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# A Dynamic model of Induction Generators for Wind Power Studies

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**Abstract:** This paper presents an induction generator model that can be used for simulations to investigate and evaluate the control strategies for variable speed operation of doubly fed induction generators driven by wind turbines in transient conditions. The model makes use of rotor reference frame.

**Keywords:** Induction generator, reference frame theory, variable speed operation, wind machine.

## I. INTRODUCTION

In the US, a total of 1,700 MW of wind power were added in the year 2001, according to the American Wind Energy Association. The rest of the world received more than 4,800 MW of wind power capacity in the same year.

So why is wind getting so much attention lately? Wind-generated electricity has become more economical to produce in the past 10 years, dropping from as much as 30 cents per kilowatt-hour to 4 to 6 cents, making it more competitive with other energy sources. The cost to develop and build a wind energy facility is approximately \$1 million per megawatt, compared to a cost for gas-fired energy generation of \$550,000 to \$700,000 per MW. Also with tax credits and other state and federal incentives, wind can become major players in the future energy delivery infrastructure. The federally sponsored wind production tax credit provides 1.8 cents per kilowatt-hour generated for the first 10 years of a wind turbine's output. So, turbines built by the end of 2003 will qualify for the tax credit for 10 years.

The wind energy sector is expected to grow about 20 percent a year, with an emphasis on Europe, the U.S. and Latin America.

With increasing penetration of wind-derived power in interconnected power systems, it has become necessary to model the complete wind energy systems in order to study their impact and also to study wind plant control. This paper presents a model of the induction generator written in appropriate d-q reference frame to facilitate investigation of control strategies.

Wind turbine power depends on both rotor speed and wind speed [1, 2]. Aerodynamic power available in the wind can be calculated using Eq. (1).

$$P = 0.5\rho AC_p v^3 \quad (1)$$

where, P = power in watts

$\rho$  = air density (about 1.225 Kg/m<sup>3</sup> at sea level)

A = rotor swept area; v = wind speed in m/sec

$C_p$  = Coefficient of performance (typically 0.35)

The relationship between rotor speed and wind speed can be

given by:  $\lambda = \frac{\omega_m R}{v}$  (2)

where R = wind machine rotor radius,

$\omega_m$  = rotor speed in rad/sec, and

$\lambda$  = tip speed ratio = ratio between the linear speed of the tip of the blade with respect to the wind speed.

Thus Eq. (1) becomes:

$$P_{max} = 0.5\rho AC_{p,max} \left[ \frac{R}{\lambda_{max}} \right] \omega_m^3 \quad (3)$$

where  $C_{p,max}$  is the value of  $C_p$  at a specific value of  $\lambda$ .

There are two basic options for wind power conversion – fixed speed and variable speed operation. In fixed speed operation, the aeroturbine can be operated at a constant speed by blade-pitch control of the wind turbine even under varying wind speeds. This option was very common because of the cost involved with the power converter needed in the variable speed generation to convert the variable frequency to match the constant grid frequency. In variable speed operation, the aeroturbine rotational speed can be allowed to vary with wind to maintain a constant and optimum tip speed ratio. A constant speed wind turbine is designed to obtain a maximum efficiency at one wind speed that will give the optimum tip speed to wind speed ratio for the rotor airfoil. The variable-speed operation by active pitch angle control allows optimum efficiency operation of the turbine over a wide range of wind speeds, resulting in increased power outputs [3].

Fig. 1 shows how variable speed operation will allow a wind turbine to capture more energy from the wind [4, 5].

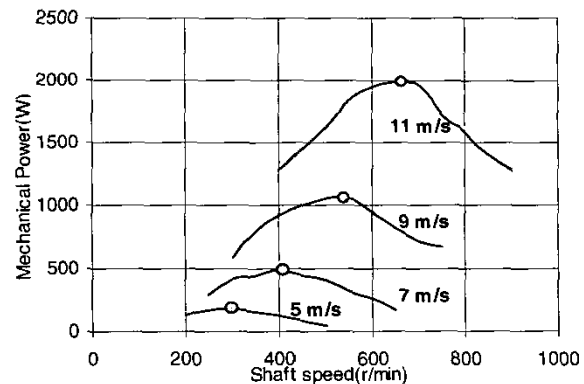


Fig. 1. Wind turbine power characteristics

As one can see, the maximum power follows a cubic relationship.

For variable speed generation, an induction generator is considered attractive due to its flexible rotor speed characteristic in contrast to the constant speed characteristic of synchronous generator. Doubly fed induction generator configuration is best suited for variable speed generation since it can be controlled from rotor side as well as stator side [6]. This is possible since rotor circuit is capable of bidirectional power flow. The doubly fed machine can be operated in generating mode in both sub-synchronous and super-synchronous modes [7]. The rotor will absorb slip power from the grid in sub synchronous operation and can feed slip power back to grid in super synchronous operation. The cost of power converter can be reduced greatly by employing doubly fed configuration such as a wound rotor machine[8], since power converter can be designed to be partially rated as it needs to carry only slip power. All these advantages make the doubly fed induction generator a favorable candidate for variable speed generation.

A commonly used model for induction generator converting power from the wind to serve the electric grid is shown in Fig. 2.

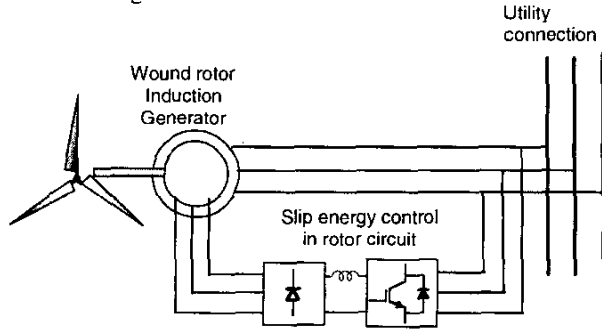


Fig. 2. Simplified model of a doubly-fed induction generator

In this paper, an attempt is made to develop a dynamic model of induction generator which can be simulated as a doubly fed generator when testing control strategies. Though the model developed in this paper can be used for simulating all types of induction generator configurations, the choice of rotor reference frame makes it particularly favorable for the simulation of doubly fed configuration in transient conditions [8].

## II. LIST OF SYMBOLS

$v_{ds}, v_{qs}, v_{dr}, v_{qr}$  =  $d-q$  axis machine voltages.

$i_{ds}, i_{qs}, i_{dr}, i_{qr}$  =  $d-q$  axis machine currents

$\Psi$  = flux linkage

$\omega_b$  = base electrical frequency (rated)

$\omega$  = angular velocity of reference frame

$\omega_r$  = angular frequency of rotor

$T_{em}$  = electromagnetic torque

$T_{mech}$  =externally applied mechanical torque (from wind turbine shaft)

$T_{damp}$  = damping torque

$r$  = resistance

$x$  = reactance

$H$  = inertia constant

Subscripts:

$d, q$  = d-q axes

$s, r$  = stator, rotor

$l$  = leakage

m=magnetizing

Superscripts:

' = referred value

## II. DYNAMIC MODEL

The equations (1)-(4) can be found in literature[8] and [9].

$$\Psi_{qs} = \omega_b \int (v_{qs} - \frac{\omega}{\omega_b} \Psi_{ds} + \frac{r_s}{x_{ls}} (\Psi_{mq} - \Psi_{qs})) \quad (1)$$

$$\Psi_{ds} = \omega_b \int (v_{ds} + \frac{\omega}{\omega_b} \Psi_{qs} + \frac{r_s}{x_{ls}} (\Psi_{md} - \Psi_{ds})) \quad (2)$$

$$\Psi'_{qr} = \omega_b \int (v'_{qr} - (\frac{\omega - \omega_r}{\omega_b}) \Psi'_{dr} + \frac{r'_r}{x'_{lr}} (\Psi'_{mq} - \Psi'_{qr})) \quad (3)$$

$$\Psi'_{dr} = \omega_b \int (v'_{dr} + (\frac{\omega - \omega_r}{\omega_b}) \Psi'_{qr} + \frac{r'_r}{x'_{lr}} (\Psi'_{md} - \Psi'_{dr})) \quad (4)$$

Using the rotor reference frame:  $\omega = \omega_r$  and  $v'_{qr} = v'_{dr} = 0$

Substituting these conditions the above equations boil down to:

$$\Psi_{qs} = \omega_b \int (v_{qs} - \frac{\omega_r}{\omega_b} \Psi_{ds} + \frac{r_s}{x_{ls}} (\Psi_{mq} - \Psi_{qs})) \quad (5)$$

$$\Psi_{ds} = \omega_b \int (v_{ds} + \frac{\omega_r}{\omega_b} \Psi_{qs} + \frac{r_s}{x_{ls}} (\Psi_{md} - \Psi_{ds})) \quad (6)$$

$$\Psi'_{qr} = \omega_b \int \frac{r'_r}{x'_{lr}} (\Psi'_{mq} - \Psi'_{qr}) \quad (7)$$

$$\Psi'_{dr} = \omega_b \int \frac{r'_r}{x'_{lr}} (\Psi'_{md} - \Psi'_{dr}) \quad (8)$$

The currents can now be calculated as:

$$i_{qs} = \frac{\Psi_{qs} - \Psi_{mq}}{x_{ls}} \quad (9)$$

$$i_{ds} = \frac{\Psi_{ds} - \Psi_{md}}{x_{ls}} \quad (10)$$

$$i'_{qr} = \frac{\Psi'_{qr} - \Psi'_{mq}}{x'_{lr}} \quad (11)$$

$$i'_{dr} = \frac{\Psi'_{dr} - \Psi'_{md}}{x'_{lr}} \quad (12)$$

$\Psi_{mq}$  and  $\Psi_{md}$  in the above four equations are calculated by:

$$\Psi_{mq} = x_M (\frac{\Psi_{qs}}{x_{ls}} + \frac{\Psi'_{qr}}{x'_{lr}}) \quad (13)$$

$$\Psi_{md} = x_M \left( \frac{\Psi_{ds}}{x_{ls}} + \frac{\Psi'_{dr}}{x'_{lr}} \right) \quad (14)$$

Where  $x_M = \frac{1}{(1/x_m + 1/x_{ls} + 1/x'_{lr})}$

Now that we know  $\Psi_{qs}$ ,  $i_{qs}$ ,  $\Psi_{ds}$  and  $i_{ds}$  the electromagnetic torque can be calculated by:

$$T_{em} = \frac{3}{2} \frac{P}{2\omega_b} (\Psi_{ds} i_{qs} - \Psi_{qs} i_{ds}) \text{ N.m} \quad (15)$$

The equation that governs the motion of rotor is obtained by equating the inertia torque to the accelerating torque:

$$J \frac{d\omega_{rm}}{dt} = T_{em} + T_{mech} - T_{damp} \text{ N.m} \quad (16)$$

Expressed in per unit values the above equation becomes

$$2H \frac{d(\omega_r / \omega_b)}{dt} = T_{em} + T_{mech} - T_{damp} \quad (\text{In pu}) \quad (17)$$

In the q-axis circuit, the Eq. (5), (7), (9), (11) and (13) are used to calculate  $\Psi_{qs}$ ,  $\Psi'_{qr}$ ,  $i_{qs}$ ,  $i'_{qr}$  and  $\Psi_{mq}$  respectively and  $\Psi_{qs}$  and  $i_{qs}$  are used in the calculation of electromagnetic torque.

In the d-axis circuit, the Eq. (6), (8), (10), (12) and (14) are used to calculate  $\Psi_{ds}$ ,  $\Psi'_{dr}$ ,  $i_{ds}$ ,  $i'_{dr}$  and  $\Psi_{md}$  respectively and  $\Psi_{ds}$  and  $i_{ds}$  are used in the calculation of electromagnetic torque.

The rotor circuit makes use of the  $\Psi_{qs}$  and  $i_{qs}$  obtained from the q-axis circuit and  $\Psi_{ds}$  and  $i_{ds}$  obtained from the d-axis circuit and calculates the electromagnetic torque using Eq. (15). The rotor circuit also takes the input mechanical torque values supplied to it and computes  $\frac{\omega_r}{\omega_b}$  from Eq. (17).

Fig. 3 shows an implementation of the above model on Simulink®

### III. METHODOLOGY

For maintaining proper flow of variables and for convenience of simulating, the above equations are separated into the q-axis, the d-axis and the rotor circuits.

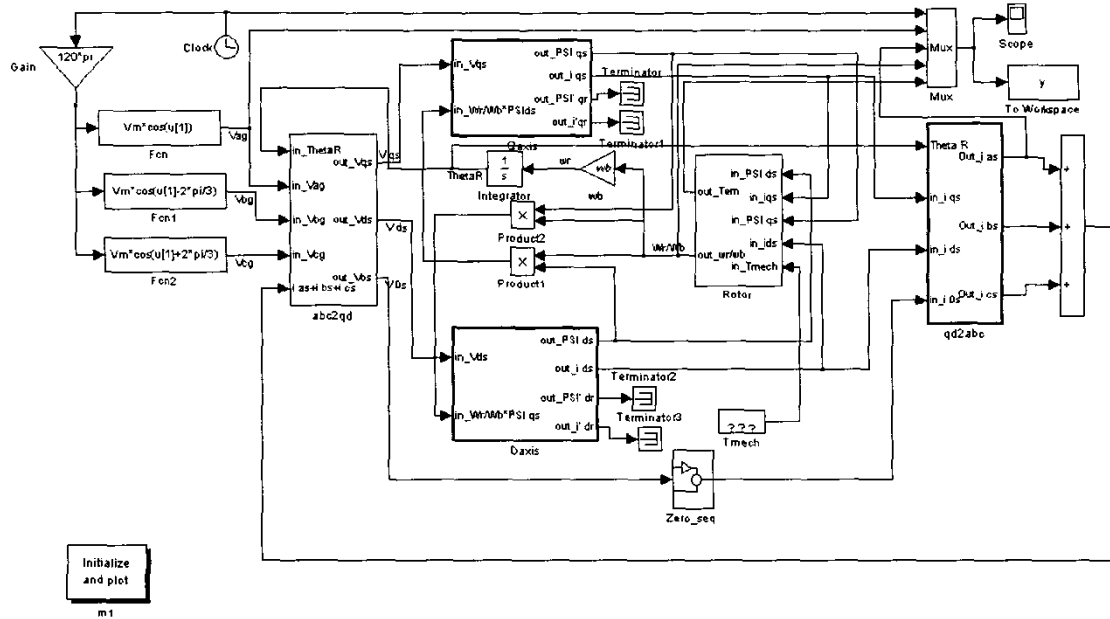


Fig. 3. Implementation of the induction generator model in Simulink®

### IV. RESULTS

The Simulink model is simulated with parameters as shown in Table I.

$V_{ag}$ ,  $V_{bg}$ ,  $V_{cg}$  are the applied voltages to the stator from the grid. The wind turbine is assumed to be

operated with variable speed so that it will operate in the peak power tracking mode. A varying wind speed profile is applied to the generator to investigate its performance. The wind speeds were changed at  $t=0.6$ ,  $0.8$ ,  $1.2$  and  $1.6$  seconds to  $5\text{ m/s}$ ,  $7\text{ m/s}$  and  $11\text{ m/s}$  respectively. The peak powers corresponding to wind speeds of  $5\text{ m/s}$ ,  $7\text{ m/s}$  and  $11\text{ m/s}$  were  $180\text{ W}$ ,  $480$

W, 1060 W and 1980 W respectively. This profile is shown in Table II.

The transient response of variables  $i_{as}$ ,  $T_{em}$  and  $\omega_r$  to the applied load torque are plotted in Fig. 4. It can be seen that the stator current, electromagnetic torque and rotor speed reach steady state by 0.6 sec. After that these values change only when the induction generator is made to adjust its speed to operate in peak power mode. The negative value of the electromagnetic torque and negative slip indicate that the machine is working in generating mode.

TABLE I. INDUCTION GENERATOR CHARACTERISTICS

Induction Generator Characteristic	Value
Number of poles	4
Rated Speed	1800rpm
Rated Voltage	200 V
Rated Output Power	750 W
Stator winding resistance	3.35 ohm
Stator leakage reactance	2.616 ohm
Rotor resistance as referred to stator	1.99 ohm
Rotor leakage reactance	2.616 ohm
Rotor inertia	0.1 Kg-m <sup>2</sup>

TABLE II. MECHANICAL POWER CORRESPONDING TO THE OPTIMUM WIND SPEED AND ROTOR SPEED

Time (Sec)	Wind Speed (m/sec)	Mechanical Power (W)
0	0	0
0.6	5	180
0.8	7	480
1.2	9	1060
1.6	11	1980

## V. CONCLUSIONS

The choice of the reference frame will affect the waveforms of all dq variables. It will also affect the simulation speed and in certain cases the accuracy of the results. Generally the conditions of operation will determine the most convenient reference frame for analysis. The following guidelines are adopted for choosing the reference frame [8]:

Case 1: Stationary reference frame should be used if the stator voltages are unbalanced and the rotor voltages are balanced.

Case 2: Rotor reference frame should be used if the rotor voltages are unbalanced and the stator voltages are balanced.

Case 3: Either the stationary or synchronous reference frame can be used if all voltages are balanced and continuous.

Since our objective is to develop a dynamic model of induction generator for rotor side control studies, Case 2 applies to our model and rotor reference frame is chosen accordingly with acceptable results.

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## Biographies

**Srinivas Chellapilla** received his BTech degree in Electrical and Electronics Engineering from Jawaharlal Nehru Technological University, Kakinada, India in 2000. He is presently an MS student at the Department of Electrical Engineering, University of Missouri-Rolla.

**Badrul H. Chowdhury** obtained his M.S. and Ph.D. degrees in Electrical Engineering from Virginia Tech, Blacksburg, VA in 1983 and 1987 respectively. He is currently a Professor in the Electrical & Computer Engineering department of the University of Missouri-Rolla. From 1987 to 1998 he was with the University of Wyoming's Electrical Engineering department where he reached the rank of Professor. Dr. Chowdhury's research interests are in power system modeling, analysis and control; power electronics and drives.

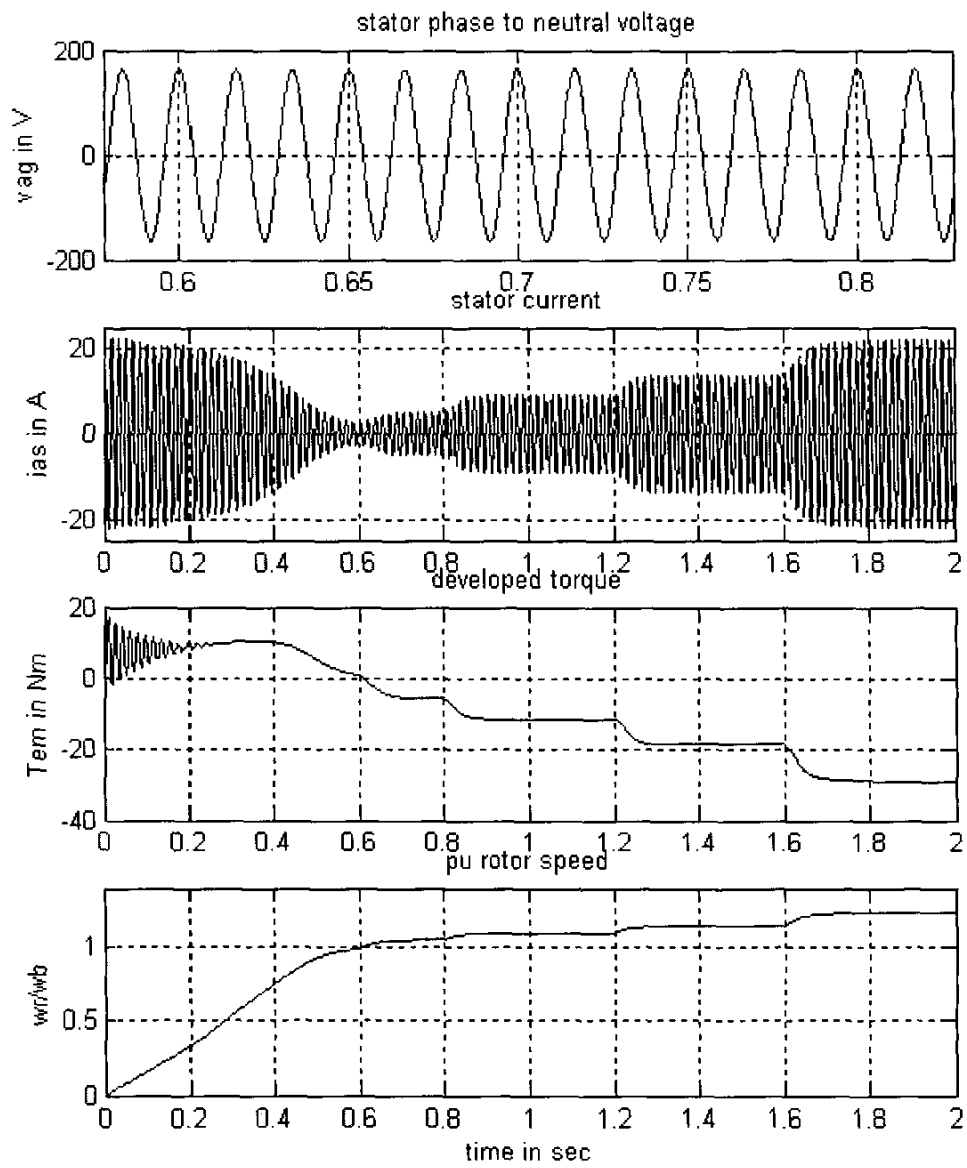


Fig. 4. Dynamic response of the induction generator to variable wind speeds