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Askin Demirkol

Levent Acar
Missouri University of Science and Technology, acar@mst.edu

Robert S. Woodley
Missouri University of Science and Technology, rwoodley@mst.edu

Erol Emre

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An Alternative Target Density Function For Radar Imaging

A.Demirkol and L.Acar
University of Missouri-Rolla
Dep.of El. and Comp.Eng.
Rolla, Missouri

R.S.Woodley
21st Century Systems, Inc.
Fort Leonard Wood, Missouri

E.Emre
Sakarya University
Dept.of.Comp.Eng.
Sakarya,Turkey

Abstract

In this paper, an alternative Target Density Function(TDF) is proposed to image the radar targets in a dense target environment. It is produced by wavelet theory considering a new range and angle plane different from the conventional methods. It is shown that Wavelet theory can be used as approach to imaging by active sensors by transmitting a waveform which is a kernel for this transform such as a window function. Although the imaging is obtained via the phased array radars, the problem associated with beamforming in linear phased array radar system is bypassed in this new algorithm.

1. Introduction

Imaging is a mapping process from three dimensional object to two dimensional image [1–5]. This transformation is obtained by using signal transforms such as Fourier and Wavelet transforms [1–7]. Radar imaging is based on a multi-sensor image fusion technique, which is in the form of multiple-apertures and arrays [8–13]. This imaging is a reconstruction process which extracts the radar echo signals off the targets.

Target density function(TDF) is the reflectivity of spatially, continuously distributed targets and it is an important characteristic of radar imaging. TDF is known by different names such as ambiguity function, density function, target density function, object(target), object reflectivity function, doubly-spread reflectivity function, and reflection coefficient [6, 8].

If TDF is assumed as a reflection coefficient, then it is defined as the ratio of the received signal to the transmitted. By this definition, the reflected signals from the object space are relevant to the intensities of the points on the target or objects. Thus the integration of the illuminated intensities reveal information related to the object shape.

There are two well known approaches on TDF. First one considers point scatterers reflected off the target scatterer centers. Integration of all point scatterers is able to obtain the whole object. This radar imaging technique is based on inverse Fourier transform(IFT) and used mostly in inverse synthetic aperture radar(SAR) studies [1–5, 16].

Second method on TDF is a dense target environment approach by Fowle and Naparst [14, 15]. This takes into consideration the existence of densities of the targets in a high dense target environment. It is based on the ambiguity functions with two variables as range and velocity [17, 21, 22]. Especially, the advanced function in the dense target environment by Naparst is developed in a novel way. Rather than typical radar imaging, this is an approach to measure the closeness of the targets to each other in the dense target environment. However, this provides an important contribution to the analysis of target density functions related to the radar imaging.

In this study, a new TDF is theoretically developed by a new approach on a range-scanning angle plane different from the early approaches. In section 4, a slight modification of the wavelet transform suited to active sensor is described. Then, it is shown how the image of a target area can be obtained by transmitting a wavelet waveform by an active phased array. In particular, the reflectivity of spatially, continuously distributed targets (target density function) is estimated utilizing the properties of a wavelet and wavelet transform. It is shown that the backscattered signal received by the array is the wavelet transform of the target density function. Thus, the inverse wavelet transform of the received signals at the sensor output yields the target density function in forms of an image.

While this is obtained via a phased array radar system, the problem associated with beamforming is bypassed.

2. Preliminaries Of Density Functions

In this section, the background of the target density functions are studied by the following techniques;

- SAR-ISAR reflectivity functions
- Naparst's target density functions

2.1. SAR - ISAR Reflectivity Functions

Synthetic aperture radar(SAR) and inverse synthetic aperture radar (ISAR) are well known radar imaging techniques used for earth surface imaging [18, 19]. However, they have different configuration. In SAR imaging, the radar is flying in space and the object is stationary, while in ISAR imaging, the object is moving and the radar is stationary [1, 4, 5, 20].

ISAR is considered as an inverse Fourier transform(IFT) of a 3-D object on a 2-D. If the target is composed of continuum point targets(scatterers), after demodulation and some pre-filtering processes, by the superposition principle, the echo(reflected signal) $r(t)$, from such a target at x, y points is:

$$r(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y) e^{-j2\pi f_c \frac{2R_p(t)}{c}} dx dy \qquad (1)$$

for $2R_p(t)/c \le t \le T_{PRI} + 2R_p(t)/c$. Here, $g(x,y)$ is the target reflectivity function, T_{PRI} is pulse interval repetition, c is the speed of light, f_c is carrier frequency, and $R_p(t)$ is the range from the radar to the point-scatterer.

If Inverse Fourier Transform is applied to Equation (1), the image $g(x,y)$ is obtained as a 2-D form of 3-D object [1, 4, 5].

$$g(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} r(f_x, f_y) e^{j2\pi(xf_x - yf_y)\frac{2R_p(t)}{c}} df_x df_y \quad (2)$$

where

$$f_x = \frac{2f_0}{c} cos\theta(t), \ f_y = \frac{2f_0}{c} sin\theta(t)$$

2.2. Target Density Functions

First *Density* term related to the target density function is called by Fowle et all [14]. Fowle is focused on the problem of the detection and resolution in two dimensions of a large number of targets in a fixed part of the target space and, he is inspired of ambiguity functions. Then, *Dense target environment* term is used by Naparst's paper [15] by taking advantage of Fowle work. His new approach is based on ambiguity and cross-ambiguity functions. In this work, the dense-target environment is defined the closeness of a lot of targets at a distance, which their velocities are so close to each other.

Definition by Naparst, density of targets at distance u and velocity v is $g(u,v)$. In this case, the echo or the reflected signal from targets is

$$r(t) = \int_0^{\infty} \int_{-\infty}^{\infty} g(u,v) \sqrt{v} s(v(t-u)) du dv \qquad (3)$$

In this approach, it is assumed that all targets are illuminated equally. As stated, the target density function is a function of the range and velocity variables similar to the ambiguity functions.

Reconstruction of the target density function in Naparst algorithm is finalized as follows(the details in Ref [15]);

$$g(u,v) = \sum_{n,m=0}^{\infty} <r_n, s_m> A_{nm}(u,v) \qquad (4)$$

where s_m are signals sent out and r_n are their echoes. The cross-ambiguity function of the signals sent out $(s_1, s_2, ...)$ is $A_{nm}(u,v)$ as $\int_{-\infty}^{\infty} s_n(v(t-u)) s_m^*(t) dt$.

3. Active Sensor Imaging by An Alternative Target Density Function

In this paper, an alternative target density function (TDF) for active sensor imaging is studied by wavelet theory. The new target density function is generated by a linear phased array radar system and considering a novel target-radar plane as a range-scanning angle. Thus, the target density function $g(R, \beta)$ composed of the points in a direction angle β and at a range R, is considered as below.

Definition: Target Density Function is the limit of the ratio of the amplitude of the signal reflected from an infinitesimally neighborhood about the point (R, β) to the amplitude of the incoming signal.

By this definition, the new target density function $g(R, \beta)$ is;

$$g(R, \beta) = \lim_{d(\Omega) \to 0} \frac{A_r}{A_i} \qquad (5)$$

where $d(\Omega)$ is the diameter of the disc about the point $(R, \beta) \in \Omega$, A_r and A_i are the amplitudes of the reflected and the incoming signals, respectively.

In this definition, the target density function(TDF) is relevant to the the reflectivity of spatially, continuously distributed targets. This approach is different from the conventional target density function definitions stated early. Instead of ambiguity functions based on range-velocity variables, the imaging is taken by a new target density function with the range and scanning angle. The definition above and Figure 1 are considered in the generation of the following technique

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3.1. Imaging by Wavelet Theory

In this section, it is shown how wavelet theory can be used in imaging by active sensors by transmitting a waveform which is a kernel for this transform such as a wavelet window function. The new target density function aided with wavelet transform is composed of the following steps.

Let us consider the target plane shown in Figure 1, where β is $cos\theta$ and R is the range from the target to the radar, and the sensor elements in the linear phased array radar system are located equally. As seen in Figure 1,
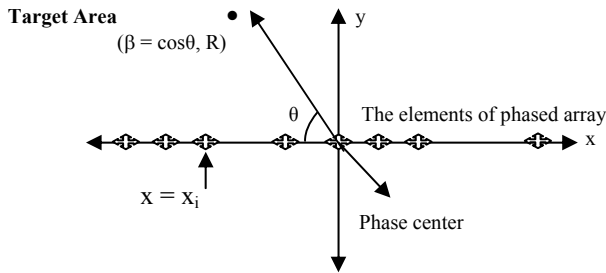

Figure 1. Phased array imaging.

the target density function is a function of the spatial coordinates (R, β) in the upper semi-plane.

Now, let us obtain the target density function. Let $P(t)$ be any periodic function of time, such as a train of pulses, where

$$p(t) = \sum_{k=-\infty}^{\infty} \alpha_k \, e^{jk\omega_0 t} \qquad (6)$$

$$\omega_0 = 2\pi \times \text{PRF}, \qquad (7)$$

where PRF is the pulse repetition frequency.

$$W(t) = e^{j\omega_c t} \qquad (8)$$

Where $W(t)$ is the carrier signal.

$$m(t) = p(t)W(t) \qquad (9)$$

Where $m(t)$ is the modulated signal.

The reflectivity of one point at $g(R, \beta)$

$$r(t,x) = m(t - 2R/c - \beta x/c)g(R,\beta) \qquad (10)$$

Let us generalize Equation 10 for the whole radar-target semi upper plane by superpositioning principle considering all point scatterers related to the range-angle.

If $g(R,\beta)$ is the reflectivity of the point (R,β), and R_1 is the maximum range of interest target area; then

$$
\begin{aligned}
r(t,x) &= \int_{-1}^{1}\int_{0}^{R_1} m(t - 2R/c - \beta x/c)g(R,\beta)dRd\beta \\
&= \int_{-1}^{1}\int_{0}^{R_1} p(t - 2R/c - \beta x/c) \\
&\quad \times W(t - (2R/c + \beta x/c))g(R,\beta)dRd\beta \quad (11)
\end{aligned}
$$

where $r(t,x)$ is the output of the sensor located at center (the feature space), and c is the speed of light.

The algorithm is as follows,

$$
\begin{aligned}
r(t,x) &= \sum_{k=-\infty}^{\infty} \alpha_k e^{jk\omega_0 t} \int_{-1}^{1}\int_{0}^{R_1} e^{-jk\omega_0 \frac{2R+\beta x}{c}} \\
&\quad \times W(t - (2R/c + \beta x/c))g(R,\beta)dRd\beta \quad (12)
\end{aligned}
$$

Then, demodulation of the equation (12) via $e^{-jk\omega_0 t}$ yields,

$$
\begin{aligned}
e_k(t,x) &= e^{-jk\omega_0 \frac{2R+\beta x}{c}} \int_{-1}^{1}\int_{0}^{R_1} W(t - (2R/c + \beta x/c)) \\
&\quad \times g(R,\beta)dRd\beta \quad (13)
\end{aligned}
$$

Let us take a look at Wavelet Theory as Wavelet transform and Inverse Wavelet transform pairs

$$\Psi(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(t)\psi^*(\frac{t-b}{a})dt \qquad (14)$$

and

$$f(t) = \frac{1}{C_\psi}\frac{1}{a^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{\sqrt{a}}\Psi a,b(t)\psi(\frac{t-b}{a})dadb \qquad (15)$$

where

$$C_\psi = \int_{0}^{\infty} \frac{\psi^*(x)\Psi x}{x}dx \qquad (16)$$

Equation 13 seems equivalence to Wavelet transform in Equation 14. Where $*$ denotes the complex conjugate. Let produce a wavelet function and assume, $W(t)$ be a function in Equation 8. Let $V(\omega)$ be its Fourier transform. Then, $W(t)$ is a wavelet if and only if

$$\gamma \equiv \int_{-\infty}^{\infty} \frac{V(\omega)V^*(\omega)}{|\omega|}d\omega < \infty \qquad (17)$$

Considering a linear phased array of point sensors in Figure 1, let define $g(\beta)$ to be the reflectivity of the point at a fixed range, R, from the phase center of a linear array in the direction.

β is the direction cosine of the line joining the point and the phase center. Thus $g(\beta)$ represents the values of the reflectivity as a function of the direction at the fixed range, R. Hence it represents the image.

Let us formulate this definition. The direction density function $g(R,\beta)$ at a fixed range $R = R_0$,

$$\tilde{g}(R,\beta)_{R=R_0} = g(\beta) \tag{18}$$

By this definition, Equation 13 is rewritten as follows.

$$e_k(t,x) \equiv e^{jk\omega_0 \frac{\beta x}{c}} \int_{-1}^{1} W(t - \beta x/c) g(\beta) d\beta \tag{19}$$

Equation 19 is a slightly modified form of Wavelet transform. The additional delay term of $2R/c$ due to the round trip is omitted simplicity. Thus, the knowledge of the sensor output for all x and t readily yields the wavelet transform of $g(\beta)$, and the desired image for range R. If this is done for each range, one obtains the complete image by taking the inverse wavelet transform of the signals from the sensor outputs.

Finally, analogous to the obtaining of a function in form of wavelet transform, $g(\beta)$ is produced by taking inverse Wavelet transform of Equation 19 as follows;

$$g(\beta) = \frac{1}{\gamma} e^{-jk\omega_0 \frac{\beta x}{c}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e_k(t,x) W^*(t - \beta x/c) dx dt \tag{20}$$

Equation 20 is a slightly modified form of inverse Wavelet transform.

In conclusion, if the transmitted waveform $W(t)$, is a wavelet, then the nature takes the Wavelet Transform of g; the sensor outputs are evaluations of the Wavelet Transform of g. This leads to a number of imaging techniques via Wavelet Theory and can be studied in multiresolution analysis.

As realized that although a phased array radar system is used during the producing of $g(\beta)_R$, the problem associated with beamforming is bypassed.

Infinity of k can be optimized by some filtration, compressing or estimation methods.

3.2. Comparison

The present TDF is generated partly by analogy to Fowle-Naparst and SAR-ISAR approaches.

- *Comparing to Fowle-Naparst:* As an advanced work of Fowle, Naparst target density function is developed for a high dense target environment with multiple targets, whose velocities are close to each other. This TDF acts like a separator rather than an imaging function for the targets at the distance with a given velocity. However, the contribution of especially Naparst, to the new target density function studies are quite remarkable.

TDF proposed here is obtained by a scanning angle and range in a high dense target environment. The main difference is in the imaging approach, which is capable of sensor imaging the targets in a dense target environment via phased array radar system.

- *Comparing to ISAR:* Main difference between the target density function in this study and ISAR arises from the utilized techniques. Instead of Fourier technique used for SAR, in this study is taken advantage of wavelet transform which is quite plausible for acquisition of image parts. Beside it, while ISAR imaging is based on multi-aperture principle, the present imaging method is a multi-sensor image fusion technique based on the phased array radar system. TDF in this study is similar to the reflectivity function in conventional ISAR imaging. However, ISAR reflectivity function is obtained by the integration of the point scatterers on the target, while our target density function is produced by the integration of ranges and scanning angles.

4. Summary and Conclusion

In this paper, an active sensor imaging is studied. An alternative target density function(TDF) is obtained by a new algorithm based on wavelet theory. The proposed target density function with the range and angle information is different from conventional approaches. Main contributions of this study are as follows;

- *A novel target density function plane:* New imaging target density functions were presented in a novel range and scanning angle plane.

- *Direction-Range target density Function:* A new target density function was defined to imaging by active sensors in a variable direction angle and at a range.

- *A proposed target density function algorithm:* Target density function (TDF) is represented by wavelet theory that is capable of producing the radar images by desired scanning angle and at the fixed range plane.

- *Imaging by Wavelet Theory:* It is shown that Wavelet theory can be used as approach to imaging by active sensors by transmitting a waveform which is a kernel for this transform such as a window function. if the transmitted waveform, is a wavelet, then the nature takes the Wavelet Transform of the target density function; the sensor outputs are evaluations of the Wavelet Transform of

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TDF. This leads to a number of imaging techniques via Wavelet Theory and can be studied in multiresolution analysis.

- *Bypassing the beam-forming problem:* Second contribution of this study is provided by the phased array radar system. Although the new TDF is produced via the phased array radar, the problem associated with beamforming is bypassed.

Future work will concentrate on the use of the extrapolation of this theoretical study in order to obtain improvements in implementation of the developed target density function.

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