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An Online Approximator-based Fault Detection Framework for Nonlinear Discrete-time Systems

Balaje T. Thumati and S. Jagannathan

Abstract— in this paper, a fault detection scheme is developed for nonlinear discrete time systems. The changes in the system dynamics due to incipient failures are modeled as a nonlinear function of state and input variables while the time profile of the failures is assumed to be exponentially developing. The fault is detected by monitoring the system and is approximated by using online approximators. A stable adaptation law in discrete-time is developed in order to characterize the faults. The robustness of the diagnosis scheme is shown by extensive mathematical analysis and simulation results.

I. Introduction

The process of fault diagnosis consists of three steps: (a) detection deals with determining if a malfunction has occurred in the system; (b) diagnosis considers the problem of root cause and location of the fault; and (c) accommodation attempts to correct a particular failure, through reconfiguration of the control decision.

Some of the earlier techniques [1-3] dealt with the linear modeling of the nonlinear industrial systems and by assuming the presence of simple additive faults. Also the unmodeled dynamics and disturbances in the system were not taken into account and these terms can cause deviations of the process variables creating degraded performance and false alarms. Consequently, robust failure detection algorithms, which could overcome the unavoidable errors due to modeling [2], were attempted. A robust diagnosis algorithm is expected to avoid false and missed alarms.

With the development of advance nonlinear modeling techniques [4], it is now possible to model nonlinear faults, which occur in the dynamic system. This helps to understand the type of fault and develop a maintenance schedule. However, most of the available schemes [3] for fault detection have been for continuous-time systems. There has been limited previous work on fault diagnosis of discrete time system [5], but has mainly been on simple faults rather than complex faults. Due to the difficulty of mathematical rigor involved in showing the robustness of the diagnostic schemes, not many [5] have been developed for discrete time systems. It is not possible to directly extend the fault detection schemes in continuous-time to discrete-time similar to control [9]. The design and analysis of robust failure diagnosis based on nonlinear modeling techniques in discrete-time require investigation since no known results are reported in the literature.

In this paper, a novel fault detection scheme is developed for a class of nonlinear discrete time systems using mild assumptions such as full state availability and a priori bounds on certain uncertainties. These assumptions are commonly found in the fault detection and diagnosis literature [6-7]. The faults considered are nonlinear and incipient in nature rather than simple additive or abrupt faults. Nonlinear estimator is designed using the online approximation approach in discrete-time (OLAD) [4] with an adaptive scheme for the adjustable parameters in order to capture the fault characteristics.

Finally, it is important to note that schemes developed in continuous-time cannot be directly converted to discrete-time systems [8].

The paper is organized as follows: Section II outlines the type of dynamic system under study and describes the nonlinear estimator along with the failure model. In Section III, the synthesis of the fault diagnosis scheme is introduced. The robustness of the diagnosis scheme is shown extensively with mathematical proofs using Lyapunov theory in Section IV. In Section V, the fault detection scheme is simulated on a simple mass damper system.

II. Problem Formulation

The objective of a diagnostic scheme is to detect any incipient faults, and to approximate the nonlinear behavior of faults using online approximation models like neural networks. To capture some of the characteristics of practical failure situations, in this section we present a nonlinear modeling framework in discrete-time for representing failures and developing estimation schemes. The faults are detected by monitoring deviations in the system dynamics.

The discrete time system under consideration is described by

$$x(k+1) = \zeta(x(k), u(k)) + \Pi(k-k_0) f(x(k), u(k)) \qquad (1)$$

where $x \in \Re^n$ is the state vector, $u \in \Re^m$ is the input vector, $\zeta, f : \Re^n \times \Re^m \to \Re^n$ are smooth vector fields, $k_0 \geq 0$ is the starting time of the failure, $\zeta(x(k), u(k))$ represents the nonlinear dynamics, $f(x(k), u(k))$ represents the incipient failure and $\Pi(k-k_0)$, a $n \times n$ square matrix function

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representing the time profiles of failures.

The time profiles of the incipient faults are modeled by $\Pi(k-k_0) = diag(\Omega_1(k-k_0), \Omega_2(k-k_0), ..., \Omega_n(k-k_0))$ where

$$\Omega_i(\tau) = \begin{cases} 0 \text{ if } \tau < 0 \\ 1 - e^{-\kappa_i \tau}, \text{ if } \tau \geq 0 \end{cases} \quad i=1, 2\ldots n \tag{2}$$

and $\kappa_i > 0$ is an unknown constant that represents the rate at which the failure in the state x_i occurs. For large values of κ_i, the time profile function $\Omega_i(\tau)$ approaches a step function to model an abrupt failure.

Remark 1: The failure representation described by (1) provides a general framework for characterizing a wide class of faults since the magnitude of faults in practical applications depends upon the system state and input [6]. The nonlinear failure representation in (1) captures the interdependencies of f on the state x and the input u.

Remark 2: Since the failure representation given by (1) is a function of input u, the fault detection scheme works even for the case when the feedback control compensates the effect of small incipient faults on the system output which is similar to the case of continuous-time [6].

Remark 3: Nonlinear fault diagnosis techniques are required in order to approximate unknown nonlinear functions during modeling of large class of failures.

Assumption 1: The fault detection scheme is based on the assumption that the state and the input vectors are bounded before and after the fault, which is a standard assumption commonly found in the literature [6]. In other words, there exist two compact sets $\chi \subset \Re^n, U \subset \Re^m$, such that $x(t) \in \chi$ and $u(t) \in U$ for all $k \geq 0$.

Assumption 2: States are assumed to be measurable.

The diagnostic algorithm developed in this paper deals only with detection and not fault accommodation. Past work on fault accommodation could be found elsewhere [7]. Under normal operation of the system i.e. without any faults present the healthy system described by (1) can be written as

$$x^{nl}(k+1) = \zeta(x^{nl}(k), u(k))$$
$$= \zeta(x^{nl}(k), u(k)) := \zeta_0(x^{nl}(k), u(k)) + \tilde{\zeta}(x^{nl}(k), u(k))$$

where the superscript nl means that the states are under "normal" operation, $\zeta_0(x^{nl}(k), u(k))$ represents the known nominal dynamics and $\tilde{\zeta}(x^{nl}(k), u(k))$ represents the modeling errors, which may arise due to the discrepancy between the nominal model and the actual nonlinear system.

The general approach of robust fault detection is to use a small threshold in the residual error to account for modeling uncertainties, and if the system dynamics change above the predefined threshold, then a failure is declared. On the other hand, another approach attempts to decouple the effects of faults and modeling errors as a way of improving robustness. In this paper, we consider the two cases where the modeling errors are assumed to be zero i.e. $\tilde{\zeta}(x^{nl}(k), u(k)) \equiv 0$ for the first scenario whereas it is assumed to be bounded above for the second case such that (Frobenius norm [11]) $\left\|\tilde{\zeta}(x^{nl}(k), u(k))\right\| \leq \bar{\zeta}_0,\ \forall (x,u) \in (\chi \times U)$, where $\bar{\zeta}_0 \geq 0$ is a known constant. Some of the diagnostic schemes for continuous time systems are already reported in the literature [8] whereas this paper deals with such schemes in discrete-time.

In many applications, there are often more state variables than sensors. Therefore the availability of full state feedback vector $x(k)$ as highlighted in Assumption 2 is a critical and limiting assumption. Next, we present the fault diagnosis scheme in discrete-time.

III. Fault Detection Scheme

Consider the following nonlinear estimator

$$\hat{x}(k+1) = A\hat{x}(k) + \zeta_0(x(k), u(k)) + \hat{f}(x(k), u(k); \hat{\theta}(k)) - Ax(k) \tag{3}$$

where $\hat{x} \in \Re^n$ is the estimated state vector, $\hat{f}$ is the online approximation approach in discrete-time (OLAD), $\hat{\theta} \in \Re^q$ is a set of adjustable parameters, and A is $n \times n$ a constant design matrix chosen by the user.

The initial conditions for the estimated model (3) $\hat{x}(0) = x_0$ and $\hat{\theta}(0) = \hat{\theta}_0$, are selected so that $\hat{f}(x, u, \hat{\theta}_0) = 0$ for all $x \in \chi$ and $u \in U$. Given the initial conditions, the next step involves the development of an adaptive law for the unknown parameters $\hat{\theta}(k)$, so that the online approximator $\hat{f}(x(k), u(k); \hat{\theta}(k))$ approximates the failure function $\Pi(k-k_0) f(x(k), u(k))$. Accurate construction of models of the nonlinear system would enable to track any system changes and helps in developing a robust diagnostic algorithm.

For the online approximation based models, (x,u) is the input vector to the model, $\hat{\theta}(k)$ is the vector of adjustable parameters, and $\hat{f}(x, u; \hat{\theta})$ is the output. In this paper, we consider a general class of sufficiently smooth online approximators; that is $\hat{f} \in C^{\infty}$.

Remark 4: Once an approximator achieves close approximation of the failure dynamics, this online approximator $\hat{f}$ may be used not only to detect but also to diagnose the failures. In some cases, the approximator can be used for failure accommodation.

Remark 5: In this paper, the failure mode described by f are considered unknown.

Next define the state estimation error as $e = x - \hat{x}$. Under the ideal conditions with no modeling errors, a fault is declared active whenever the output of the online approximator $\hat{f}(x(k),u(k);\hat{\theta}(k))$ becomes nonzero. An intuitive way of generating robustness with respect to modeling uncertainties is to start the adaptation whenever the state estimation error is above a certain threshold. This can be easily implemented by using a dead-zone operator $D[.]$, which is defined for improving robustness of the fault diagnosis scheme as

$$D[e(k)] = \begin{cases} 0, \text{ if } \|e(k)\| \le \varepsilon \\ e(k), \text{if } \|e(k)\| > \varepsilon \end{cases}$$

where $e(k)$ is the state estimation error in the current time instant and $\varepsilon > 0$ is a design constant similar to the case of continuous-time [6]. However, the adaptive update will be different between the continuous and discrete-time cases. The selection of the dead-zone size ε clearly provides a tradeoff between reducing the possibility of false alarms (robustness) and improving the sensitivity of the faults. In the next section, the dead-zone size ε (in terms of modeling uncertainty bound $\tilde{\zeta}_0$) is derived that guarantees robustness in the presence of modeling uncertainties satisfying the given bound.

IV. Stability and Performance Analysis

The fault diagnosis scheme described above has interesting stability properties, performance and robustness properties which are discussed in this section by using novel parameter update law and dead-zone operator. These results are obtained for the case of incipient failures which occur at some unknown time k_0 and develop with unknown rates κ_i. The incipient failure changes the dynamics of the system but it is assumed to retain the boundedness of the state and input variables [6] (Assumption 1).

In an ideal case, where there is no modeling errors and prior to the occurrence of a fault i.e. $k \in [0, k_0)$, the state estimation error is given by

$$e(k+1) = Ae(k) - \hat{f}(x(k),u(k);\hat{\theta}(k)) \quad (4)$$

and the parameter estimate $\hat{\theta}$ can be selected as

$$\hat{\theta}(k+1) = \hat{\theta}(k) + \alpha Z e(k+1)$$

where $\alpha > 0$ is the learning rate or adaptation gain and Z is a $q \times n$ matrix defined as

$$Z = \left[\frac{\partial \hat{f}(x,u;\hat{\theta})}{\partial \hat{\theta}}\right]^T \quad (5)$$

Since $\hat{\theta}_0$ is chosen such that $\hat{f}(x,u,\hat{\theta}_0) = 0$ for all x and u, the vector $(e,\hat{\theta}_0) = (0,\hat{\theta}_0)$ is an equilibrium point for the system in (4). Therefore, $e(k) = 0$ and $\hat{\theta}(k) = \hat{\theta}_0$ for $k \in [0, k_0)$.

Similarly, in the presence of modeling errors, (4) becomes

$$e(k+1) = Ae(k) + \tilde{\zeta}(x(k),u(k)) - \hat{f}(x(k),u(k);\hat{\theta}(k)) \quad (6)$$

According to the robust adaptive law due to the dead-zone operator

$$\hat{\theta}(k+1) = \hat{\theta}(k) + \alpha Z\ D[e(k+1)] \quad (7)$$

The output of the online approximator remains zero as long as $\|e(k)\| \le \varepsilon$. To determine an appropriate value for ε, we derive an upper bound for $e(k)$ in the case $\hat{f}(x,u,\hat{\theta}_0) = 0$. From (6), we have $e(k) = \sum_{j=1}^{k} A^{k-j}\tilde{\zeta}(x(j-1),u(j-1))$. Since the matrix A is stable, there exist two positive constants μ and λ such that (Frobenius norm) $\|A^k\| \le \lambda\mu^k \le 1$. Therefore $\|e(k)\| \le \lambda\tilde{\zeta}_0 \frac{(1-\mu^k)}{(1-\mu)}$. This implies that if the size of the dead-zone is selected as $\varepsilon = \frac{\lambda\tilde{\zeta}_0}{(1-\mu)}$, $e(k)$ remains within the dead zone for all $k \le k_0$ and the output of the approximator remains zero. Therefore, the adaptive scheme given by (7) is robust in the sense that it is not affected by modeling errors provided $\|\tilde{\zeta}(x(k),u(k))\| \le \tilde{\zeta}_0$. By letting $\|\tilde{\zeta}(x(k),u(k))\| = \tilde{\zeta}_0$ for all time k, it is easy to verify that the selected bound for the dead-zone size ε is not conservative.

Next during the time interval $k \ge k_0$, after the occurrence of the fault, using (1) and (3), the state estimation error satisfies

$$\begin{aligned} e(k+1) &= Ae(k) + \tilde{\zeta}(x(k),u(k)) \\ &\quad + \Pi(k-k_0) f(x(k),u(k)) - \hat{f}(x(k),u(k);\hat{\theta}(k)) \\ &= Ae(k) + \tilde{\zeta}(x(k),u(k)) + \Pi(k-k_0)\hat{f}(x(k),u(k),\theta^*) \\ &\quad - \hat{f}(x(k),u(k);\hat{\theta}(k)) + \upsilon(k) \end{aligned} \quad (8)$$

where $\upsilon(k)$ is the approximation error given by

$$\upsilon(k) = \Pi(k-k_0)[f(x(k),u(k)) - \hat{f}(x(k),u(k),\theta^*)] \quad (9)$$

and θ^* is an optimal value chosen such that it minimizes the L_2 norm distance between $\hat{f}(x,u;\hat{\theta})$ and $f(x,u)$ for all $(x, u) \in \chi \times U$ provided θ^* is constrained to a compact set $W \subset \Re^q$. Based on the smooth assumptions on $\hat{f}(x,u,\hat{\theta})$ [5], (8) can be expressed as

$$\begin{aligned} e(k+1) &= Ae(k) + \tilde{\zeta}(x(k),u(k)) - [I - \Pi(k-k_0)]\hat{f}(x(k),u(k),\theta^*) \\ &\quad + \frac{\partial \hat{f}(x,u;\hat{\theta})}{\partial \hat{\theta}}(\hat{\theta} - \theta^*) + \Delta(x,u;\hat{\theta},\theta^*) + \upsilon(k) \end{aligned} \quad (10)$$

where

$$\Delta(x,u;\hat{\theta},\theta^*) = \hat{f}(x,u,\theta^*) - \hat{f}(x,u;\hat{\theta}) - \frac{\partial \hat{f}(x,u;\hat{\theta})}{\partial \hat{\theta}}(\hat{\theta}-\theta^*) \quad (11)$$

with $\Delta(x,u;\hat{\theta},\theta^*)$ represents the higher order terms of the Taylor series expansion of $\hat{f}(x,u;\hat{\theta})$ w.r.t to $\hat{\theta}$. Let $\tilde{\theta} = \theta^* - \hat{\theta}$ and $\delta(k) = \Delta(x,u;\hat{\theta},\theta^*) - [I - \Pi(k-k_0)]\hat{f}(x(k),u(k),\theta^*) + \tilde{\zeta}(x(k),u(k)) + \upsilon(k)$, then error equation (10) becomes

$$e(k+1) = Ae(k) + Z^T\tilde{\theta} + \delta(k) \quad (12)$$

In a special case of linearly parameterized approximators the higher order term is identically equal to zero [6]. A fault is declared when the output $\hat{f}(x,u;\hat{\theta})$ is non-zero. Next the following result is stated regarding the performance of the fault detection scheme. For the following, it is taken that $\|e(k)\| > \varepsilon$.

Remark 6: In the following text, improved parameter tuning schemes for the fault detection scheme is presented so that the PE condition is not required.

Theorem 1: (PE condition not required) let the initial conditions for the nonlinear estimator be bounded in a compact set $S \subset \Re^n$. In the presence of modeling and fault dynamics reconstruction errors, consider the parameter update law given by

$$\hat{\theta}(k+1) = \hat{\theta}(k) + \alpha Z\, \mathrm{D}[e^T(k+1)] - \gamma\left\|I - \alpha ZZ^T\right\|\hat{\theta}(k) \quad (13)$$

where $0 < \gamma < 1$ is a design parameter. Then there exist two constants d_e and d_θ, denoted as the uniform ultimate bounds for the estimation error $e(k)$ and parameter error $\tilde{\theta}(k)$ respectively of the nonlinear estimator given by

$$d_e = \frac{1}{1-\sigma A_{\max}^2}\left[\xi A_{\max} + \sqrt{\rho(1-\sigma A_{\max}^2)}\right] \quad (14)$$

$$d_\theta = \frac{\gamma(1-\gamma)\theta_{\max} + \sqrt{\gamma^2(1-\gamma)^2\theta_{\max}^2 + \gamma(2-\gamma)\tilde{\theta}}}{\gamma(2-\gamma)} \quad (15)$$

provided the following conditions hold

$$\alpha\|Z\|^2 < 1 \quad (16)$$

$$0 < \gamma < 1 \quad (17)$$

$$A_{\max} = \lambda_{\max}(A) < \frac{1}{\sqrt{\sigma}} \quad (18)$$

$$\sigma = \eta + \frac{1}{1-\alpha A_{\max}^2}\left[\gamma^2(1-\alpha A_{\max}^2)^2 + 2\alpha\gamma A_{\max}^2(1-\alpha A_{\max}^2)\right] \quad (19)$$

where $\|Z\| \le Z_{\max}$, η in (19) is given by $\eta = \dfrac{1}{1-\alpha\|Z\|^2}$, ξ and ρ are given in (24) and (25).

Proof: Consider a Lyapunov candidate as

$$V = e^T(k)e(k) + \frac{1}{\alpha}tr[\tilde{\theta}^T(k)\tilde{\theta}(k)]$$

The first difference is given by

$$\Delta V = \Delta V_1 + \Delta V_2 = e^T(k+1)e(k+1) - e^T(k)e(k) + \frac{1}{\alpha}tr[\tilde{\theta}^T(k+1)\tilde{\theta}(k+1) - \tilde{\theta}^T(k)\tilde{\theta}(k)]$$

Consider the first term in the first difference ΔV, substituting equation (12), using $\Psi(k) = Z^T\tilde{\theta}(k)$ and combining terms, we get

$$\begin{aligned}\Delta V_1 = {} & e^T(k)A^TAe(k) + 2[Ae(k)]^T\Psi(k) + \Psi^T(k)\Psi(k)\\ & +[\delta(k)]^T[\delta(k)] + 2[Ae(k)]^T[\delta(k)]\\ & +2[\delta(k)]^T\Psi(k) - e^T(k)e(k) \quad (20)\end{aligned}$$

Next consider the second term and obtaining its first difference ΔV_2, as

$$\Delta V_2 = \frac{1}{\alpha}tr[\tilde{\theta}^T(k+1)\tilde{\theta}(k+1) - \tilde{\theta}^T(k)\tilde{\theta}(k)]$$

by using the parameter update law (13), dead-zone operator and defining $\tilde{\theta} = \theta^* - \hat{\theta}$, one obtains

$$\begin{aligned}\Delta V_2 = {} & \frac{1}{\alpha}tr\{[\tilde{\theta}^T(k) + \alpha e^T(k)A^TZ^T + \alpha\,\theta^T(k)ZZ^T\\ & +\alpha\delta^T(k)Z^T - \gamma\left\|I-\alpha ZZ^T\right\|\hat{\theta}^T(k)]\\ & \times[\tilde{\theta}(k) + \alpha ZAe(k) + ZZ^T\theta(k) + \alpha Z\delta(k)\\ & -\gamma\left\|I-\alpha ZZ^T\right\|\hat{\theta}(k) - \tilde{\theta}^T(k)\tilde{\theta}(k)\} \quad (21)\end{aligned}$$

Combining ΔV_1 from (20) and ΔV_2 from (21), applying $tr(xx^T) = x^Tx$, adding and subtracting $\alpha^{-1}tr[\left\|I-\alpha ZZ^T\right\|^2\tilde{\theta}^T(k)\tilde{\theta}(k)]$, we get

$$\begin{aligned}\Delta V \le {} & -e^T(k)(I - A^TA)e(k) + 2[Ae(k)]^T\Psi(k) +\\ & 2[Ae(k)]^T[\delta(k)] + \Psi^T(k)\Psi(k) + 2[\delta(k)]^T\Psi(k)\\ & +[\delta(k)]^T[\delta(k)] - 2[1-\alpha ZZ^T][Ae(k)]^T\Psi(k)\\ & -[2-\alpha ZZ^T]\Psi^T(k)\Psi(k)\\ & -2[1-\alpha ZZ^T]\Psi^T(k)[\delta(k)] + \alpha ZZ^T\{e^T(k)A^TAe(k)\\ & +2[Ae(k)]^T[\delta(k)] + [\delta(k)]^T[\delta(k)]\}\\ & -\frac{1}{\alpha}\left\|I-\alpha ZZ^T\right\|^2[\gamma(2-\gamma)\left\|\tilde{\theta}(k)\right\|^2 2\gamma(1-\gamma)\left\|\tilde{\theta}(k)\right\|\times\theta_{\max}\\ & -\gamma^2\theta_{\max}^2] + 2\gamma\left\|I-\alpha ZZ^T\right\|[Ae(k)]^T\Psi(k)\\ & +2\gamma\left\|I-\alpha ZZ^T\right\|\Psi(k)[\delta(k)] + 2\gamma A_{\max}\left\|I-\alpha ZZ^T\right\|\times\\ & \|Z\|\theta_{\max}\|e(k)\| + 2\gamma\left\|I-\alpha ZZ^T\right\|(\delta(k))\|Z\|\theta_{\max} \quad (22)\end{aligned}$$

and by completing the squares for $\Psi(k)$ in (22), one obtains

$$\Delta V \le -(1-\sigma A_{\max}^2)\|e(k)\|^2 - [1-\alpha Z^TZ]\times$$

$$\left\| \Psi(k) - \frac{1}{1-\alpha Z^T Z}[\alpha Z^T Z + 2\gamma[I - \alpha ZZ^T]] \times [Ae(k) + \delta(k)] \right\|^2$$

$$2\xi A_{\max} \|e(k)\| + \rho - \frac{1}{\alpha}\left\| I - \alpha ZZ^T \right\|^2 \times [\gamma(2-\gamma)\|\tilde{\theta}(k)\|^2$$

$$-2\gamma(1-\gamma)\|\tilde{\theta}(k)\|\theta_{\max} - \gamma^2\theta_{\max}^2] \quad (23)$$

where ξ and ρ are given as

$$\xi = \eta\delta_N + \gamma(1 - \alpha Z_{\max}^2) Z_{\max}\theta_{\max} \quad (24)$$

and

$$\rho = \eta\delta_N^2 + 2\gamma(1 - \alpha Z_{\max}^2) Z_{\max}\theta_{\max}\delta_N \quad (25)$$

with $\|\delta(k)\| \le \delta_N$ as the uniformly bound [6]. To show the bound of the estimation error $e(k)$ and the parameter error $\tilde{\theta}(k)$, completing the squares for $\|\tilde{\theta}(k)\|$ using (23), we get

$$\Delta V \le -(1-\sigma A_{\max}^2)[\|e(k)\|^2 - \frac{2\xi A_{\max}}{(1-\sigma A_{\max}^2)}\|e(k)\| - \frac{\bar{\rho}}{(1-\sigma A_{\max}^2)}]$$

$$-[1 - \alpha Z^T Z] \times$$

$$\left\| \Psi(k) - \frac{1}{1-\alpha Z^T Z}[\alpha Z^T Z + 2\gamma[I - \alpha ZZ^T]] \times [Ae(k) + \delta(k)] \right\|^2$$

$$-\frac{1}{\alpha}\left\| I - \alpha ZZ^T \right\|^2 \times \gamma(2-\gamma)\left[\|\tilde{\theta}\| - \frac{2(1-\gamma)}{(2-\gamma)}\theta_{\max}\right]^2 \quad (26)$$

where $\bar{\rho} = \rho + \frac{1}{\alpha}\left\| I - \alpha ZZ^T \right\| \times \left[\frac{\gamma(1-\gamma)^2}{(2-\gamma)}\theta_{\max}^2 + \gamma^2\theta_{\max}^2\right]$

Then $\Delta V \le 0$ as long as the conditions in (16)-(18), hold and the quadratic term for $e(k)$ in (26) is positive, which is guaranteed when

$$\|e(k)\| > \frac{1}{(1-\sigma A_{\max}^2)} \times \left[\xi A_{\max} + \sqrt{\xi^2 A_{\max}^2 + \bar{\rho}(1-\sigma A_{\max}^2)}\right] \quad (27)$$

Similarly, completing the squares for $\|e(k)\|$ using (26), we get

$$\Delta V \le -(1-\sigma A_{\max}^2)\left[\|e(k)\| - \frac{\xi A_{\max}}{(1-\sigma A_{\max}^2)}\right]^2 - [1-\alpha Z^T Z] \times$$

$$\left\| \Psi(k) - \frac{1}{1-\alpha Z^T Z}[\alpha Z^T Z + 2\gamma[I - \alpha ZZ^T]] \times [Ae(k) + \delta(k)] \right\|^2$$

$$-\frac{1}{\alpha}\left\| I - \alpha ZZ^T \right\|^2 \left[\gamma(2-\gamma)\|\tilde{\theta}\|^2 - 2\gamma(1-\gamma)\theta_{\max}\|\tilde{\theta}\| - \tilde{\rho}\right] \quad (28)$$

where

$$\tilde{\rho} = -\frac{\left[\rho - \frac{\xi^2 A_{\max}^2}{(1-\sigma A_{\max}^2)}\right]}{\frac{1}{\alpha}\left\| I - \alpha ZZ^T \right\|^2} + \gamma^2\theta_{\max}^2$$

Then $\Delta V \le 0$ as long as (16)-(18) hold and the quadratic term for $\tilde{\theta}(k)$ in (28) is positive, which is guaranteed when

$$\|\tilde{\theta}(k)\| > \frac{\gamma(1-\gamma)\theta_{\max} + \sqrt{\gamma^2(1-\gamma)^2\theta_{\max}^2 + \gamma(2-\gamma)\tilde{\rho}}}{\gamma(2-\gamma)} \quad (29)$$

From (27) and (29), ΔV is negative outside a compact set M. According to a standard Lyapunov theorem extension [11], it can be concluded that the state estimation error $e(k)$ and the error in parameter estimate $\tilde{\theta}(k)$ are uniformly ultimately bounded.

Remark 7: The output of the online approximator $\hat{f}(x,u;\hat{\theta})$ tuned with the update law in (13) remains zero as long as $|e(k)| \le \varepsilon$ (dead zone). Hence a failure is identified when the bounded error $e(k)$ exceeds the dead zone.

V. EXAMPLE AND DISCUSSION

The fault detection developed is tested onto a simple mass damper system [6]. The discrete time states space model equivalent to a continuous time mass damper system is given as

$$x_1(k+1) = Tx_2(k) + x_1(k)$$

$$x_2(k+1) = \frac{1}{m}\{T(F - c_1x_2(k) - k_1x_1(k) - \Pi(k-k_0)\delta x^3{}_1(k)\} + x_2(k)$$

$$(30)$$

Where $x_1(k)$ and $x_2(k)$ are the states of the system and represent the displacement and velocity term of the mass damper system. The external force (input) applied to the system is defined as $F = 5\sin(kT)$. The term $\Pi(k-k_0)\delta x^3{}_1(k)$ is the actual failure term, and in this simulation, we assume a fault of incipient nature. Also $\delta = k_1 a_0{}^2$, $\Pi(k-k_0) = H(k-k_0)(1-e^{-\kappa(k-k_0)})$ and where H is the unit step function. The actual system given in (30) is studied using the following nonlinear estimator scheme

$$\hat{x}_1(k+1) = T\hat{x}_2(k) + \hat{x}_1(k)$$

$$\hat{x}_2(k+1) = \frac{1}{m}\{T(F - c_1x_2(k) - k_1x_1(k) + c_2(x_2(k) - \hat{x}_2(k))$$

$$+k_2(x_1(k) - \hat{x}_1(k)) - \hat{f}(x_1,\hat{\theta}))\} + \hat{x}_2(k) \quad (31)$$

where $\hat{x}_1(k)$ and $\hat{x}_2(k)$ are estimated states of $x_1(k)$ and $x_2(k)$. The values of the parameters for the actual system and the estimator are given as follows

$m = 1$, $c_1 = 0.5$, $k_1 = 0.5$, $c_2 = 5.0$, $k_2 = 0.55$, $a_0 = 1$, $\kappa = 0.1$, $x_1(0) = 0.5$, $x_2(0) = 0.1$, and $T = 0.01$. The failure is assumed to occur at $k = 10$ sec, and a spring stiffness fault (spring hardening) is induced in the actual system. The online approximator (OLAD) used is a single layer radial basis function network with ten neurons, $\hat{f}(x_1,\hat{\theta}) = \sum_{i=1}^{N=10} \hat{\theta}_i \exp(-|x_1 - c_i|^2 / \sigma^2)$. The centers c_i are randomly chosen in the interval [-9, 9] and widths as $\sigma = 0.911$.

Figure 1 shows the normalized norm of the state estimation error, prior to the time instant $k = 10$ sec., the error is small. At the instant of the fault, the error increases to a large value. Hence the fault is detected and the online approximator is triggered to learn the occurring fault in the system. Once the OLAD adapts to the actual failure term, the state estimation error attains a uniform bound. Figure 2 shows the behavior of the system prior to and after the fault occurs. The term $mx_{2f}(k+1) + c_1 x_2(k) + k_1 x_1(k) - F$ is simulated in Fig. 2, where $x_{2f}(k+1)$, is the value of $x_2(k+1)$ i.e.

$$x_{2f}(k+1) = ((1/\tau)x_2(k+1)) - ((1/\tau)x_2(k)) - ((T/\tau)x_{2f}(k)) + x_{2f}(k)$$

where, in this simulation it is taken that $\tau = 0.1$. Hence it could be seen that the system behavior changes significantly after the failure. Hence by using Figures 1 and 2 the fault occurring in the system could be detected.

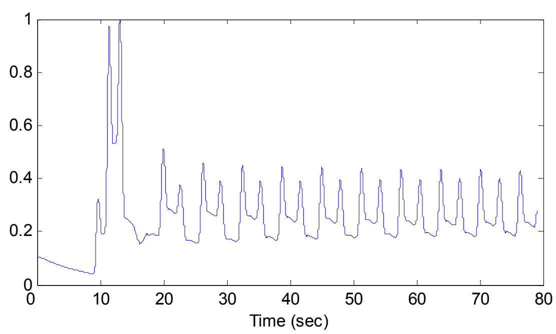

Figure 1: Normalized Euclidean norm of the state estimation error.

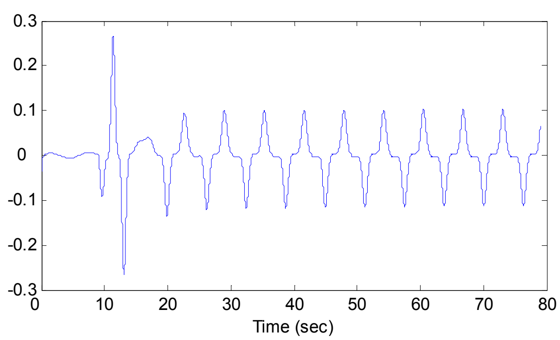

Figure 2: Evolution of $mx_{2f}(k+1) + c_1 x_2(k) + k_1 x_1(k) - F$.

Figure 3 shows the evolution of the actual failure term (solid line) and the response of the online approximator (dashed line) scheme in discrete-time. The OLAD scheme is tuned using the weight update law given in (13). The values of the parameters used for the parameter update law are: $\alpha = 0.01$ and $\gamma = 0.454$.

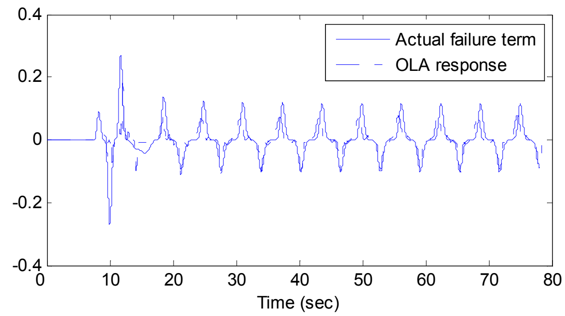

Figure 3: Evolution of the actual failure term (solid line) and online approximator (dashed line).

From Fig. 3 it is evident that the OLAD scheme learns the fault satisfactorily. Hence the scheme not only detects the fault but also learns the fault occurring in the system satisfactorily.

The above simulation results show the implementation, robustness and the performance of the fault detection scheme. Also the boundness of the estimation error is shown in the result. Further based on the mathematical proofs and the simulation results, it was seen that the proposed scheme could be used as a robust fault detection tool for nonlinear discrete time systems.

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