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Shahab Mehraeen

Jagannathan Sarangapani
Missouri University of Science and Technology, sarangap@mst.edu

Mariesa Crow
Missouri University of Science and Technology, crow@mst.edu

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Novel Dynamic Representation and Control of Power Networks Embedded with FACTS Devices

S. Mehraeen, S. Jagannathan, M. L. Crow[1]
Department of Electrical and Computer Engineering
Missouri University of Science and Technology
Rolla, MO 65409, USA

***Abstract*— FACTS devices have been shown to be powerful in damping power system oscillations caused by faults; however, in the multi machine control using FACTS, the control problem involves solving differential-algebraic equations of a power network which renders the available control schemes ineffective due to heuristic design and lack of know how to incorporate FACTS into the network. A method to generate nonlinear dynamic representation of a power system consisting of differential equations alone with universal power flow controller (UPFC) is introduced since differential equations are typically preferred for controller development. Subsequently, backstepping methodology is utilized to reduce the generator oscillations by using a FACTS device after a fault has occurred. Finally, we use neural networks to approximate the nonlinear network dynamics for controller design. The net result is a representation that could be potentially utilized for studying the placement and number of FACTS devices as well as to design a better control scheme for FACTS given a power network. Simulation results justify theoretical conjectures.**

***Keywords* – Power System Control. Nonlinear Control, Neural Networks, FACTS.**

I. Introduction

The analysis of a power network includes studying transient behavior after a fault has occurred, dynamic stability, and demonstration of an acceptable performance by a controller. In order to analyze the network performance with a controller, the power network with several generators is normally modeled using a combination of differential and algebraic equations. The differential equations represent generator angles and speeds whereas algebraic equations represent nodal active and reactive power balance relationships. Solving the differential-algebraic equations becomes extremely difficult during the transient analysis and control design particularly when the network is controlled by unified power flow control (UPFC) type FACTS devices since it is not clearly understood how to develop a representation when a power network is embedded with FACTS devices and lack of know-how to generate an algebraic free representation.

[1] Contact:sm347@mst.edu
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On the other hand, controller designs normally require a network given in the form of differential equations.

Several approaches have been introduced to mitigate this problem. Past work [1] has attempted to linearize the differential-algebraic equations to solve for the needed variables. Then linear control methods can be easily applied to the linear power system; however, an assumption is made such that the variables are in a neighborhood of an operating point. In addition, the placement and number of UPFC devices is accomplished heuristically. In contrast, in other works [2-3], an infinite bus is assumed in order to apply nonlinear control schemes. However, if all the generators in the power system are to be included in the control, this assumption is no longer valid since this method assumes that only a small portion of power system is affected by disturbance.

Nonlinear control of a multi-machine power system has been proposed using backstepping [4]. However, this approach omits FACTS devices and uses standard generator controllers such as steam and excitation. In [5-8], power system energy functions which include FACTS devices were used whereas the calculation of the derivatives of power system nodal voltages and phases that are needed for UPFC control is not obtained i.e. numerical differentiators or approximations have been utilized.

In this paper we address the following to overcome the above mentioned challenges. First, a new nonlinear dynamical representation of a power network with UPFC as a controller that is free of algebraic equations is introduced. This representation can be utilized to model a nonlinear power network with several FACTS devices. Second, the problem of multi-machine damping control using UPFC is addressed using nonlinear control schemes (i.e the case when the number of inputs is less than that of outputs) by utilizing the aforementioned dynamical model. Third, neural network approximation property is asserted to relax the need for knowing the power system topology (i.e Ybus) and to calculate the nonlinear uncertainties.

Here we have obtained a nonlinear state space representation of power network dynamics which is useful for power network transient analysis as well as control design. Our approach contains obtaining a set of nonlinear dynamical representation using the swing and nodal power balance equations. The advantage of this approach is that no algebraic equations are involved in the control design while retaining nonlinear behavior. In this approach, we use the power system

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classical model where generators' internal voltages are constant; whereas, the rotor angels are time varying variables. The state variables in the differential equations are the generator angles and speeds as well as the nodal voltages and phase angles of the power network. Then, a nonlinear control scheme is developed to stabilize and control the power network subject to a disturbance. Finally, we have deployed universal approximation property of neural networks to approximate the power system uncertainties and relaxed the need for complex calculations and a prori knowledge of the uncertainties.

This paper is organized as follows. First the power system differential-algebraic equations are reviewed in Section II. Then in Section III and IV power system dynamic equations are derived. Control strategy is introduced in Section V and finally neural network implementation is shown in Section VI.

II. Power System Differential-algebraic Model

The classical generator representation has been found to be sufficient for stability analysis. The advantage of this model is that the mechanical rotor angle deviation with respect to the synchronous reference frame is directly equivalent to the electrical angle and yields the equivalent circuit known as the voltage behind the transient reactance model [5]. Also, for simplicity of development, the resistance of power network lines is ignored. By ignoring friction term, a set of dynamic equations for describing rotor motion in a multi machine power system is described by

$$\begin{aligned} \dot{\bar{\delta}}_i &= \bar{\omega}_i - \bar{\omega}_0 \\ M_i \dot{\bar{\omega}}_i &= P_{mi} - B_{i,i+n} E_{gi} V_{i+n} \sin(\bar{\delta}_i - \bar{\psi}_{i+n}) \qquad i = 1,\ldots,n \end{aligned} \tag{1}$$

where $\bar{\delta}_i$ is the rotor angle of the i th machine, $\bar{\omega}_i$ is the angular speed, $\bar{\omega}_0$ is the synchronous angular speed, B represents the admittance matrix, E_{gi} is the i th machine internal voltage, n is the number of generators in the power system, $M_i = 2H/\omega_0$ is the i th machine inertia, P_{mi} is the mechanical power, and V_{i+n} and $\bar{\psi}_{i+n}$ are the generator bus voltage and phase angle, respectively, according to Fig. 1. Also, we define N to be the number of non-generator buses in the power system.

For stability analysis, it is more convenient to represent the dynamical equations (1) in the Center of Inertia (COI) coordinates, which yield the following

$$\begin{aligned} \dot{\delta}_i &= \omega_i \\ M_i \dot{\omega}_i &= P_{mi} - \frac{M_i}{M_T} P_{COI} - B_{i,i+n} E_{gi} V_{i+n} \sin(\delta_i - \psi_{i+n}); i = 1,\ldots,n \end{aligned} \tag{2}$$

where $\delta_i = \bar{\delta}_i - \delta_0, \omega_i = \bar{\omega}_i - \omega_0, \psi_i = \bar{\psi}_i - \delta_0$, $M_T = \sum_{i=1}^{n} M_i$, $\delta_0 = \frac{1}{M_T}\sum_{i=1}^{n} M_i \bar{\delta}_i$, $\omega_0 = \frac{1}{M_T}\sum_{i=1}^{n} M_i \bar{\omega}_i$, and $P_{COI} = \sum_{i=1}^{n} P_{mi} - \sum_{i=n+1}^{n+N} P_{Li}$ with P_{Li} being the active load at each bus.

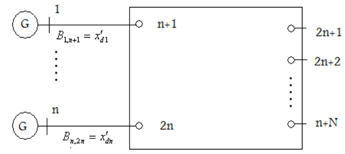

Fig.1-Power System

The nodal voltages and phase angles of all the power system buses are constrained by the following set of algebraic equations.

$$\begin{aligned} P_{Li} + \sum_{j=1}^{N+n} B_{ij} V_i V_j \sin(\psi_i - \psi_j) &= S_{Pi} = 0 \\ Q_{Li} - \sum_{j=1}^{N+n} B_{ij} V_i V_j \cos(\psi_i - \psi_j) &= S_{Qi} = 0; i = 1,\ldots,n+N \end{aligned} \tag{3}$$

where P_{Li} and Q_{Li} are the active and reactive loads on the i th bus. Also, we define $\psi_i = \delta_i; \quad 1 \le i \le n$

III. New Dynamic Representation of Power Networks

Equation (2) together with (3) is referred to as the power system equations which typically are differential-algebraic set of equations. Usually, a controller design in a differential-algebraic environment is difficult to attempt, thus we need to substitute the set of equations (3) with a more appropriate one. One way to have a pure dynamical system is to take derivative of equations (3) to obtain $\dot{V}_i$ and $\dot{\psi}_i$ terms. Thus, we have

$$\frac{\partial S_{Pi}}{\partial t} = \frac{\partial S_{Pi}}{\partial V}\dot{V} + \frac{\partial S_{Pi}}{\partial \psi}\dot{\psi} + \frac{\partial S_{Pi}}{\partial \delta}\dot{\delta} = 0 \tag{4}$$

$$\frac{\partial S_{Qi}}{\partial t} = \frac{\partial S_{Qi}}{\partial V}\dot{V} + \frac{\partial S_{Qi}}{\partial \psi}\dot{\psi} + \frac{\partial S_{Qi}}{\partial \delta}\dot{\delta} = 0 \,; i = 1,\ldots,n+N \tag{5}$$

Solving equations (4) and (5) for $\dot{V}_i$ and $\dot{\psi}_i$, we obtain a new set of dynamic equations as

$$\begin{bmatrix} \dot{V} \\ \dot{\psi} \end{bmatrix} = -\begin{bmatrix} \bar{A}(x_S) & \bar{B}(x_S) \\ \bar{D}(x_S) & \bar{E}(x_S) \end{bmatrix}^{-1} \begin{bmatrix} C(x_S) \\ F(x_S) \end{bmatrix} \omega \tag{6}$$

where $V = [V_{n+1} \quad V_{n+2} \quad \ldots \quad V_{n+N}]^T$, $\psi = [\psi_{n+1} \quad \psi_{n+2} \quad \ldots \quad \psi_{n+N}]^T$, and $\omega = [\omega_1 \quad \omega_2 \quad \ldots \quad \omega_n]^T$. Also we define $\delta = [\delta_1 \quad \delta_2 \quad \ldots \quad \delta_n]^T$ and $x_S = [\delta, \omega, V, \psi]^T$. Assuming P_{Li} and Q_{Li} to be constant, we obtain $\bar{A}_{N\times N} = \partial S_P / \partial V$, $\bar{B}_{N\times N} = \partial S_P / \partial \psi$, $C_{N\times n} = \partial S_P / \partial \delta$, $\bar{D}_{N\times N} = \partial S_Q / \partial V$, $\bar{E}_{N\times N} = \partial S_Q / \partial \psi$, and $F_{N\times n} = \partial S_Q / \partial \delta$

IV. UPFC as a Nonlinear Controller

In the proposed effort, the UPFC is connected between two power system buses to reduce the system oscillations after a fault occurs. As illustrated in Fig. 2a, the UPFC shunt transformer is connected to bus $t+n$ and the series transformer is connected between buses $t+n$ and $h+n$. The effect of the UPFC on the power system can be represented as injected powers to the connecting buses [9] as shown in Fig. 2b.

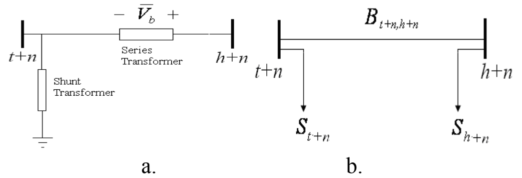

Fig.2 a)UPFC connected between two network nodes; b) Injected powers to the connected buses

The injected active and reactive powers are in turn equal to

$$
\begin{aligned}
&P_{t+n} = B_{t+n,h+n}V_b V_{h+n}\sin(\psi_{t+n}-\psi_{h+n}+\theta)\\
&P_{h+n} = -B_{t+n,h+n}V_b V_{h+n}\sin(\psi_{t+n}-\psi_{h+n}+\theta)\\
&Q_{t+n} = B_{t+n,h+n}V_b V_{t+n}\cos(\theta)\\
&Q_{h+n} = -B_{t+n,h+n}V_b V_{h+n}\cos(\psi_{t+n}-\psi_{h+n}+\theta)
\end{aligned} \tag{7}
$$

where $\overline{V}_b = V_b\angle(\psi_{t+n}+\theta)$ is the voltage produced by the series transformer and can be assumed to be a function of time. Thus, the power flow equation at buses $t+n$ and $h+n$ can be represented by (8) as

$$
\begin{aligned}
&P_{\mathrm{OLD}t+n} + B_{t+n,h+n}V_{h+n}[\gamma\sin(\psi_{t+n}-\psi_{h+n})+\mu\cos(\psi_{t+n}-\psi_{h+n})]=0\\
&P_{\mathrm{OLD}h+n} - B_{t+n,h+n}V_{h+n}[\gamma\sin(\psi_{t+n}-\psi_{h+n})+\mu\cos(\psi_{t+n}-\psi_{h+n})]=0\\
&Q_{\mathrm{OLD}t+n} + B_{t+n,h+n}V_{t+n}\gamma=0\\
&Q_{\mathrm{OLD}h+n} - B_{t+n,h+n}V_{h+n}[\gamma\cos(\psi_{t+n}-\psi_{h+n})-\mu\sin(\psi_{t+n}-\psi_{h+n})]=0
\end{aligned} \tag{8}
$$

where $\gamma = V_b\cos\theta$, $\mu = V_b\sin\theta$, and P_{OLD} and Q_{OLD} represent the left hand side of equations (3) i.e. $P_{\mathrm{OLD}i} = S_{\mathrm{P}i}$ and $Q_{\mathrm{OLD}i} = S_{\mathrm{Q}i}$ for $i=t+n$ and $i=h+n$.

By taking derivative of (8) certain new terms such as (9a) and (9b) will be added to the left hand side of (4), on the buses $t+n$ and $h+n$.

At bus $t+n$, we get

$$
\begin{aligned}
&+B_{t+n,h+n}[\gamma\sin(\psi_{t+n}-\psi_{h+n})+\mu\cos(\psi_{t+n}-\psi_{h+n})]\dot{V}_{h+n}\\
&+B_{t+n,h+n}V_{h+n}[\gamma\cos(\psi_{t+n}-\psi_{h+n})-\mu\sin(\psi_{t+n}-\psi_{h+n})](\dot{\psi}_{t+n}-\dot{\psi}_{h+n})\\
&+B_{t+n,h+n}V_{h+n}[\dot{\gamma}\sin(\psi_{t+n}-\psi_{h+n})+\dot{\mu}\cos(\psi_{t+n}-\psi_{h+n})]
\end{aligned} \tag{9a}
$$

At bus $h+n$, we get

$$
\begin{aligned}
&-B_{t+n,h+n}[\gamma\sin(\psi_{t+n}-\psi_{h+n})+\mu\cos(\psi_{t+n}-\psi_{h+n})]\dot{V}_{h+n}\\
&-B_{t+n,h+n}V_{h+n}[\gamma\cos(\psi_{t+n}-\psi_{h+n})-\mu\sin(\psi_{t+n}-\psi_{h+n})](\dot{\psi}_{t+n}-\dot{\psi}_{h+n})\\
&-B_{t+n,h+n}V_{h+n}[\dot{\gamma}\sin(\psi_{t+n}-\psi_{h+n})+\dot{\mu}\cos(\psi_{t+n}-\psi_{h+n})]
\end{aligned} \tag{9b}
$$

Similarly, terms are added to the left hand side of (5) at buses $t+n$ and $h+n$.

At bus $t+n$, we get

$$B_{t+n,h+n}\gamma\dot{V}_{t+n} + B_{t+n,h+n}V_{t+n}\dot{\gamma} \tag{10a}$$

At bus $h+n$, we get

$$
\begin{aligned}
&-B_{t+n,h+n}[\gamma\cos(\psi_{t+n}-\psi_{h+n})-\mu\sin(\psi_{t+n}-\psi_{h+n})]\dot{V}_{h+n}\\
&-B_{t+n,h+n}V_{h+n}[-\gamma\sin(\psi_{t+n}-\psi_{h+n})-\mu\cos(\psi_{t+n}-\psi_{h+n})](\dot{\psi}_{t+n}-\dot{\psi}_{h+n})\\
&-B_{t+n,h+n}V_{h+n}[\dot{\gamma}\cos(\psi_{t+n}-\psi_{h+n})-\dot{\mu}\sin(\psi_{t+n}-\psi_{h+n})]
\end{aligned} \tag{10b}
$$

Following the same procedure as in the previous section reveals that with changing some of the entries of matrices $\overline{A}$, $\overline{B}$, $\overline{D}$, and $\overline{E}$ due to the addition of terms, new matrices A, B, D, and E are obtained; whereas, matrices C and F remain unchanged. Consequently, (6) is changed as

$$\begin{bmatrix} A(x) & B(x) \\ D(x) & E(x)\end{bmatrix}\begin{bmatrix}\dot{V}\\ \dot{\psi}\end{bmatrix} = -\begin{bmatrix}C(x)\\ F(x)\end{bmatrix}\omega - G(x) \tag{11}$$

where $x=[x_S^T,\gamma,\mu]^T$ and vector G represents additional terms in (9) and (10) which are dependent on $\dot{\gamma}$ and $\dot{\mu}$. We define $u_1=\dot{\lambda}$ and $u_2=\dot{\mu}$ and obtain the entries of vector $G\in R^{2N}$ as (12).

$$
\begin{cases}
G_t = +B_{t+n,h+n}V_{h+n}[u_1\sin(\psi_{t+n}-\psi_{h+n})+u_2\cos(\psi_{t+n}-\psi_{h+n})]\\
G_h = -B_{t+n,h+n}V_{h+n}[u_1\sin(\psi_{t+n}-\psi_{h+n})+u_2\cos(\psi_{t+n}-\psi_{h+n})]\\
G_{t+N} = +B_{t+n,h+n}V_{t+n}u_1\\
G_{h+N} = -B_{t+n,h+n}V_{h+n}[u_1\cos(\psi_{t+n}-\psi_{h+n})-u_2\sin(\psi_{t+n}-\psi_{h+n})]\\
G_i = 0; \quad 1\le i\le 2N; i\notin\{t,h,t+n,h+n\}
\end{cases} \tag{12}
$$

By solving (12) for $\dot{V}$ and $\dot{\psi}$, we obtain the set of nonlinear equations

$$\begin{bmatrix}\dot{V}\\ \dot{\psi}\end{bmatrix} = \begin{bmatrix}\bar{f}_1(x)\\ \bar{f}_2(x)\end{bmatrix} + \begin{bmatrix}\bar{g}_1(x) & \bar{g}_2(x)\\ \bar{g}_3(x) & \bar{g}_4(x)\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix} \tag{13}$$

where

$$\begin{bmatrix}\bar{f}_1(x)\\ \bar{f}_2(x)\end{bmatrix} = -\begin{bmatrix}A & B\\ D & E\end{bmatrix}^{-1}\begin{bmatrix}C\\ F\end{bmatrix},\ \begin{bmatrix}\bar{g}_1(x) & \bar{g}_2(x)\\ \bar{g}_3(x) & \bar{g}_4(x)\end{bmatrix} = -\begin{bmatrix}A & B\\ D & E\end{bmatrix}^{-1}\overline{G},$$

$\overline{G}_{2N\times 2}$ satisfies $G=\overline{G}[u_1 \quad u_2]^T$, and $\bar{f}_1,\bar{f}_2,\bar{g}_1,\bar{g}_2,\bar{g}_3,\bar{g}_4\in R^N$.

Equation (13) provides a nonlinear affine differential equation in terms of control inputs u_1 and u_2. Once the control inputs are defined, we can easily obtain the UPFC control parameters γ and μ by taking integral of the inputs. Including the swing equations (2), we obtain the total system dynamic equations as

$$
\begin{cases}
\dot{\delta}_i = \omega_i\\
M_i\dot{\omega}_i = P_{mi} - \dfrac{M_i}{M_T}P_{COI} - B_{i,i+n}E_{gi}V_{i+n}\sin(\delta_i-\psi_{i+n}); i=1,\ldots,n\\
\begin{bmatrix}\dot{V}\\ \dot{\psi}\end{bmatrix} = \begin{bmatrix}\bar{f}_1(x)\\ \bar{f}_2(x)\end{bmatrix} + \begin{bmatrix}\bar{g}_1(x) & \bar{g}_2(x)\\ \bar{g}_3(x) & \bar{g}_4(x)\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}\\
\dot{\gamma}=u_1; \dot{\mu}=u_2
\end{cases} \tag{14}
$$

Remark 1. In the case that multiple UPFCs are inserted as controllers in the network, equations (7) through (12) are repeated for each pair of UPFC buses t_j and h_j for all $1\le j\le k$, where k is the total number of UPFCs. Accordingly, the corresponding entries of matrices A, B, D, and E change following the same logic described for equation (11). Moreover, vector G has entries corresponding to each UPFC. Consequently, the resulting differential equation is affine in terms of all UPFC control inputs as shown next

$$\begin{bmatrix}\dot{V}\\ \dot{\psi}\end{bmatrix} = \begin{bmatrix}\bar{f}_{T1}(x_T)\\ \bar{f}_{T2}(x_T)\end{bmatrix} + \sum_{j=1}^{k}\begin{bmatrix}\bar{g}_{1j}(x_T) & \bar{g}_{2j}(x_T)\\ \bar{g}_{3j}(x_T) & \bar{g}_{4j}(x_T)\end{bmatrix}\begin{bmatrix}u_{1j}\\ u_{2j}\end{bmatrix} \tag{15}$$

where k is the number of UPFCs and $x_T=[x_S^T,\gamma_1,\mu_1,\cdots,\gamma_k,\mu_k]^T$. Also, we have $\bar{f}_{T1},\bar{f}_{T2},\bar{g}_{1j},\bar{g}_{2j},\bar{g}_{3j},\bar{g}_{4j}\in R^N$.

V. Controller Design

The conventional approach in damping oscillations in an interconnected power system deals with mitigating the inter area oscillation modes. Usually, the linear control approaches are utilized to achieve this goal [10]. In contrast, we target the generators' stability in a nonlinear sense by defining an appropriate Lyapunov function.

A close look at equation (14) reveals that back stepping method could be utilized to control the generators angles and speeds. We restrict our design to the case with constant loads. Also, we assume that the mechanical power P_{mi} is a slow changing function of time compared to the other control variables; thus, $\dot{P}_{mi} \approx 0$. For the purpose of convenience we define the new state variables as

$$
\begin{aligned}
x_{1i} &= \delta_i - \delta_{i0} \\
x_{2i} &= \omega_i \\
x_{3i} &= V_{i+n}\sin(\delta_i - \psi_{i+n})
\end{aligned}
\tag{16}
$$

where δ_{i0} is the generator angle before fault occurs. Defining $\bar{u}_j = [u_{1j} \quad u_{2j}]^T$ and using (15), we obtain $\dot{x}_{3i}$ as (17) for the case of multiple UPFCs.

$$
\begin{aligned}
\dot{x}_{3i} &= \dot{V}_{i+n}\sin(\delta_i - \psi_{i+n}) + V_{i+n}(\omega_i - \dot{\psi}_{i+n})\cos(\delta_i - \psi_{i+n}) \\
&= \left(\bar{f}_{T1i}(x_T) + \sum_{j=1}^{k}[\bar{g}_{1ij}(x_T) \quad \bar{g}_{2ij}(x_T)]\bar{u}_j\right)\sin(\delta_i - \psi_{i+n}) + V_{i+n}\omega_i\cos(\delta_i - \psi_{i+n}) \\
&\quad - V_{i+n}\left(\bar{f}_{T2i}(x_T) + \sum_{j=1}^{k}[\bar{g}_{3ij}(x_T) \quad \bar{g}_{4ij}(x_T)]\bar{u}_j\right)\cos(\delta_i - \psi_{i+n}) \\
&= \mathrm{f}_{Ti}(x_T) + \sum_{j=1}^{k}\mathrm{a}_{1ij}(x_T)u_{1j} + \mathrm{a}_{2ij}(x_T)u_{2j}
\end{aligned}
\tag{17}
$$

where $\bar{f}_{T1i}$, $\bar{f}_{T2i}$, $\bar{g}_{1ij}$, $\bar{g}_{2ij}$, $\bar{g}_{3ij}$, and $\bar{g}_{4ij}$ are the i th elements of $\bar{f}_{T1}$, $\bar{f}_{T2}$, $\bar{g}_{1j}$, $\bar{g}_{2j}$, $\bar{g}_{3j}$, and $\bar{g}_{4j}$, respectively. Also, k is the total number of UPFCs and j is the UPFC number.

Single generator control. Here for simplicity and without loss of generality we design a stable controller for a one-UPFC power system using back stepping design method. The design can be generalized to the case of multiple UPFC. From (2), (16) and (17) we construct the new set of state equations as

$$
\begin{cases}
\dot{x}_{1i} = x_{2i} \\
M_i\dot{x}_{2i} = P_{mi} - \dfrac{M_i}{M_T}P_{COI} - B_{i,i+n}E_{gi}x_{3i} \\
\dot{x}_{3i} = \mathrm{f}_i(x) + \mathrm{a}_{1i}(x)u_1 + \mathrm{a}_{2i}(x)u_2; i = 1, \ldots, n
\end{cases}
\tag{18}
$$

where $\mathrm{f}_i(x) = \mathrm{f}_{Ti}(x_T)$, $\mathrm{a}_{1i}(x) = \mathrm{a}_{1ij}(x_T)$, and $\mathrm{a}_{2i}(x) = \mathrm{a}_{2ij}(x_T)$ from (17) when there is only one UPFC in the power network. Introducing $K_{\delta i}$ and K_{Z1i} as design constants, we have

$$\dot{x}_{1i} = -K_{\delta i}x_{1i} + z_{1i} \tag{19}$$

where $z_{1i} = x_{2i} + K_{\delta i}x_{1i}$. As a result

$$M_i\dot{z}_{1i} = P_{mi} - \frac{M_i}{M_T}P_{COI} + M_iK_{\delta i}x_{2i} - B_{i,i+n}E_{gi}x_{3si} - B_{i,i+n}E_{gi}z_{2i} \tag{20}$$

and

$$\dot{z}_{2i} = \mathrm{f}_i(x) + \mathrm{a}_{1i}(x)u_1 + \mathrm{a}_{2i}(x)u_2 - \dot{x}_{3si} \tag{21}$$

where $z_{2i} = (x_{3i} - x_{3si})$. Here

$$x_{3si} = \frac{1}{-B_{i,i+n}E_{gi}} \times [-x_{1i} - P_{mi} + \frac{M_i}{M_T}P_{COI} + M_iK_{\delta i}x_{2i} - K_{Z1i}z_{1i}] \tag{22}$$

Defining the following Lyapunov function

$$\mathrm{L}_{2i} = K_{\mathrm{atten}i}(\frac{1}{2}x_{1i}^2 + \frac{1}{2}M_iz_{1i}^2) + \frac{1}{2}z_{2i}^2 \tag{23}$$

with $K_{\mathrm{atten}i}$ being a design constant, we can easily show that $\dot{\mathrm{L}}_{2i} < 0$ provided that $\dot{z}_{2i} = \nu_i$ is chosen to be as (24).

$$\nu_i = K_{\mathrm{atten}i}z_{1i}B_{i,i+n}E_{gi} - K_{Z2i}z_{2i} \tag{24}$$

Equation (21) along with $\dot{z}_{2i} = \nu_i$ and (24) provides a linear relationship between the inputs u_1, u_2, and nonlinear functions of states as

$$\mathrm{a}_{1i}(x)u_1 + \mathrm{a}_{2i}(x)u_2 = \nu_i - \mathrm{f}_i(x) + \dot{x}_{3si}. \tag{25}$$

Remark 2. Equation (25) renders a relationship between u_1 and u_2. In order to identify these, we have to use another relationship between the mentioned inputs from other control requirements. For our simulations we have considered them equal, i.e. $u_1 = u_2$.

Multiple generator control. Due to the inconsistency that occurs in the calculation of the control inputs u_1 and u_2 for different generators, the scheme deployed for single generator control cannot be repeated for multiple generators to be controlled. Thus, for the case of multiple generator control, we propose the Lyapunov function in previous section as

$$\mathrm{L} = K_{\mathrm{atten}i}\sum_{i=1}^{n-1}\mathrm{L}_{1i} + \frac{1}{2}(\sum_{i=1}^{n-1}z_{2i})^2 \tag{26}$$

where $n-1$ generators are chosen to be controlled. Since the n generators are located in one interconnected power network the last one is normally forced to be controlled if the remaining $n-1$ are controlled. Taking derivative of (26), we have

$$\dot{\mathrm{L}} = K_{\mathrm{atten}i}\sum_{i=1}^{n-1}\dot{\mathrm{L}}_{1i} + (\sum_{i=1}^{n-1}\dot{z}_{2i})(\sum_{i=1}^{n-1}z_{2i}). \tag{27}$$

Using the following equation

$$\sum_{i=1}^{n-1}\dot{z}_{2i} = \sum_{i=1}^{n-1}\nu_i \tag{28}$$

causes the second term in (27) to be as (29)

$$(\sum_{i=1}^{n-1}K_{\mathrm{atten}i}z_{1i}B_{i,i+n}E_{gi})(\sum_{i=1}^{n-1}z_{2i}) - (\sum_{i=1}^{n-1}K_{Z2i}z_{2i})(\sum_{i=1}^{n-1}z_{2i}) \tag{29}$$

This in turn causes the Lyapunov function's derivative (27) to become as (30) by choosing $K_{Z2i} = K_{Z2}$ and $K_{\mathrm{atten}i} = K_{\mathrm{atten}}$ for all $1 \le i \le n-1$.

$$\dot{\mathrm{L}} \le \sum_{i=1}^{n-1}K_{\mathrm{atten}}(-K_{\delta i}x_{1i}^2 - K_{\mathbf{Z1}i}M_iz_{1i}^2) - K_{Z2}(\sum_{i=1}^{n-1}z_{2i})^2 + \sum_{i=1}^{n-1}\left(\left|K_{\mathrm{atten}}z_{1i}B_{i,i+n}E_{gi}\right| \cdot \sum_{j=1}^{n-1}\left|z_{2j}\right|\right) \tag{30}$$

The derivative of the Lyapunov function can be made negative by choosing large K_{Z2} and small K_{atten}. This will lead to bounded stability of the power network with the bound shown below.

$$2\overline{\eta}\|z_1\|^2 + \|z_2\|^2 \le \frac{2}{(n-1)K_{\text{atten}}}\left(\sum_{i=1}^{n-1} K_{\text{atten}} K_{\delta i} x_{1i}^2 + K_{Z2}(\sum_{i=1}^{n-1} z_{2i})^2\right) \qquad (31)$$

where $z_1 = [z_{11} \ \cdots \ z_{1,n-1}]^T$, $z_2 = [z_{21} \ \cdots \ z_{2,n-1}]^T$, $\overline{\eta} = \sqrt{\sum_{i=1}^{n-1} (\eta - K_{Z1i} M_i)^2}$, and $\eta = \frac{1}{2} \max_{1 \le i \le n-1} (B_{i,i+n} E_{gi}) > 0$.

In order to obtain u_1 and u_2 in the case of multiple generator control equation (21) is changed as (32) by using (28)

$$\sum_{i=1}^{n-1} v_i = \sum_{i=1}^{n-1} \left(\mathrm{f}_i(x) + \mathrm{a}_{1i}(x) u_1 + \mathrm{a}_{2i}(x) u_2 - \dot{x}_{3si}\right) \qquad (32)$$

VI. NN CONTROL

Although equation (32) provides the UPFC control inputs, finding the analytical and/or numerical nonlinear control inputs in practice (for fast computing) is an issue especially for large power systems. Moreover, in order to implement the control law, a complete knowledge of the total power system topology (i.e. Ybus.) is needed. However, by using neural network approximation property for nonlinear functions, we are able to approximate the nonlinear terms involved in the system dynamics, here known as unknown dynamics, thus relax the need for system description as well as burdensome function calculation.

Without loss of generality we assume $u_1 = u_2$. Starting with set of equations (19), (20), and (21) for the i th generator in a single-UPFC power system, we have

$$\dot{x}_{1i} = -K_{\delta i} x_{1i} + z_{1i} \qquad (33)$$
$$M_i \dot{z}_{1i} = f_{1i}(x_{1i}, x_{2i}) + g_{1i}(x_{1i}, x_{2i}) z_{2i}$$
$$\dot{z}_{2i} = g_{2i}\left(\frac{f_{2i}(x)}{g_{2i}(x)} + u_1\right)$$

where x is the vector of the global parameters as defined earlier.

Assumption 1. Here we assume that $g_{2i}(x)$ is positive and bounded away from zero which is a valid practical assumption. This claim is supported by the fact that if $g_{2i}(x) = \mathrm{a}_{1i}(x) + \mathrm{a}_{2i}(x)$ changes sign, due to its continuity, it must pass through the origin for which case equation (25) encounters singularity tending to make $[\gamma \quad \mu]^T$ infinitely large. By selecting a proper place for UPFC and setting appropriate design gains, we can avoid large control inputs.

Multiple generator control. The entire Lyapunov function in this case is proposed as

$$\mathrm{L} = K_{\text{atten}i} \sum_{i=1}^{n-1} \mathrm{L}_{1i} + \left(\sum_{i=1}^{n-1} z_{2i}\right)^2 \Big/ 2\sum_{i=1}^{n-1} g_{2i} + \frac{1}{2} \widetilde{W}^T \Gamma_i^{-1} \widetilde{W} \qquad (34)$$

which leads to the energy function derivative (35)

$$\dot{\mathrm{L}} = K_{\text{atten}i}(-K_{\delta i} x_{1i}^2 - K_{Z1i} M_i z_{1i}^2) - K_{\text{atten}i} z_{1i} B_{i,i+n} E_{gi} z_{2i}$$
$$+ \left(\sum_{i=1}^{n-1} f_{2i} \Big/ \sum_{i=1}^{n-1} g_{2i} + u_1 - \sum_{i=1}^{n-1} \dot{g}_{2i} \sum_{i=1}^{n-1} z_{2i} \Big/ 2\left(\sum_{i=1}^{n-1} g_{2i}\right)^2\right) \sum_{i=1}^{n-1} z_{2i} + \widetilde{W}^T \Gamma^{-1} \dot{\hat{W}} \qquad (35)$$

Here we define the control input u_1 as

$$u_1 = \sum_{i=1}^{n-1} v_i - \left(\sum_{i=1}^{n-1} f_{2i} \Big/ \sum_{i=1}^{n-1} g_{2i} - \sum_{i=1}^{n-1} \dot{g}_{2i} \sum_{i=1}^{n-1} z_{2i} \Big/ 2\left(\sum_{i=1}^{n-1} g_{2i}\right)^2\right) \approx \sum_{i=1}^{n-1} v_i - \hat{W}^T \varphi(\overline{V}^T x) \qquad (36)$$

where the second term is approximated by the neural network as $W^T \phi(\overline{V}^T x) + \varepsilon$ with $\varepsilon \le \varepsilon_M$ being the approximation error. The weight matrix $\overline{V}_i$ is randomly chosen and fixed during the entire control time and W_i is the unknown ideal weight matrix to be determined [11]. Since the ideal weights are not known, the estimated weight matrix $\hat{W}_i$ is utilized to approximate u_1. Implementing (36) into (35), using (24), and assuming $K_{Z2i} = K_{Z2}$ for $1 \le i \le n-1$, we obtain the Lyapunov function's derivative $\dot{\mathrm{L}}$ to be

$$\dot{\mathrm{L}} = K_{\text{atten}i}(-K_{\delta i} x_{1i}^2 - K_{Z1i} M_i z_{1i}^2) - K_{Z2}\left(\sum_{i=1}^{n-1} z_{2i}\right)^2 \qquad (37)$$
$$+ \sum_{i=1}^{n-1} \sum_{\substack{j=1 \\ j \ne i}}^{n-1} \left(K_{\text{atten}i} z_{1i} B_{i,i+n} E_{gi} g_{2i}\right) z_{2j} - \widetilde{W}^T \varphi(\overline{V}^T x)\left(\sum_{i=1}^{n-1} z_{2i}\right) + \varepsilon\left(\sum_{i=1}^{n-1} z_{2i}\right) + \widetilde{W}^T \Gamma^{-1} \dot{\hat{W}}$$

Now we define the weight estimation matrix update law as (38) [11] resulting in the energy function derivative as (39).

$$\dot{\hat{W}} = \Gamma \phi(\overline{V}^T x) \sum_{i=1}^{n-1} z_{2i} - \alpha \Gamma \hat{W} \qquad (38)$$

$$\dot{\mathrm{L}} \le -\alpha \widetilde{W}^T \widetilde{W} - \varepsilon \sum_{i=1}^{n-1} z_{2i} - \alpha \widetilde{W}^T W^T - K_{Z2}(\sum_{i=1}^{n-1} z_{2i})^2 \qquad (39)$$
$$- \sum_{i=1}^{n-1} K_{\text{atten}}(-K_{\delta i} x_{1i}^2 - K_{Z1i} M_i z_{1i}^2) + \sum_{i=1}^{n-1}\left(\left|K_{\text{atten}} z_{1i} B_{i,i+n} E_{gi}\right| \cdot \sum_{j=1}^{n-1} \left|z_{2j}\right|\right)$$

The Lyapunov function derivative (39) can be made negative by choosing a large K_{Z2} and small K_{atten}.

Remark 3. Equation (34) needs the term $\sum_{i=1}^{n-1} g_{2i}$ to be bounded away from zero. If it is not zero, based on Assumption 1, this can be easily achieved when each z_{2i} is replaced with $K_{MZ2i} z_{2i}$ in the energy function (34) where K_{MZ2i} is a proper modification factor. In our design we assume $K_{MZ2i} = 1$ for $1 \le i \le n-1$.

Remark 4. We can see from (36) and (38) that the control and update laws are only functions of power system states, generators data, and loads. Although for the controller design δ_{i0} is needed, this parameter can be achieved by knowing the present generator working conditions. Thus, we relax the need for obtaining Ybus by using the neural network controller. This in turn relaxes a priori knowledge about network topology if the topology after the fault is cleared is same as that of the case prior to the fault.

VII. SIMULATION RESULTS

For simulation, a 14-bus, 5-generator system is selected as shown in Fig. 3. The generator classical model is applied for simulating. All the generators have turbine speed controllers and the UPFC controller is implemented by means of UPFC active and reactive power injections at the UPFC terminals. The power system loads are all constant. The control objective is to damp the generators oscillations after the fault is cleared. In the system of Fig. 3 the UPFC is installed on bus 6 between buses 6 and 9.

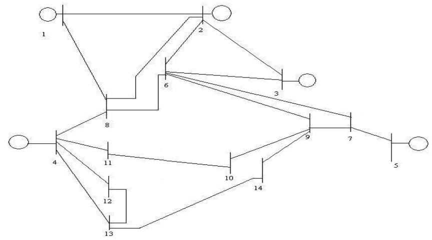

Fig.3-The IEEE 14-bus, 5-generator power system

The line resistances are not ignored in this simulation. A three phase short circuit fault is occurred on bus 6 at $t = 0$ and lasts for 0.2 seconds. The UPFC is activated after the fault is cleared and parameters γ and μ are initially set to zero. Here generators1 through 4 are chosen to be controlled and as a result fifth one is forced to be controlled automatically. The generators data is given as follows. $x'_{di} = 0.006; 1 \le i \le 5; H_1 = 5; H_2 = 1; H_3 = 1; H_4 = 5; H_5 = 5;$

The simulations are accomplished as in the following cases.

Case 1. All power system dynamics assumed to be available for the control design and equation (33) along with the assumption $u_1 = u_2$ are used to design the controller. The design gains are chosen as follows. K_{atten} =0.0001, $K_{\delta 1}$=.1, $K_{\delta 2}$=.2, $K_{\delta 3}$=.1, $K_{\delta 4}$=.1, K_{Z11} through K_{Z14}=.1, K_{Z2}=500

Case 2. Power system dynamics are assumed unavailable. Ten neurons are selected for the NN hidden layer and design gains are chosen as follows. K_{atten} =1e-4, $K_{\delta 1}$=.1, $K_{\delta 2}$=.2, $K_{\delta 3}$ & $K_{\delta 4}$=.1, K_{Z11} thru K_{Z14}=.1, K_{Z2}=500, α=1e-4, Γ=5e5

The simulations results show significant oscillation damping can be achieved for medium size power networks by using a single UPFC as a damping controller. The generator speeds decay; however, the generator angles as well as UPFC control parameters γ and μ do not go back to pre fault situations as the size of the power system increases. This can be explained by the fact that the total system has bounded stability. The simulations with the neural network controller show almost the same significance in damping the oscillations. Although the UPFC injected powers are a little larger in the case of neural network controller, they are in an acceptable range.

VIII. Conclusions

We have introduced a general nonlinear dynamical model for power systems with UPFC as stabilizing controller using power system classical model. This model is free of algebraic equations, thus conventional nonlinear control strategies are applicable to stabilize the power system after fault occurrence. We have addressed a multi machine control scheme in which the number of control inputs is less than the number of outputs. Furthermore, we have utilized neural networks approximation property to relax burdensome nonlinear function calculation and a priori knowledge about the power system topology needed for control design. Our analytical approach as well as our simulation results shows the effectiveness of our approach.

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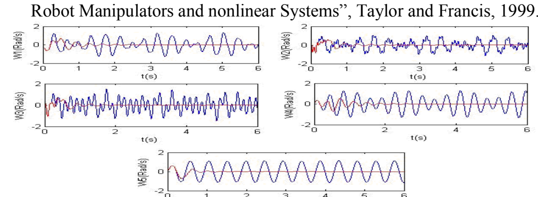

Fig.4-Generator speeds as compared to the case with turbine control only; Case 1

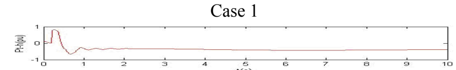

Fig.5-Active power flow from bus 6 to 9; Case 1

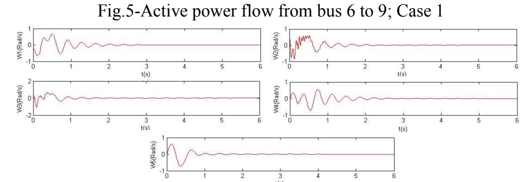

Fig.6-Generator speeds; Case 2

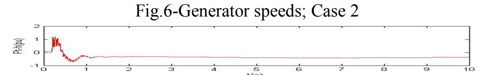

Fig.7-Active power flow from bus 6 to 9; Case 2