01 Feb 2008

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Jing Huang

Keith Corzine
Missouri University of Science and Technology

Shuai Lu

Chris M. Hutson

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Jing Huang[1], *Student Member, IEEE*, Keith A. Corzine[1], *Senior Member, IEEE*, Chris M. Hutson[1], and Shuai Lu[2], *Member, IEEE*

[1]Missouri University of Science and Technology, 1870 Miner Circle, Rolla, MO 65409
[2]MTS System Corporation, 14000 Technology Drive, Eden Prairie, MN 55344

***Abstract* - As multi-phase (defined as greater than three-phase) drives become more popular and practical, new research in this area investigates potential advantages including lower torque ripple and better power density. The added dimensions of a multi-phase machine leads to a completely different operating nature than standard three-phase machines. It can be shown physically and mathematically that certain harmonics do not contribute to torque production and therefore the torque is not directly tied to the current wave-shape. This paper utilizes this property to demonstrate a substantial increase in voltage range and a reduction in torque ripple through the use of added voltage harmonics. An analysis of a five-phase motor is presented followed by a range of modulation techniques. It is shown that by proper selection of third, fifth, and seventh harmonics, the required dc voltage can be reduced by eighteen percent and the torque ripple can be reduced by nearly sixty percent over traditional methods at the expense of higher current THD; which may not be a disadvantage in certain applications. Further investigation is then carried out in applying a unique space-vector modulation patter to the five-phase motor drive. This further reduces the torque ripple. Detailed simulation and laboratory tests are used to demonstrate this concept.**

I. INTRODUCTION

Recently, the multi-phase motor drive system concept has become popular in some specialized applications such as electric ship propulsion, electric/hybrid vehicle, traction, etc. Five-phase motor drive systems are gaining attention due to the trade-off of cost and potential advantages over the three-phase drive, such as higher power density where the weight and volume are critical and fault tolerance for better performance [1-11]. The five-phase system has some inherent characteristics which are completely different from standard three-phase systems. Most importantly, the existence of two *q-d* planes by transforming machine variables to *q-d* variables in the arbitrary reference frame splits the harmonics of system. This property could be used to design a controller for specific applications.

High torque ripple is the major consideration in applications where minimal acoustic noise is required. The application of a multi-phase system itself is a good way to reduce torque ripple [1-2]. Proper motor design is another popular way for torque ripple minimization [3]. Different pulse width modulation (PWM) methods have been developed in three-phase systems. However, in these systems, the torque ripple is directly related to the current wave-shape which is not the case in five-phase systems. For a five-phase system without a zero sequence current, it is possible to have components in the currents that do not contribute to the torque. This property is helpful in lowering the torque ripple. Recent research shows that a proper vector combination can be used to reduce the torque ripple with space vector PWM (SVPWM) [4]. Although this will introduce more current THD, the current distortion may not be a disadvantage in some specific application where the torque ripple performance is the major concern. Since the SVPWM method needs different vector identification and sequence selection, different look-up tables are needed to determine the dwell time at different modulation indices, which increases the complexity of practical implementation. Therefore, carrier-based sine-triangle method which could reach the same torque performance becomes desirable and is the main focus of this paper.

A wide range of PWM strategies have been proposed for studying multi-phase systems [7-13]. Current research focuses on the SVPWM control in which the main objective is to generate the voltage-source inverter (VSI) with output as close to sinusoidal waveforms as possible. It can be shown that the maximum modulation index in the five-phase drive is 1.2311. However, some carrier-based PWM [12,13] methods can not utilize the full dc bus voltage. In this paper, a modulation method with 3^{rd}, 5^{th} and 7^{th} harmonic injection is proposed. The advantages include simple practical implementation, full utilization of dc voltage supply, and lower torque ripple.

A three-level five-phase inverter is used in this paper to demonstrate the proposed concepts. This converter integrates multilevel and multi-phase technologies. It combines the advantages of both and has better performance than the two-level five-phase system [4]. Simulation and laboratory tests of this topology are used to validate the proposed ideas.

II. FIVE-PHASE INDUCTION MACHINE MODEL

Compared with the traditional three-phase induction machine, the multi-phase induction machine provides some

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distinct features, such as the ability to start and run even with some phases open circuited. This makes the five-phase machine attractive in systems where very high reliability is required [5]. High power density can be more easily achieved than in the case of three-phase machines by increasing the number of phases without increasing the rated current. Torque ripple behavior is also inherently superior to that of a three-phase drive system [6].

To simplify the development of the five-phase induction machine model, the stator windings of all phases are assumed to be identical and coils are sinusoidally distributed. Furthermore, a linear magnetic core is considered. The inherent 72º stator winding structure causes unsymmetrical line-to-line voltages (i.e. v_{ab} and v_{ac}). For this reason the vector space does not appear in evenly distributed layers as in the case of a three-phase machine. The five-phase induction machine model used herein will be based on the arbitrary reference frame transformation [4]

$$\begin{bmatrix} f_{qs1} \\ f_{ds1} \\ f_{qs2} \\ f_{ds2} \\ f_{0s} \end{bmatrix} = \frac{2}{5} \begin{bmatrix} \cos(\theta) & \cos\left(\theta - \frac{2\pi}{5}\right) & \cos\left(\theta - \frac{4\pi}{5}\right) & \cos\left(\theta + \frac{4\pi}{5}\right) & \cos\left(\theta + \frac{2\pi}{5}\right) \\ \sin(\theta) & \sin\left(\theta - \frac{2\pi}{5}\right) & \sin\left(\theta - \frac{4\pi}{5}\right) & \sin\left(\theta + \frac{4\pi}{5}\right) & \sin\left(\theta + \frac{2\pi}{5}\right) \\ \cos(\theta) & \cos\left(\theta + \frac{4\pi}{5}\right) & \cos\left(\theta - \frac{2\pi}{5}\right) & \cos\left(\theta + \frac{2\pi}{5}\right) & \cos\left(\theta - \frac{4\pi}{5}\right) \\ \sin(\theta) & \sin\left(\theta + \frac{4\pi}{5}\right) & \sin\left(\theta - \frac{2\pi}{5}\right) & \sin\left(\theta + \frac{2\pi}{5}\right) & \sin\left(\theta - \frac{4\pi}{5}\right) \\ 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} f_{as} \\ f_{bs} \\ f_{cs} \\ f_{ds} \\ f_{es} \end{bmatrix} \quad \textbf{(1)}$$

where f may represent voltage, current, or flux linkage. Transforming the machine-variable voltage and flux linkage equations to the arbitrary reference frame gives a traditional q-d model which has two q-d axes and one zero sequence that are used to represent the original five phases. It can be shown from (1) that applied voltages of harmonic $5k \pm 1$ appear in the $q1$-$d1$ plane and harmonics of $5k \pm 2$ appear in the $q2$-$d2$ plane (where k is the integer harmonic number). Furthermore, the $q2$-$d2$ plane does not contain back-EMF terms which is also noted by the torque equation

$$T_e = \frac{5}{2}\frac{P}{2}\left(\lambda_{qr1} i_{dr1} - \lambda_{dr1} i_{qr1}\right) \quad \textbf{(2)}$$

Therefore, harmonics such as the 3rd, 7th, 11th, 13th, etc. do not contribute to electromagnetic torque. This property will be utilized below to increase the voltage capability of the five-phase drive.

III. Five-Phase Inverter Model

A number of research work has been directed to two voltage levels per phase. In many specific applications where acoustic noise caused by torque pulsation is a major concern, the multilevel drive can be used due to the more discrete voltage levels per phase to achieve the target waveform. A typical five-phase three-level diode-clamped inverter is composed of five phase legs with four transistors each as shown in Figure 1.

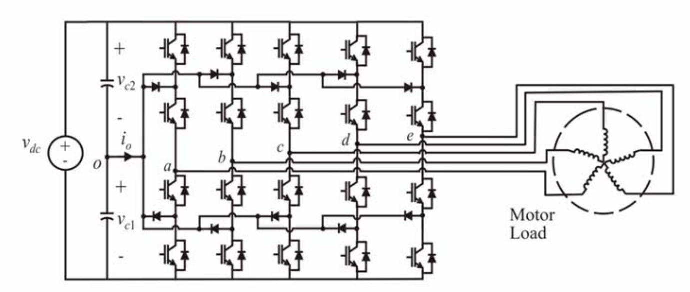

Figure 1. Three-level five-phase motor drive system.

With the star-connected motor, it can be shown that the motor voltages are related to the inverter line-to-ground voltage by

$$\begin{bmatrix} v_{as} \\ v_{bs} \\ v_{cs} \\ v_{ds} \\ v_{es} \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 4 & -1 & -1 & -1 & -1 \\ -1 & 4 & -1 & -1 & -1 \\ -1 & -1 & 4 & -1 & -1 \\ -1 & -1 & -1 & 4 & -1 \\ -1 & -1 & -1 & -1 & 4 \end{bmatrix} \begin{bmatrix} v_{ag} \\ v_{bg} \\ v_{cg} \\ v_{dg} \\ v_{eg} \end{bmatrix} \quad \textbf{(3)}$$

The inverter modulation operates by commanding the per-phase line-to-ground voltage as

$$v_{xg}^{*} = v_{dc}\left[\frac{1}{2} + m\sum_{k=1}^{n}\frac{m_k}{2}\cos\left(k\left(\theta_e - x\frac{2\pi}{5}\right)\right)\right] \quad \textbf{(4)}$$

where x represents phase that can be a through e. The phase shift x is 0 for the a-phase, 1 for the b-phase, and so on. In this general form, the fundamental component and a number of harmonics can be added; each with a modulation factor $m \cdot m_k$. Where k is the harmonic number which can have values up to n.

IV. Maximum Modulation Index of a Multi-Phase Drive

The maximum modulation index of a multi-phase drive is the maximum value of m that can be used in (4) without having the commanded voltage exceed the dc link voltage. The modulation index is a ratio of the magnitude of ac voltage to the dc voltage and, it could be used to indicate the minimum dc bus voltage of inverter for a given motor voltage. The maximum modulation index may differ depending on the modulation methods. *However, it has a limited value for a system of a specified number of phases.* It can be easily observed from the three-phase two-level vector space that two zero-states are located at the origin and the remaining six states form a hexagon around the origin. The maximum modulation index is determined by the radius of an inscribed circle inside the hexagon, which can be expressed as

$$\frac{2}{3} v_{dc} \cos(30°) = \frac{m_{\max}}{2} v_{dc} \quad \textbf{(5)}$$

For multi-phase multi-level systems, the number of switching states increases dramatically, and there are more redundant states in the vector space. However, the maximum modulation index can still be obtained by calculating the radius of the inscribed circle of the states in the outer layer of the vector space. For example, for five-phase systems, the maximum modulation index is calculated from

$$0.6472\, v_{dc} \cos(18°) = \frac{m_{\max}}{2} v_{dc} \qquad \textbf{(6)}$$

A study of the vector spaces of multi-phase drives reveals that some switching vectors at the outer layer can be easily found. Table I shows one such vector for each of the systems with up to fifteen phases. Several specific examples are plotted in Figure 2. Therein, the blue dots are the space vectors and the red dots present vectors under study.

Table 1. Maximum mod. index for various phase numbers.

Phase number	Switching states	Maximum mod. index
3	100	1.1547
5	11000	1.2311
7	111000	1.2518
9	111100000	1.2603
11	11111000000	1.2646
13	1111110000000	1.2670
15	1111111000000000	1.2686

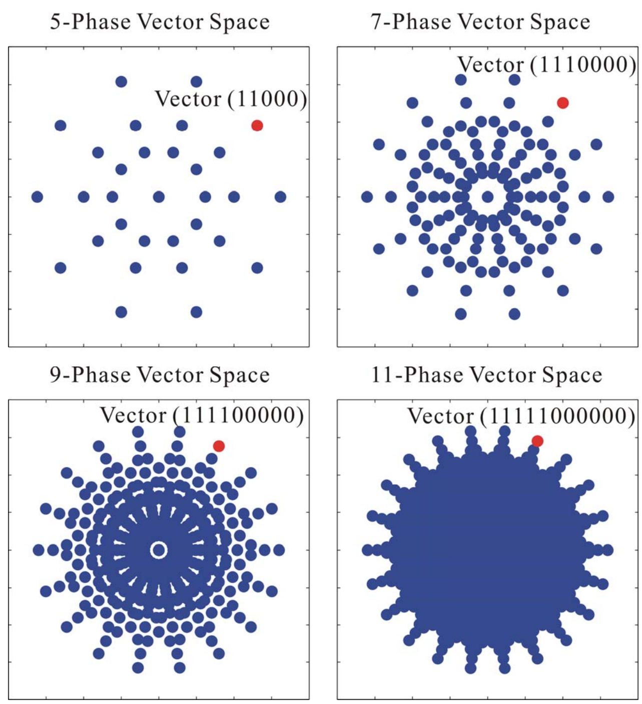

Figure 2. Space-vector plots for various multi-phase drives.

Generally, for a two-level system with 2N+1 phases, where N = 1, 2, …, the maximum modulation index could be expressed as

$$m_{\max} = \left| \sum_{k=1}^{(N-1)/2} e^{-\frac{(k-1)2\pi}{N} j} \right| \frac{4}{N} \cos\left(\frac{\pi}{2} \frac{1}{N} \right) \qquad \textbf{(7)}$$

Based on (7), Table I shows the maximum modulation indices for several systems. It can be observed that the higher the number of phases, the larger the maximum modulation index is. For systems with very large number of phases, the gain in modulation index is marginal and the largest number is 1.2732. Figure 3 shows the trend of values of modulation indices versus phase number. Although machines with very large numbers of phases are not practical, this analysis provides a mathematical basis of maximum modulation index in multi-phase systems.

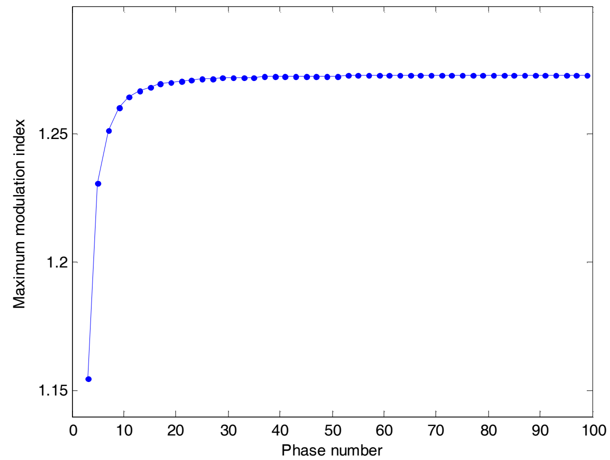

Figure 3. Maximum modulation index verse phase number.

In three-phase systems, adding a third harmonic component to the output of each phase [14] or the discontinuous clamping modulation method [15] can produce a 15% increase in the output voltage of PWM inverters. In other words, the dc link voltage can be reduced by 13%. In five-phase systems, the maximum modulation index is 1.2311. However, the traditional adding of zero-sequence terms could not obtain the minimal dc link voltage as in three-phase drives. The following section will discuss how the different forms of injection of harmonics will relate with the utilization of dc voltage and torque ripple.

V. Injection of Voltage Harmonics

A. *Traditional Natural Sampled Modulation*

Utilizing commanded voltages given by (4) with only the fundamental component yields traditional natural sampled modulation. According to (4), m_1 is limited to 1.0. The maximum modulation index m is also limited to 1.0. A system with a three-level five-phase inverter connected to a five-phase machine as shown in Figure 1 was simulated in [17]. The dc supply voltage was 125 V and could create a fundamental phase voltage of 62.5 V. In the simulation, 90% of maximum modulation index is used in the example system which represents the normal working condition, and the switching frequency was set to 3600 Hz. The following figures show the voltage and current waveform, the electromagnetic torque waveform (with torque ripple), and the corresponding space vector plot.

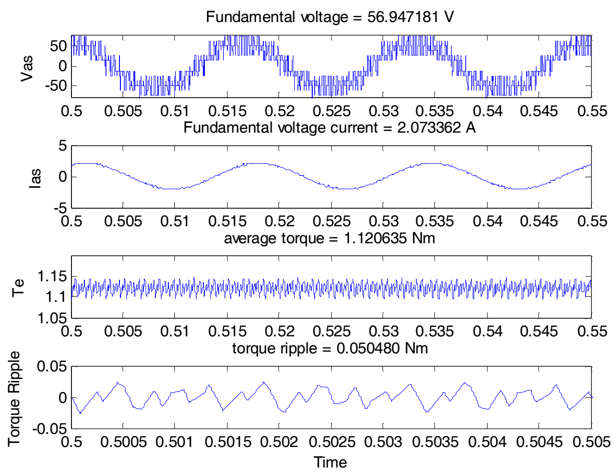

Figure 4a. Waveforms using natural sampled modulation.

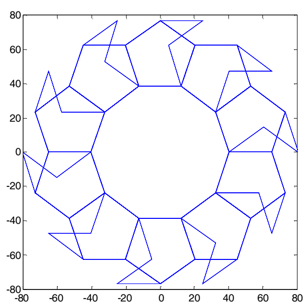

Figure 4b. Vector space of natural sampled modulation.

B. Fifth-Harmonic Injection

When a 5th harmonic term is added to (4), the resulting line-to-line voltage and phase voltage are almost the same as traditional natural sampled modulation since the 5th harmonic terms of the line-to-ground voltages will not appear on the motor windings according to (3). However, selecting a modulation index of $m_5 = -1/16$ will allow a maximum modulation index of $m = 1.05$ while still keeping (4) within the physical bounds of the dc voltage. Simulation results are shown in Figure 5. To maintain the fundamental phase voltage with 62.5 V, the required dc supply voltage is set to 119 V. The torque ripple is slightly improved. However, the maximum modulation index in this case is not the upper limit, which means the dc voltage could be lowered even further.

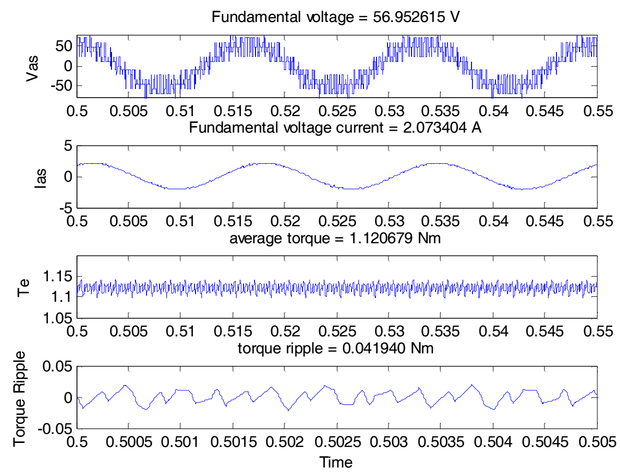

Figure 5a. Fifth-harmonic injection waveforms.

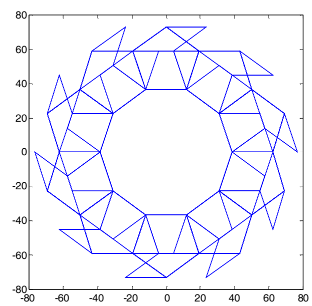

Figure 5b. Vector space of fifth-harmonic injection.

C. Additional Harmonic Injection

To determine the amount of harmonics that can produce the maximum modulation index for a five-phase system, an optimization based method was adopted. Three harmonics, 3rd, 5th and 7th, were considered, which are represented by the parameters *m*3, *m*5 and *m*7 respectively. The reference voltage with harmonics added can be expressed as

$$v_{xg}^{*} = \frac{v_{dc}}{2}\left[1 + m\begin{pmatrix}\cos(\theta_e) + m_3\cos(3\theta_e) + \\ m_5\cos(5\theta_e) + m_7\cos(7\theta_e)\end{pmatrix}\right] \quad \textbf{(8)}$$

The objective of the optimization algorithm is to determine the optimal combination of *m*3, *m*5 and *m*7 so that the summation of these terms has the smallest maximum value in a period. A multivariable unconstrained minimization algorithm is used, which is based on a simplex search method that only gives local minimal solutions near a starting point. To overcome this difficulty, the algorithm is repeatedly called to find solutions near different starting points. A total of 29,791 initial estimates were tried, and these estimated harmonic magnitude values were distributed in a range from -30% to 30% of the fundamental component. The results of the study show that the maximum modulation index that can be achieved by adding harmonics to references in a five-phase system is around 1.2311, which is consistent with the derivation described in the previous section. The corresponding harmonic contents are -26.52% for 3rd harmonics, 10.0% for 5th harmonics, and -2.92% for 7th harmonics. If the amount of 3rd harmonics is not acceptable, lower values of $k3$ can be used, but the resulting modulation index will also be lower. Figure 7 shows the trend of maximum modulation index obtained from the optimization algorithm as the parameters change. It can be clearly seen that the maximum value, 1.2311, is only achieved for a single combination of parameters. As $m3$, $m5$ and $m7$ move away from these values, the maximum modulation index decreases.

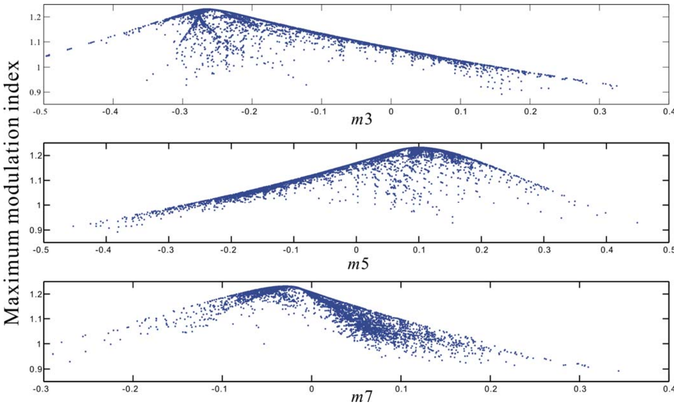

Figure 7. Coefficient selection for maximum modulation index.

Herein, addition of the 3rd and 7th harmonics is suggested to extend the voltage range of the five-phase motor drive. With the added harmonics, a dc bus voltage v_{dc} of only 102 V is needed for maintaining the same fundamental inverter output voltage, which is an 18% decrease compared with traditional natural sampled modulation. Simulation results of Figure 8 show the addition of these harmonics will indeed add low-frequency content to the motor phase voltages and currents. However, the torque ripple is only 60% of that using traditional natural sampled modulation. In applications such as Naval ship propulsion, where torque ripple handily produces acoustic noise, this modulation method may be preferred.

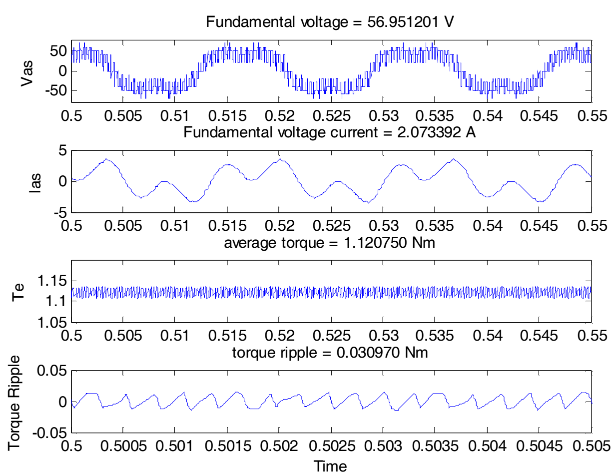

Figure 8a. Waveforms with additional harmonic injection.

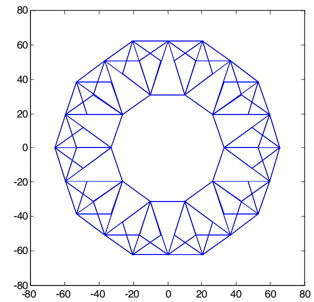

Figure 8b. Vector space of additional harmonic injection.

D. *Harmonic Injection by SVM*

The modulation techniques described above are all carrier-based modulation methods. The space-vector modulation (SVM) is another modulation technique and is commonly used for three-phase systems. It has been shown by researchers that SVM and carrier-based modulation methods are practically the same [16]. In past research, the SVM applied to multi-phase drives is adjusted so that the applied voltages are sinusoidal. In this paper, an SVM technique is adopted which is based on the concept of nearest vectors. This SVM method introduces a significant amount of harmonic components; the 3rd, 5th, and 7th being the dominant ones. The nearest triangle vectors are chosen based on the commanded voltage. Simulation results show a fixed vector sequence with a specific modulation index (90% of maximum modulation index) can produce the minimum torque ripple. The vector sequence was chosen so that the transistor switching frequency was nearly the same as with the carrier-based modulation methods described above.

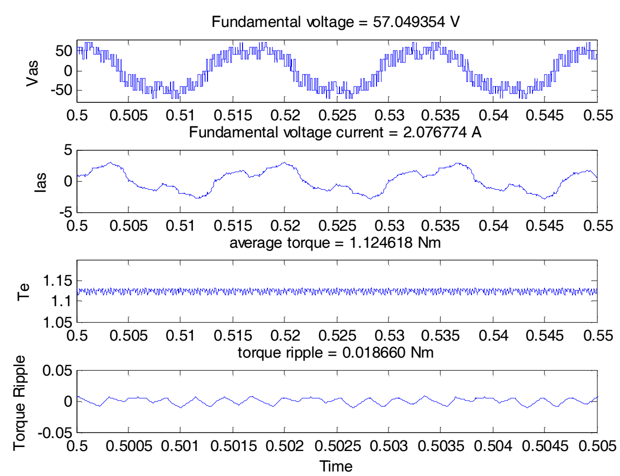

Figure 9a. Simulation results of the SVM method.

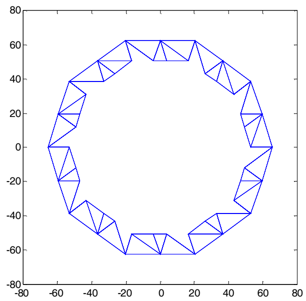

Figure 9a. Vector space of the SVM method.

The harmonic injection methods described above are not a carrier-based PWM equivalent of SVM in a five-phase machine. Simulation shows that the torque ripple in SVM is better than the additional harmonic injection modulation method. However, carrier-based PWM methods are much easier to implement in three-level five-phase system. For different modulation indices, the SVM needs to select the proper vectors and calculate the dwell time for each vector and decompose the vector to corresponding transistor signals. It is not as simple as in the three-phase two-level case where some fixed pattern is easily derived. The problem also cannot be easily solved by using pre-calculated look-up tables. Even if the time consumption of this calculation is not of a concern, the practical implementation of SVM with a microcontroller requires a complicated decision-making process to determine the right vectors and appropriate vector sequence online, which is much more difficult in five-phase three-level drives. In comparison, the additional-harmonic injection method is easy to implement in the laboratory and the full dc voltage supply can be utilized, although the torque ripple it not as good as the SVM method, it is still much better than the traditional natural sampled modulation.

VI. Laboratory Validation

The system of Figure 1 was constructed in the laboratory. The laboratory system photo is shown in Figure 10. A digital signal processor (DSP) TI-TMS320F2812 and PC interface are on the back side of the rack to provide the PWM output. A five-phase induction machine is connected to the multi-level inverter. The switching frequencies are also set to be the same as the simulation. Figure 11 shows the voltage waveforms for the four cases; case A being the sinusoidal reference waveform, case B with the 5^{th} harmonic, case C with the 3^{rd}, 5^{th}, and 7^{th} harmonics, and case D with the nearest triangle SVM. The dc supply for the four cases was 125 V, 119 V, 102 V and 102 V respectively to correspond to the simulation studies above. The measured current, measured torque, and measured vector plots are shown in Figures 12, 13, and 14 respectively. As can be seen, the wave-shapes are nearly the same as in the simulation. The torque ripple also shows a decreasing trend going from case A to B to C to D as with the simulation; with a significant drop when the 3^{rd} and 7^{th} harmonic are injected.

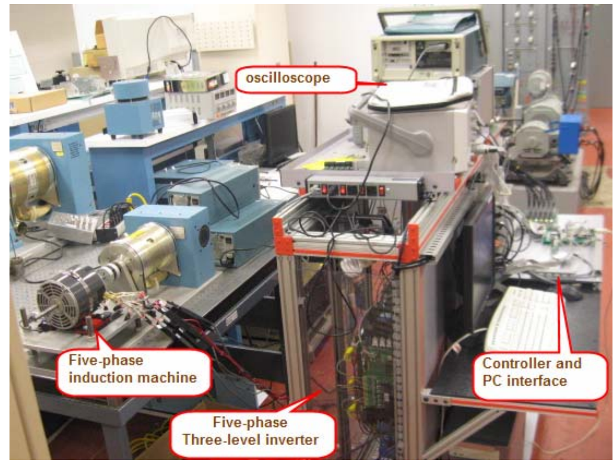

Figure 10. Five-phase drive laboratory setup.

It is shown that although different dc voltages are applied, the same fundamental voltage and current could be obtained for different cases. Also good agreement is apparent between the simulation results and laboratory results.

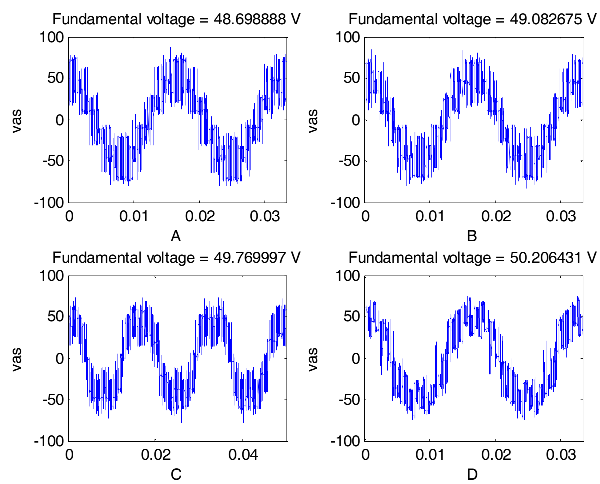

Figure 11a. Measured voltage waveforms.

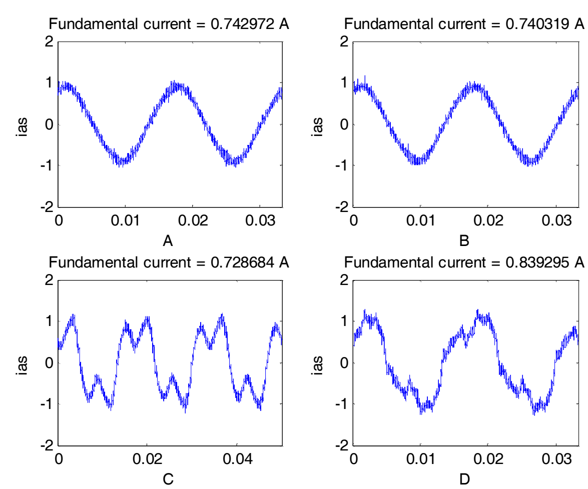

Figure 11b. Measured current waveforms.

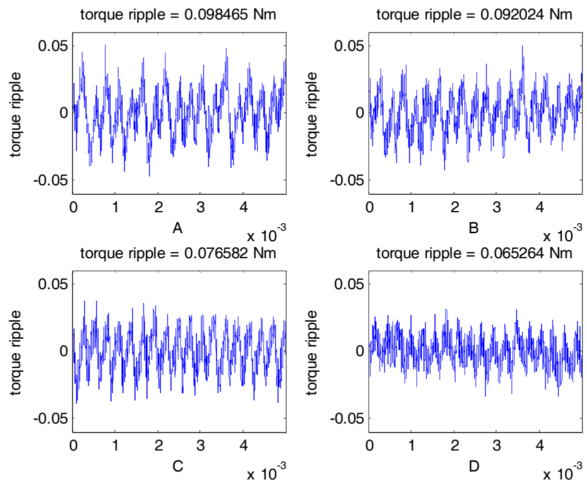

Figure 11c. Measured torque waveforms.

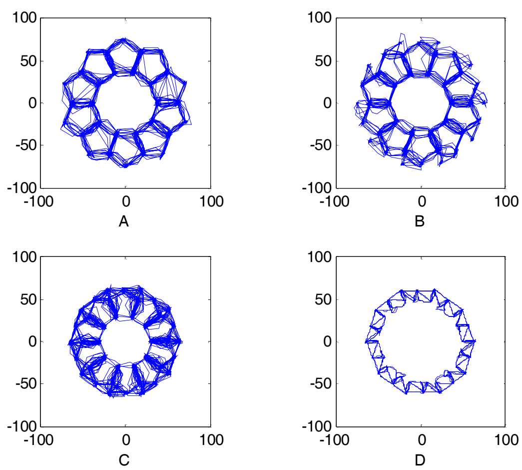

Figure 11a. Measured vector plots.

VII. CONCLUSION

Multi-phase motor drives have received much attention in recent years due to advantages of lower torque ripple and higher power density. Mathematically, the multi-phase machine differs greatly from the three-phase machine since certain harmonics do not contribute to torque production. For this reason, if torque ripple is a concern, adding harmonics to the modulation reference signal can reduce torque ripple. At the same time, significant improvements in dc voltage utilization can be realized. This paper introduced new modulation methods for the five-phase machine where dc voltage utilization was increased by about 12%, allowing the same fundamental voltage from a much lower dc voltage. Furthermore, the addition of the harmonics utilizes the outer region of the vector space and results in smaller voltage deviations and lower torque ripple. This comes at the expense of current harmonics which may not be an issue in some applications where torque ripple is more important. A unique space-vector modulation pattern, utilizing the nearest vectors, was also introduced to further improve the torque ripple. The new modulation methods were validated in simulation and with laboratory measurements.

ACKNOWLEDGEMENT

The authors wish to acknowledge Lynn Petersen for support of this project under ONR grant N00014-06-1-0845.

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