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Numerical Modelling of Whispering Gallery Modes in Parallel-Plates Type Cylindrical Anisotropic Dielectric Resonators

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Abstract — A parallel-plates-type cylindrical anisotropic dielectric resonator (DR) with whispering gallery modes (WGM's) has been investigated. Numerical modelling of the resonance frequencies, Q-factors and the electric field filling factors of the isotropic and anisotropic DR are presented. Applications of the results for sensing purposes are discussed.

I. INTRODUCTION

Dielectric resonators (DR's) with whispering gallery modes (WGM's) and devices with them have been under investigation for twenty years and they have been especially intensively studied lately [1-3]. The reason is the certain advantages of WGM DR's in microwave and millimeter wave region, such as high Q-factor, field localisation in the narrow area near the dielectric boundary.

Cylindrical DR's with WGM's are promising structures for applications. Their fabrication process can be based of standard integrated circuit technology. Moreover, they can be excited by dielectric waveguides and such waveguide-resonator structures are very appropriate for low-cost mass production. When designing such resonators, the influence of the anisotropy property on the resonant modes should be taken into consideration. Krupka *et al.* [1] rigorously analysed the WGM's in shielded anisotropic dielectric resonators. They presented some numerical results on the resonant frequencies and the Q-factors of such resonator. Tobar *et al.* [4] did the same task for an open anisotropic dielectric resonator. Kobayashi *et al.* [5] obtained the characteristic equation and mode charts for low order modes in parallel-plates type cylindrical anisotropic resonator. In a more recent study [6], Prokopenko received the analytical expressions for partial Q-factors in such resonator. At present, it is necessary to numerically investigate the WGM's in different types of parallel-plates type cylindrical resonators for optimising their performances.

In this paper, new results of the numerical modelling of parallel-plates-type cylindrical isotropic and anisotropic dielectric resonators with WGM's are presented. The dependency of resonant frequencies, Q-factors and electric field filling factors of WGM's on resonator parameters is studied numerically by using the developed computer programs. Resonators with different materials and dimensions are investigated. Finally, applications of the results for sensing purposes are discussed.

II. BACKGROUND

A. Characteristic Equation

Fig. 1 shows the resonator structure that is a parallel-plates-type resonator, where a dielectric cylinder is placed between two parallel, infinitely large conducting plates. The dielectric cylinder is assumed to have homogeneous uniaxial-anisotropic characteristic with the c-axis of the dielectric parallel to the z-axis. Defining ϵ_t and ϵ_z as the relative permittivity perpendicular and parallel to the c-axis, respectively, the relative permittivity tensor $[\epsilon_r]$ is given by

$$[\epsilon_r] = \begin{bmatrix} \epsilon_t & 0 & 0 \\ 0 & \epsilon_t & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix} \quad (1)$$

The relative permittivity of the dielectric is assumed to be $\mu_r = 1$, and the conductor is also assumed to be lossless.

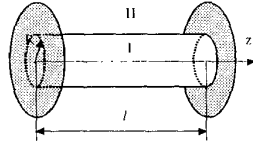


Fig. 1. Parallel-plates-type dielectric resonator

B. Q-factor and Electric Energy Filling Factor

It's well known that unloaded Q-factor of any modes of the isotropic DR is determined by dielectric losses, radiation losses and conductor losses in metal enclosure and it can be expressed as follows [1]:

$$Q^i = \gamma \tan \delta + Q_c^{-i} + Q_r^{-i} \quad (2)$$

where γ is the electric energy filling factor of DR modes (the ratio of the electric energy of the mode stored inside the resonator to the total electric energy of the mode stored in the resonant system), $\tan \delta$ is the dielectric loss tangent, $\gamma \tan \delta = Q_c^{-i}$, Q_c^{-i} is Q-factor depending on dielectric losses, Q_r^{-i} is the Q-factor depending on radiation losses and Q_c^{-i} is the Q-factor depending on conductor losses.

Partial Q-factors are defined as

$$Q_d^i = \frac{wW}{P_d} \quad Q_c^i = \frac{wW}{P_c} \quad Q_r^i = \frac{wW}{P_r} \quad (3)$$

where P_d , P_c and P_r are the dielectric, conductor and radiation power losses, respectively, w is the resonant frequency and W is the total stored energy.

For an anisotropic DR made in such manner that its symmetry axis is aligned with the anisotropy axis of the DR material, its Q-factor can be determined by [1]

$$Q^i = \gamma_1 \tan \delta_1 + \gamma_2 \tan \delta_2 + Q_c^{-i} + Q_r^{-i} \quad (4)$$

where γ_1 and γ_2 are electric energy filling factors perpendicular and parallel to DR's axis (anisotropy axis), respectively, $\tan \delta_1$ and $\tan \delta_2$ are appropriate the dielectric loss tangents.

III. RESULTS

DR's with different properties have been investigated in wide frequency band. Results are presented for $m = 0$ where the parallel-plates-type dielectric resonator supports only TM mode.

A. Isotropic DR.

Consider, first, the isotropic DR made from Teflon ($\epsilon = 2.08$, $\tan \delta = 1.78 \times 10^{-4}$). Fig. 2 shows the spectra of TM_{020} modes of the resonator. The horizontal axis gives the ratio of resonator radius, R , to wavelength, λ , the vertical axis shows the normalised slow down factor, $(U - 1)$. Information is given for two radial modes ($s = 1, 2$). For better observation, the points corresponding to the resonant frequencies of the every mode are connected by lines. It is shown that they form an almost periodic sequence of the resonances.

It can be seen from Fig. 2 that slow down factors of the lower order radial ($s = 1$) mode is greater than that of the higher order radial one ($s = 2$). This means that the fields of the latter extend further in the outer medium. Slow down factor of both modes increase with increasing frequency.

In Fig. 3 we show the dependencies of the Q-factor of the investigated modes on the parameter R/λ . The bold solid line gives a level of dielectric losses tangent, $(\tan \delta)^{-1}$. As can be seen from Fig. 3, for the investigated frequency range lower order radial mode has higher Q-factors than higher order radial one. This is a result of their different slow down factors (Fig. 2). For $R/\lambda > 5$, the dielectric losses are more essential for this lower order mode. In this region, the dependency of Q-factor to R/λ has a non-monotonous character and the maximums of Q-factor are greater than the $(\tan \delta)^{-1} = 5.6 \times 10^3$ level because of the localisation of some field outside the resonator.

For $R/\lambda < 5$, decreasing R/λ gives decreasing of Q-factor because of the leakage of the fields from the dielectric to the outside (the radiation losses).

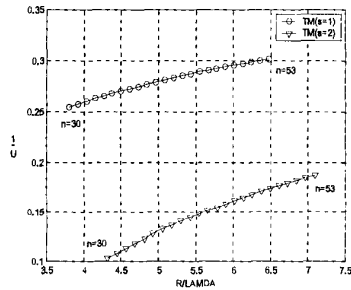


Fig. 2. Normalised slow down factor vs. R/λ .

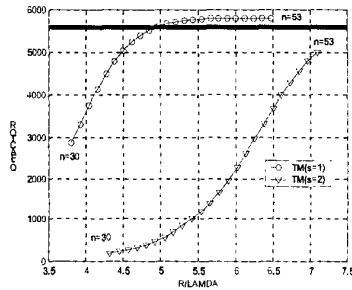


Fig. 3. Q-factor vs. R/λ .

B. Anisotropic DR

Now, consider the characteristics of the modes of the anisotropic resonator. In Fig. 4 we show the spectra of $TM_{\nu s}$ modes of the resonator ($\epsilon_z = 2.38$, $\epsilon_r = 1.78$) in the same manner as in Fig. 2.

The resonant frequencies increase almost periodically with increasing of the azimuthal number, having different periods for different modes so their identification is possible even though the frequency spectrum is dense. It can be seen from Fig. 4 that slow down factor of the TM mode with $s = 1$ is greater than that of TM with $s = 2$.

Fig. 5 shows the dependencies of the Q-factor of the investigated anisotropic DR modes on the parameter R/λ . Comparing with Fig. 3, we can note the following: In considered intervals of the parameter R/λ , the level of the Q curves are below the bold solid line meaning that both dielectric and radiation losses are present.

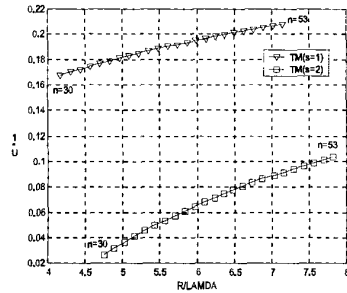


Fig. 4. Normalised slow down factor vs. R/λ .

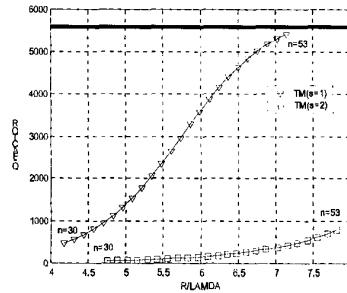


Fig. 5. Q-Factor vs. R/λ .

Fig. 6 shows the dependencies of the electric energy filling factors, γ of the investigated modes on the parameter R/λ for isotropic and anisotropic DR. We can see that the electric energy filling factor of the anisotropic DR is less than that of the isotropic DR because $\epsilon_z < \epsilon_r$. The filling factor increases with increase of R/λ for both DR's.

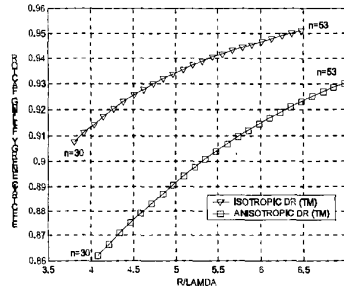


Fig. 6. Electric energy filling factor vs. R/λ .

IV. DISCUSSION ON POSSIBLE APPLICATIONS

It's possible to use the obtained results for the optimisation of the investigated resonator characteristics for specific applications. For instance, for sensing purposes, the DR has to be designed to assure a high sensitivity, that is, the resonator response has to be as high as possible for changes in the electromagnetic properties of the outer medium. This sensitivity depends on the strength and distribution of the evanescent field in the outer medium [7]. It is proportional to the ratio of the power of the WGM at the outer medium to the total power of the WGM, that is, to electric filling factor of the evanescent field ($\gamma' = 1 - \gamma$). We have shown how sensitivity can be increased by selecting the frequencies, dielectric properties and size of the resonator. To obtain a high sensitivity we must decrease the electric energy filling factor. This factor decreases by decreasing the dielectric permittivity and the radius of the resonator. It gives us more clear spectra, too. However, we are limited with the decrease of Q-factor because of radiation losses. Thus, it's necessary to use a compromise between the Q-factor and filling factors.

V. CONCLUSION

The cylindrical DR with WGM's is a promising structure for applications. Numerical results of the resonance frequencies, Q-factors and the electric energy filling factors of a parallel-plates-type cylindrical anisotropic DR with WGM's are presented. The obtained results proved the possibility of optimisation of the resonator characteristics for specific applications, namely for sensing applications.

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