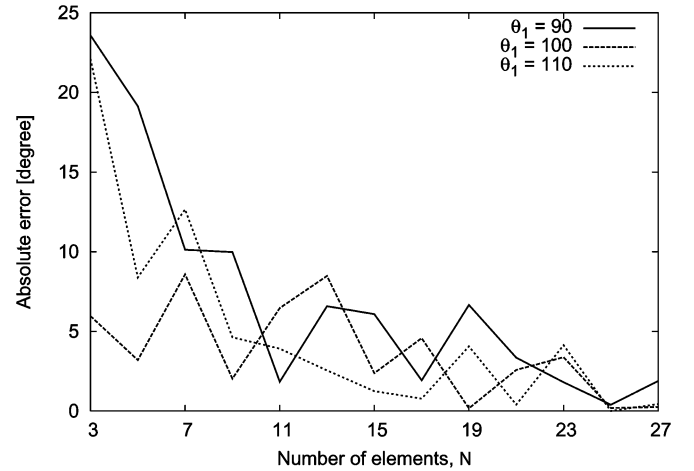

Fig. 1. Absolute error in estimating the DOAs of one arriving wave for different number of elements using the SVM approach.

to improve the performances and to provide robustness against noise/antenna variations, it would be possible to use more samples.

Finally, the parameters of the SVM were: $\gamma = 0.01$, $C = 10$, and $\epsilon = 0.03$ [8], [12], [13]. It is evident that the SVM-based approach was able to estimate the DOAs of the three incident waves with a quite good accuracy when the number of elements is large. This behavior can be related to the fact that increasing the number of elements lead to higher diversity and allows a better averaging over the noise.

To further investigate the performance of the SVM-based DOA estimation approach, the experimental setup was emulated via computer simulations. The SVM was trained using the same training set and parameters used to process the real signals. In the test phase, the total received signal at each array element was modeled as per (1). The variance of the noise added to the synthetic data has been set equal to that of the noise affecting the real data. In particular, the signal-to-noise ratio (SNR) at the output of each element of the antenna for the measured data has been estimated to be equal to 18 dB. Moreover, the considered DOAs for the synthetic plane waves were 90°, 100°, and 110°.

The estimated values of the DOAs for the cases of synthetic and real data are shown in Fig. 2. It is shown that, in both cases, when the number of elements is quite large, the SVM-based approach provided very good DOA estimates. However, in the case of synthetic signal, the approach was able to provide accurate results even for a limited number of elements. It is worth noting that this discrepancy might be related to the approximations introduced in the synthetic model, i.e., the assumptions that the arriving waves are plane waves and that the array elements are isotropic antennas.

Finally, a comparison with the well known MUSIC algorithm was performed. Fig. 3 shows the mean absolute error (i.e., the mean value of the absolute errors on the estimated values of the three considered DOAs) versus the number of elements composing the ULA. As can be seen, the results obtained by the
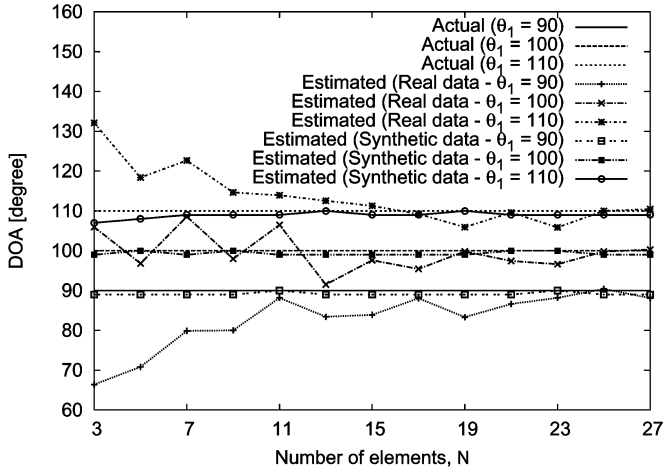
Fig. 2. Estimated values of the DOAs of one arriving wave for different number of elements composing the ULA and for the three considered angles of incidence. Comparison with the results obtained using synthetic data ($\mathrm{SNR} = 18$ dB).

Fig. 4. Absolute errors in estimating the DOAs of two incident waves for different number of elements composing the ULA and for the three considered pairs of angles of incidence.
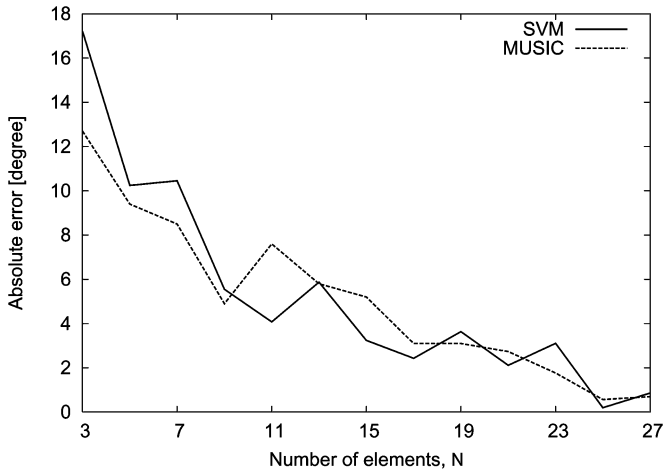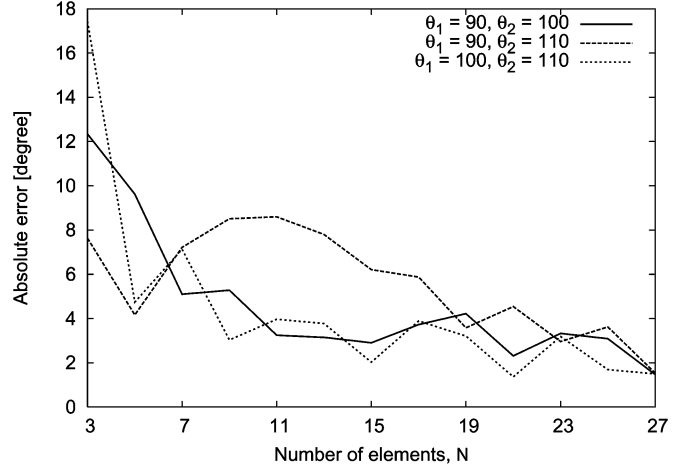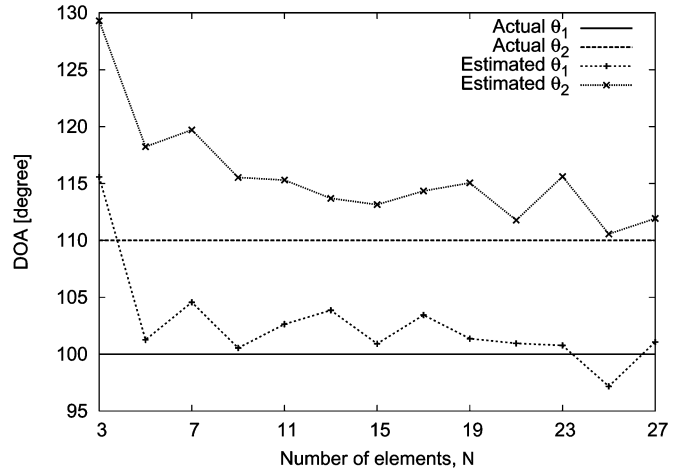

Fig. 3. Mean absolute error in estimating the DOA of one arriving wave for different number of elements composing the ULA. Comparison between the SVM-based approach and the MUSIC algorithm.

Fig. 5. Estimated values of the DOAs of two incident waves for different number of elements composing the ULA. $(\theta_1, \theta_2) = (100°, 110°)$.

SVM-based approach are comparable with those drawn by the high resolution MUSIC algorithm.

To illustrate the efficacy of the SVM-based approach in localizing simultaneous emitters, a two-transmitter environment was synthesized using the experimental setup described previously. As described in [8], the training set was composed by L fixed values for the training angles $\hat{\theta}_1$ and $\hat{\theta}_2$. In order to estimate the DOAs of waves with different angular separations $\Delta\theta^{12}$ (defined as $\Delta\theta^{12} = \theta_2 - \theta_1$), $Q = 5$ angular separations between the training angles were considered in the training phase. In particular, the values used were: $\Delta\hat{\theta}_1^{12} = 5°$, $\Delta\hat{\theta}_2^{12} = 10°$, $\Delta\hat{\theta}_3^{12} = 15°$, $\Delta\hat{\theta}_4^{12} = 20°$, and $\Delta\hat{\theta}_5^{12} = 25°$. For each value of the angular separation between the two training waves, a set of L_q samples for $\hat{\theta}_1$ and L_q samples for $\hat{\theta}_2$ was constructed. To cover the whole region of interest ($[-90°, +90°]$), the first sample of $\hat{\theta}_1$ was assumed to be equal to $-90°$ and the last

sample of $\hat{\theta}_2$ was assumed to be equal to $+90°$. With the above assumptions, the DOAs of the training samples are given by

$$\begin{aligned}
\hat{\theta}_1^{l_q} &= -90 + 0.5(l-1) \\
\hat{\theta}_2^{l_q} &= -90 + 0.5(l-1) + \Delta\hat{\theta}_q^{12} \\
l &= 1, \ldots, L_q, \quad q = 1, \ldots, Q.
\end{aligned} \tag{4}$$

The number of samples for the various angular separations was: $L_1 = 351$, $L_2 = 341$, $L_3 = 331$, $L_4 = 321$, and $L_5 = 311$. Moreover, the total number of samples for each wave was $L = 1655$. The other parameters of the SVM were fixed as before.

The SVM-based approach was tested using the following three pairs of DOAs: $(\theta_1, \theta_2) = (90°, 100°)$, $(\theta_1, \theta_2) = (90°, 110°)$, and $(\theta_1, \theta_2) = (100°, 110°)$.

Fig. 4 reports the absolute error in estimating the DOAs versus the number of elements of the ULA. Moreover, Fig. 5 shows the estimated values of the DOAs of the two incident

waves versus the number of elements of the antenna for the case $(\theta_1, \theta_2) = (100°, 110°)$.

As suggested in Figs. 4 and 5, the SVM-based approach was able to retrieve the DOAs of the incident waves accurately provided that the number of elements in ULA is sufficiently large.

Concerning the computational requirements of the approach, it should be noted that after the training phase, which is performed off line and once and for all, the DOA estimation is performed in a very short time. As an example, for the case of $M = 2$ electromagnetic waves impinging on a ULA composed by $N = 6$ elements, the CPU time needed to train each SVM in the learning phase is 12.25 s, whereas, in the test phase, any SVM takes 0.025 s to estimate each DOA value (on a PC equipped with a 2-GHz processor and 512 MB of RAM.) The CPU time needed by the MUSIC method in the same case is about 95 s.

IV. CONCLUSION

In this letter, an SVM-based DOA estimation approach was experimentally evaluated. The performance of the approach, previously assessed by means of numerical simulations, has been validated in a real environment. Furthermore, the proposed approach was also compared with the classic MUSIC algorithm. It was shown that the SVM approach projects a comparable performance to the MUSIC algorithm. The efficiency of the SVM approach stems from the fact that, after the training phase (performed off-line), the directions of arrival are identified in a very short time and with limited memory requirements, since only the support vectors and the support vector expansion coefficients must be stored.

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