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Giuseppe Selli

James L. Drewniak

Missouri University of Science and Technology, drewniak@mst.edu

David Pommerenke

Missouri University of Science and Technology, davidjp@mst.edu

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Recommended Citation

G. Selli et al., "An Extrapolation Procedure to Shorten Time Domain Simulations," *Proceedings of the IEEE International Symposium on Electromagnetic Compatibility (2004, Santa Clara, CA)*, vol. 1, pp. 113-117, Institute of Electrical and Electronics Engineers (IEEE), Aug 2004.

The definitive version is available at <https://doi.org/10.1109/ISEMC.2004.1350007>

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An Extrapolation Procedure to Shorten Time Domain Simulations

Giuseppe Selli
EMC Laboratory
University of Missouri-Rolla
Rolla MO 65410 USA
gs5xd@umr.edu

James L. Drewniak
EMC Laboratory
University of Missouri-Rolla
Rolla MO 65401 USA
drewniak@umr.edu

David J. Pommerenke
EMC Laboratory
University of Missouri-Rolla
Rolla MO 65401
davidjp@umr.edu

Abstract—Time-domain simulation algorithms are widely used in the analysis and design of electromagnetic systems. Many of them are characterized by high Q's. Thus, the simulations have to employ many time steps in order to achieve a complete characterization of these systems. This time-consuming computational effort can be avoided if the late instants of time are extrapolated by applying a parametric estimation algorithm. An optimized implementation of a time-domain extrapolation method and a stop criterion are discussed in this paper. The latter criterion is based upon a normalized squared difference between the waveforms extrapolated from two different sets of initial data and it will be used as a means to stop the time domain simulation algorithm.

Keywords—Time-domain simulations, extrapolation, eigenvalues, eigenvectors, stop-criterion.

I. INTRODUCTION

Shortening time-domain simulations enhances the efficiency in the analysis of electromagnetic systems. This purpose can be achieved by extracting frequencies and damping factors from the data recorded in the early instants of time and approximating the system response waveform as a linear combination of damped sinusoids. The problem of extracting the characteristic parameters of a truncated signal has been extensively addressed in the literature and several methods have been proposed to solve it. Different procedures can be used to extract these parameters, however, the Matrix Pencil (MP) method [1-3] is considered to be the most robust with respect to noise when compared with the alternatives [2]. This method retrieves the damping factors and the frequencies from the eigenvalues or poles associated with the solution of a pencil matrix. The choice of the number of poles M , and the pencil parameter L determines the successful application of this method. Therefore, a two-dimensional optimization procedure needs to be employed, in order to find the most suitable parameter values among the possible ones. The implementation of the matrix pencil method together with the optimizing procedure does not guarantee, if applied once, an accurate extrapolation. In fact, a convergence criterion needs to be employed as a function of the normalized squared difference between two extrapolated solutions, which are retrieved from different numbers of data samples. Once this difference drops below a certain threshold, the simulation can be terminated, since additional data samples do not significantly enhance the extrapolated solution.

II. THE MATRIX PENCIL METHOD

The Matrix Pencil method was introduced in [1], and the underlying assumption is that the response waveforms can be reconstructed as a linear combination of damped sinusoids, e.g.,

$$y[k] = \sum_{i=1}^M R_i z_i^{kT}, \quad (1)$$

where T_s indicates the sampling interval, the R_i 's represent the residues and the z_i 's are the complex or real exponentials in the Z-transform domain. Particular attention has to be given to the equality sign of (1). In fact, the sampled quantity at the left hand side is the original signal in addition to random noise, while the right-hand side is the approximation of the original signal by means of damped sinusoids. In the current application, the noise is the numerical error caused by the finite precision of the simulation algorithms.

The Matrix Pencil problem is posed in the form of a generalized eigenvalue problem, by filling two non-square Hankel matrices from N data samples given by the time simulation algorithm, i.e., [2]

$$[Y_2] - \lambda [Y_1] = \begin{pmatrix} y[0] & \dots & y[L] \\ y[1] & \dots & y[L+1] \\ \dots & \dots & \dots \\ y[N-L] & \dots & y[M] \end{pmatrix}_{(N-L) \times L} - \lambda \begin{pmatrix} y[0] & \dots & y[L-1] \\ y[1] & \dots & y[L] \\ \dots & \dots & \dots \\ y[N-L-1] & \dots & y[N-1] \end{pmatrix}_{(N-1) \times L} \quad (2)$$

where the quantity L indicates the pencil parameter. The solution to the MP method, as formulated in (2), is obtained by finding the eigenvalues associated to the square matrix $[Y_2] [Y_1]^{-1}$. Due to the ill-conditioning of this matrix, a preliminary Singular Value Decomposition (SVD) needs to be performed on the matrices $[Y_1]$ and $[Y_2]$. In fact, only M eigenvalues are relevant for the description of the system under consideration [2], as indicated in (1). The noise makes the pencil matrix to have rank L , however, the additional $L-M$ are redundant and create severe instability problems, if the direct solution of the matrix $[Y_2] [Y_1]^{-1}$ is attempted. The SVD enforces the rank M ,

but the initial Hankel structure is lost. Now, the solution of the new SVD'ed matrix pencil can be found and the M eigenvalues with the corresponding eigenvectors calculated [2]. The z_i 's of (1) correspond to the eigenvalues of the square matrix [4]

$$[S]_{(M) \times (M)} = [V_{T1}]_{(M) \times (L)}^+ [V_{T2}]_{(L) \times (M)} \quad (3)$$

where $[V_{T1}]$ and $[V_{T2}]$ are the eigenvectors associated with SVD truncated matrices $[Y_1]$ and $[Y_2]$, defined in (2). On the other hand, the residues R_i corresponding to each z_i can be easily calculated with a least square approximation [2], and all these parameters can be used to construct the closed form expression of (1), and the original waveform can be finally extrapolated at each instant of time.

III. THE OPTIMIZATION OF THE MP METHOD AND THE STOP CRITERION

It is shown [2] that the SVD on the following matrix $[Y]$

$$[Y] = \begin{pmatrix} \mathcal{Y}[0] & \mathcal{Y}[1] & \dots & \mathcal{Y}[L] \\ \mathcal{Y}[1] & \mathcal{Y}[2] & \dots & \mathcal{Y}[L+1] \\ \dots & \dots & \dots & \dots \\ \mathcal{Y}[N-L-1] & \mathcal{Y}[N-L] & \dots & \mathcal{Y}[N] \end{pmatrix}_{(N-L) \times (L+1)} \quad (4)$$

is equivalent to the SVD on the matrices $[Y_1]$ and $[Y_2]$. Although the matrix $[Y]$ has $L+1$ eigenvalues, only the largest M are related to the signal to be extrapolated, while the reminders are related to the noise added. Hence, the ratio of each eigenvalue to the largest one can be taken, and a criterion based on these ratio values can be employed to discard the smallest eigenvalues [2]. It is also known that the parameter L can span approximately $N/6$ values in the range between $N/3$ and $N/2$ [3], and approximately $N/6$ different curves are obtained, if the normalized eigenvalues for each $(N-1) \times (L+1)$ $[Y]$ matrix are plotted as a function of L , as in Fig. 1.

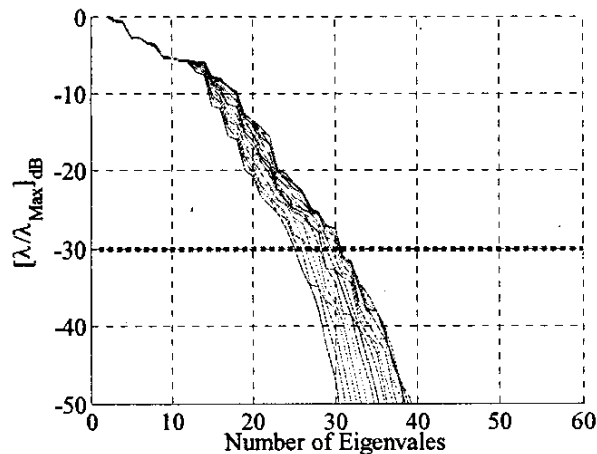


Fig. 1. Normalized eigenvalues of the matrix Y defined in (4) for a small value of the total number of samples employed N .

The various intersections of all the curves with the threshold line, e.g., -30 dB, help to find an initial range where

the M of (1) may lie. It is interesting to observe that as the number of data samples N is increased, all these curves tend to converge to one, until they overlap, as shown in Fig. 2.

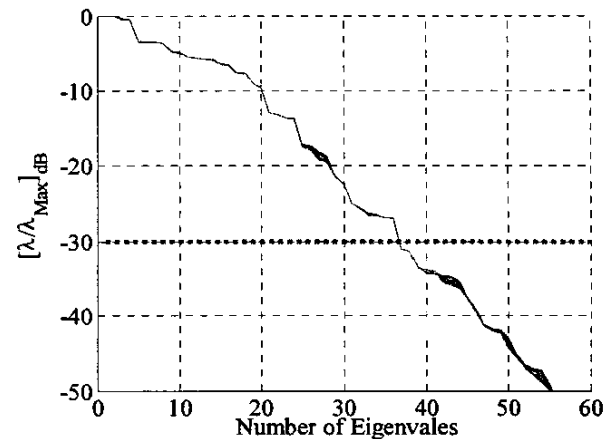


Fig. 2. Normalized eigenvalues of the matrix Y defined in (4) for a large value of the total number of samples employed N .

The N samples employed for plotting the curves in Fig. 2 are much more (4 times) than those used in Fig. 1. Hence, the computational effort needed to obtain a unique value for M is not worth the investment. In fact, valuable information is embedded in the range indicated in Fig. 1, by the lowest and the largest intersection with the threshold line.

The eigenvalues associated with the squared matrix $[S]$ are calculated as a function of the parameter L , still varying in the range between $N/3$ and $N/2$, and the number M , now spanning the range indicated by the outer intersections in Fig. 1. Approximately $N/6$ sets of eigenvalues can be calculated for each M in the range specified. Each set corresponds to a solution, and it can be labeled unstable, if at least one of the eigenvalues is greater than one [5]; the ratio of the number of unstable solutions over the total number can set the criterion to uniquely identify M among all the possible values.

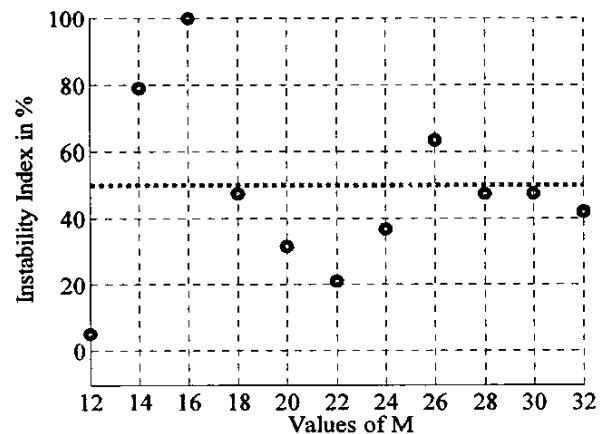


Fig. 3. Instability index as a function of M .

The most suitable value for M can be chosen by looking at Fig. 3. Different criteria can be employed for this choice, however, experience has shown that the largest M which has less than 50 % instability is a good value. Usually, more data needs to be recorded, if all the solutions found for each M are unstable. On the other hand, it is possible to extrapolate the waveform, if all the solutions are not unstable at least for some M , although the reconstructed result may be very distorted.

Once the number M is uniquely defined, several solutions are still available as a function of L . The normalized squared difference between each possible solution and the original data can be used to find the best among the available solutions, i.e.,

$$NSD = \left\{ \frac{\sum_{k=1}^N \left[y[k] - \sum_{i=1}^M R_i z_i^k \right]^2}{\left[\sum_{k=1}^N y[k] \right]^2} \right\} \quad (5)$$

the set, which minimizes the normalized squared difference as defined in (5), is chosen as the final solution [6].

The MP method with the optimizing procedure does not guarantee, if applied one time, an accurate extrapolation according to (1). In fact, a comparison needs to be performed between the final solutions obtained with different sets of data and it needs to be established whether the enhancement in the accuracy of the solutions is worth the additional computational effort or not. A frequency-domain normalized squared difference between two final solutions obtained with different sets of data can be employed as a stop criterion [5], i.e.,

$$\Delta_{dB} = \left[\frac{\sum \left[|y_{\tau+1}(\omega)|^2 - |y_{\tau}(\omega)|^2 \right]}{\sum \left[|y_{\tau}(\omega)|^2 \right]} \right]_{dB} \quad (6)$$

The time domain algorithm can be stopped as soon as this difference drops below a certain threshold. Fig. 4 shows the implementation of this criterion as function of the data samples employed.

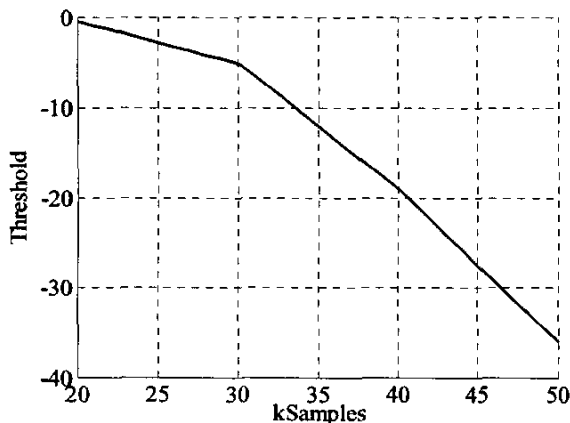


Fig. 4. The stop criterion defined in (6) as a function of the number of samples employed.

The practical application of this optimized method does not necessarily require the employment of all the samples given by the simulation, and usually some sort of decimation scheme can be applied. In fact, the inherent requirements of the simulation algorithms usually force the sampling interval Δt to be much smaller than the limit provided by the Nyquist theorem. In the FDTD algorithms, for example, the time interval is related to the cell size through the Courant stability condition; therefore, the presence of small features in the geometry under consideration always determines the time interval. The implementation of a decimation scheme allows the representation of the same temporal interval with less samples, as long as the new Δt meets the Nyquist theorem.

IV. APPLICATION OF THE OPTIMIZED MP METHOD

The employment of the MP method is very suitable for parameter extraction and time domain extrapolation of those systems characterized by long time responses, e.g., resonant metal enclosures with one or multiple slots, EM fields induced between the different ground and power planes in printed circuit boards due to currents injected along the vias, and EM fields induced by narrowband antennas, e.g., patch antennas.

Fig. 5 shows a model under test, which is a 22 cm x 14 cm x 22 cm enclosure, with two longitudinal slots along the z direction, the geometry is fed through a 50 Ω semirigid coaxial cable terminated on the interior of the cavity with a resistor. The geometry was thoroughly investigated in [7]. The electric field measured at a location 3 meters away from the enclosure has a long time response due to the energy bouncing back and forth inside the cavity, and slowly radiated from the two slots.

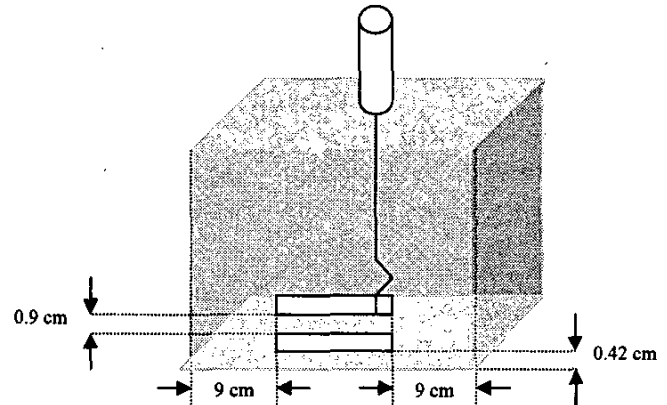


Fig. 5. Enclosure with two parallel slots.

The time domain extrapolation is applied to several sets of data given by the simulation algorithm. The number of samples indicated correspond to the data effectively employed in the MP method, in fact, a decimation factor of 6 is used. The application of the stop criterion shows that the time-domain simulation can be terminated after 840 samples ($6 \cdot 140$), because the gain in accuracy is very small. The comparison between the extrapolated waveform and the simulated one given in Fig. 7 shows that the parameters

extracted are sufficient to provide a good representation of the simulated waveform.

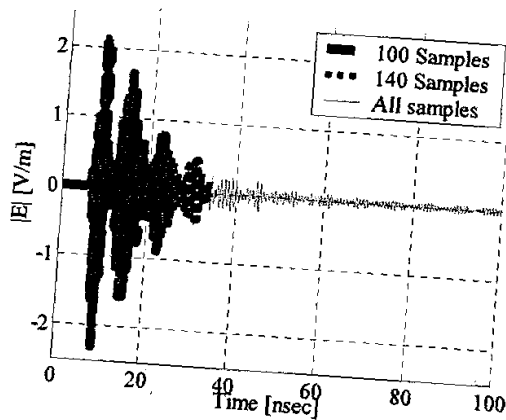


Fig. 6. Truncated waveforms vs. simulated waveform.

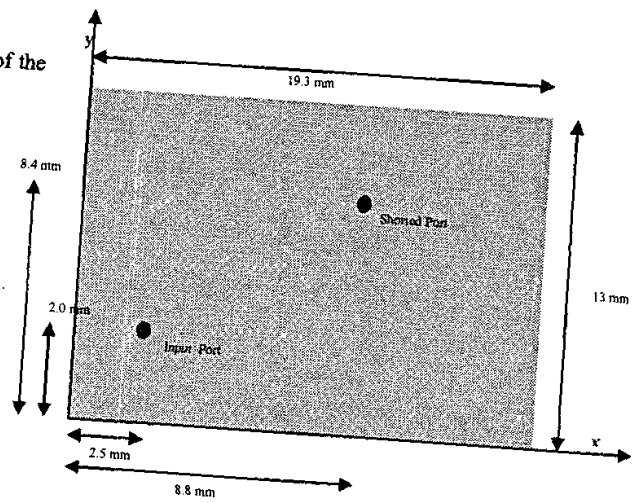


Fig. 9. Top view of the PCB under analysis

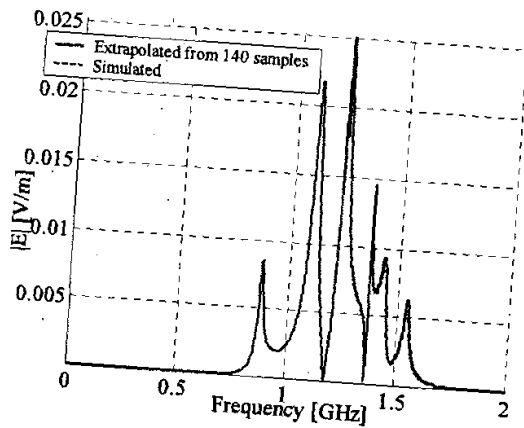


Fig. 7. Comparison of the extracted waveform with the simulated one.

Another geometry considered is shown in Fig. 8 and Fig. 9. It is a 2-layer board with a shorted via, and fed by a voltage source at the input port. This system has a long time response due to the bouncing of the electromagnetic energy between the two plates. Fig. 10 shows several sets of data, which the extrapolation algorithm is applied to. The decimation factor employed is very large, because the time step of the simulation algorithm is forced to be very small due to the features of the geometry to be simulated.

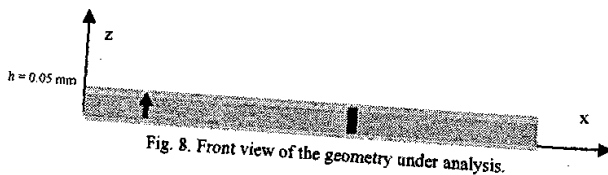


Fig. 8. Front view of the geometry under analysis.

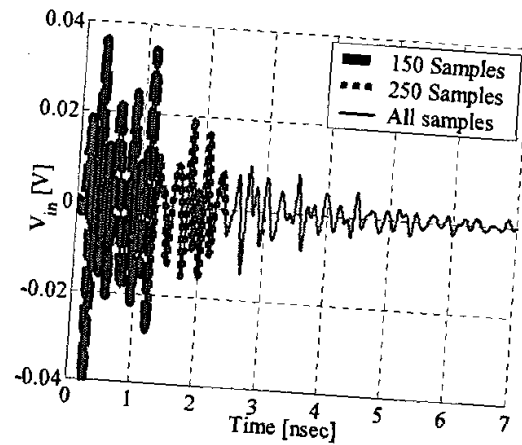


Fig. 10. Truncated waveforms vs. simulated waveform.

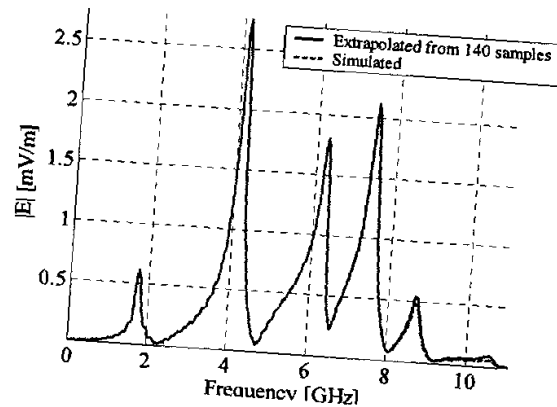


Fig. 11. Comparison of the extracted waveform with the simulated one.

The implementation of the stop criterion shows that the parameters extracted from the 250 decimated samples are sufficient to reconstruct the simulated waveform, as shown in Fig. 11.

V. CONCLUSION

A method for terminating time domain simulations has been presented in this paper. The method is based upon an optimization of the well known Matrix Pencil method and the implementation of a stop criterion based upon the normalized squared difference between two waveforms extrapolated from different sets of data. Several criteria based upon experience have been proposed, however these criteria need to be thoroughly investigated, in order to establish general rules to be applied when dealing with different classes of problems.

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