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Jagannathan Sarangapani
Missouri University of Science and Technology, sarangap@mst.edu

Travis Alan Dierks

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Neural Network Control of Robot Formations using RISE Feedback

Travis Dierks and S. Jagannathan

Abstract—***In this paper, a combined kinematic/torque control law is developed for leader-follower based formation control using backstepping in order to accommodate the dynamics of the robots and the formation in contrast with kinematic-based formation controllers that are widely reported in the literature. A neural network (NN) is introduced along with robust integral of the sign of the error (RISE) feedback to approximate the dynamics of the follower as well as its leader using online weight tuning. It is shown using Lyapunov theory that the errors for the entire formation are asymptotically stable and the NN weights are bounded as opposed to uniformly ultimately bounded (UUB) stability which is typical with most NN controllers. Theoretical results are demonstrated using numerical simulations.***

Index Terms —**Neural network, formation control, Lyapunov method, kinematic/dynamic controller, RISE.**

I. Introduction

Over the past decade, the attention has shifted from the control of a single nonholonomic mobile robot [1] to the control of multiple mobile robots because of the advantages a team of robots offers for complex tasks like search and rescue operations, mapping unknown or hazardous environments, security and bomb sniffing.

However, a characteristic that is commonly found in many formation control papers [2-5] is the design of a kinematic controller requiring a perfect velocity tracking assumption as a result of ignoring the robot dynamics and the formation dynamics. However, in [6], the follower robot dynamics are considered using a neural network (NN) whereas the formation dynamics are still ignored.

In this paper, the framework developed for controlling single nonholonomic mobile robots is expanded to leader follower formation control by incorporating the dynamics of the robots and that of the formation in the controller design. A two-layer NN with one tunable layer is introduced to learn the dynamics of the follower robots well as their leaders' online, and is combined with a recently developed robust integral of sign of the error (RISE) feedback method originating in [8]. Both velocity feedback control inputs and velocity tracking control laws are presented, and the *asymptotic stability* of the entire formation as well as the boundedness of the NN weights is shown using Lyapunov methods as opposed to only uniform ultimately boundedness, a result common in the NN controls literature [7][10].

The RISE method [8] is designed to reject bounded unmodeled disturbances, like NN functional reconstruction errors, to yield asymptotic tracking. An approach to blend a multilayer NN with RISE feedback for a single rigid robot control is taken in [9]. Boundedness of the actual NN weights is shown separately using projection algorithm and convergence of the tracking errors is then demonstrated by using constant controller gains. Selection of the predefined convex set in the projection algorithm to prevent the NN weights from diverging is a challenging task since the convex set must be carefully chosen to contain the ideal weights.

By contrast, in this work the constant bounds and gains in [9] are replaced for formation control with time varying functions allowing bounds and gains to be determined with more certainty, and a novel weight tuning is used instead of the projection algorithm [9]. An additional advantage of using the proposed NN weight tuning as opposed to the projection algorithm is less decision making in the NN learning process, which can lead to reduced system delays. Further, Lyapunov analysis is presented to show the asymptotic convergence of the tracking errors and boundedness of the NN weights simultaneously. Finally, the bounds and gains developed here also applicable to single rigid robot control [9] besides formation control. No offline training is needed for the NNs. Simulation results justify theoretical concepts.

II. Leader-Follower Formation Control

The two popular techniques in leader-follower formation control include separation-separation and separation-bearing [5]. The goal of separation-bearing formation control is to find a velocity control input such that

$$\lim_{t\to\infty}(L_{ijd}-L_{ij})=0 \quad \text{and} \quad \lim_{t\to\infty}(\Psi_{ijd}-\Psi_{ij})=0 \tag{1}$$

where L_{ij} and ψ_{ij} are the measured separation and bearing of the follower robot with L_{ijd} and ψ_{ijd} represent desired distance and angles respectively [5]. Only separation-bearing techniques are considered, but our approach can be extended to separation-separation control. To avoid collisions, separation distances are measured from the back of the leader to the front of the follower, and the kinematic equations for the front of the j^{th} follower robot can be written as

$$\begin{bmatrix}\dot{x}_j\\ \dot{y}_j\\ \dot{\theta}_j\end{bmatrix}=\begin{bmatrix}\cos\theta_j & -d_j\sin\theta_j\\ \sin\theta_j & d_j\cos\theta_j\\ 0 & 1\end{bmatrix}\begin{bmatrix}v_j\\ \omega_j\end{bmatrix} \tag{2}$$

where d_j is the distance from the rear axle to the to front of the robot, x_j , y_j , and θ_j are actual Cartesian position and orientation of the physical robot, and, v_j , and ω_j are linear and angular velocities respectively. Many robotic systems can be characterized as a robotic system having an

Authors are with the Department of Electrical & Computer Engineering, University of Missouri, Rolla, MO, 65401 USA (e-mail: tad5x4@ umr.edu).
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n-dimensional configuration space $\mathcal{C}$ with generalized coordinates $(q_1, \ldots q_n)$ and subject to m constraints described in detail in [1] and mathematically after applying the transformation described in [1] as

$$\overline{M}_j(q_j)\dot{v}_j + \overline{V}_{m_j}(q_j,\dot{q}_j)v_j + \overline{F}_j(v_j) + \overline{\tau}_{d_j} = \overline{B}_j(q_j)\tau_j. \tag{3}$$

where $\overline{M}_j \in \Re^{rxr}$ is a symmetric positive definite inertia matrix, $\overline{V}_{mj} \in \Re^{rxr}$ is the centripetal and coriolis matrix, $\overline{F}_j \in \Re^{rx1}$ is the friction vector, $\overline{\tau}_{dj}$ represents unknown bounded disturbances, and $\overline{\tau}_j = \overline{B}_j \tau \in \Re^{rx1}$ is the input vector.

Backstepping Controller Design: The behavior of a mobile robot is described by equations (2) and (3), and standard approaches [2-5] to leader follower formation control deal only with (2) and assume perfect velocity tracking holds by ignoring the dynamics of (3). To remove this assumption, integrator backstepping is applied. The contribution in this paper lies in deriving an alternative control velocity, $v_{jc}(t)$, for separation-bearing leader follower formation control and designing an augmented NN/RISE based torque controller such that (2) and (3) exhibit the desired behavior for a given control velocity $v_c(t)$, thus incorporating the dynamics of the leader i and follower j, and removing perfect velocity tracking assumptions.

Kinematic Controller Design: Contributions in single robot frameworks are now considered and expanded upon in the development a kinematic controller for the separation-bearing formation control technique. Consider the tracking controller error system presented in [1] used to control a single robot as

$$\begin{bmatrix} e_{j1} \\ e_{j2} \\ e_{j3} \end{bmatrix} = \begin{bmatrix} \cos\theta_j & \sin\theta_j & 0 \\ -\sin\theta_j & \cos\theta_j & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{jr} - x_j \\ y_{jr} - y_j \\ \theta_{jr} - \theta_j \end{bmatrix} \tag{4}$$

$$\dot{x}_{jr} = v_{jr}\sin\theta_{jr} \quad \dot{y}_{jr} = v_{jr}\sin\theta_{jr} \quad \dot{\theta}_{jr} = \omega_{jr} \quad \dot{q}_{jr} = \begin{bmatrix} \dot{x}_{jr} & \dot{y}_{jr} & \dot{\theta}_{jr} \end{bmatrix}^T \tag{5}$$

where x_{jr}, y_{jr}, θ_{jr}, v_{jr}, and ω_{jr} are the position, orientation and linear and angular velocities respectively of a virtual reference cart robot j seeks to follow.

Here, the virtual reference cart is replaced with a physical mobile robot acting as the leader i, and x_{jr} and y_{jr} are defined as points at a distance L_{ijd} at a desired angle Ψ_{ijd} from the lead robot as follows. Let there be a leader i subject to kinematic constraints in the form of (2) for robot j to follow. Define the desired location and orientation for follower j as

$$\begin{aligned} x_{jr} &= x_i - d_i\cos\theta_i + L_{ijd}\cos(\Psi_{ijd} + \theta_i) \\ y_{jr} &= y_i - d_i\sin\theta_i + L_{ijd}\sin(\Psi_{ijd} + \theta_i) \\ \theta_{jr} &= \theta_i \end{aligned} \tag{6}$$

and the actual position and orientation of follower j as

$$\begin{aligned} x_j &= x_i - d_i\cos\theta_i + L_{ij}\cos(\Psi_{ij} + \theta_i) \\ y_j &= y_i - d_i\sin\theta_i + L_{ij}\sin(\Psi_{ij} + \theta_i) \\ \theta_j &= \theta_j \end{aligned} \tag{7}$$

where L_{ij} and Ψ_{ij} are the actual separation and bearing of follower j. Next, define the reference velocity as $v_{jr} = [|v_i| \quad |\omega_i|]^T$ where $|v_i|$ and $|\omega_i|$ are the time varying linear and angular speeds of the leader such that $v_{jr} > 0$ for all time.

Substitution of (6) and (7) into (4) and applying simple trigonometric identities [10], the error system (4) can be written as

$$e_j = \begin{bmatrix} e_{j1} \\ e_{j2} \\ e_{j3} \end{bmatrix} = \begin{bmatrix} L_{ijd}\cos(\Psi_{ijd} + e_{j3}) - L_{ij}\cos(\Psi_{ij} + e_{j3}) \\ L_{ijd}\sin(\Psi_{ijd} + e_{j3}) - L_{ij}\sin(\Psi_{ij} + e_{j3}) \\ \theta_i - \theta_j \end{bmatrix}. \tag{8}$$

The transformed error system now acts as a formation tracking controller which not only seeks to remain at a fixed desired distance L_{ijd} with a desired angle ψ_{ijd} relative to the lead robot i, but also achieves the same orientation as the lead robot which is desirable when $\omega_i = 0$. In order to calculate the error dynamics given in (8), it is necessary to calculate the derivatives of L_{ij} and ψ_{ij}, and it is assumed that L_{ijd} and ψ_{ijd} are constant. It is shown in our previous work [10] that the derivatives of the separation and bearing are consistent with [5] even when using kinematic equations (2) such that

$$\begin{aligned} \dot{L}_{ij} &= v_j\cos\gamma_j - v_i\cos\Psi_{ij} + d_j w_j \sin\gamma_j \\ \dot{\Psi}_{ij} &= \frac{1}{L_{ij}}(v_i\sin\Psi_{ij} - v_j\sin\gamma_j + d_j w_j\cos\gamma_j - L_{ij} w_i) \end{aligned} \tag{9}$$

where $\gamma_j = \Psi_{ij} + e_{j3}$. Now, using (9) to calculate the derivative of (8) and applying trigonometric identities, the error dynamics are written as

$$\dot{e}_j = \begin{bmatrix} \dot{e}_{j1} \\ \dot{e}_{j2} \\ \dot{e}_{j3} \end{bmatrix} = \begin{bmatrix} -v_j + v_i\cos e_{j3} + \omega_j e_{j2} - \omega_i L_{ijd}\sin(\Psi_{ijd} + e_{j3}) \\ -\omega_j e_{j1} + v_i\sin e_{j3} - d_j\omega_j + \omega_i L_{ijd}\cos(\Psi_{ijd} + e_{j3}) \\ \omega_i - \omega_j \end{bmatrix}. \tag{10}$$

To stabilize the kinematic system, we propose the following velocity control inputs for follower robot j to achieve the desired position and orientation with respect to leader i as

$$v_{jc} = \begin{bmatrix} v_i\cos e_{j3} + k_1 e_{j1} - \omega_i L_{ijd}\sin(\Psi_{12d} + e_{j3}) \\ \omega_i + (v_i + 1)k_2 e_{j2} + (v_i + 1)k_3\sin e_{j3} + \gamma_{\omega jc} \end{bmatrix} \tag{11}$$

where

$$\gamma_{\omega jc} = -\frac{|e_{j2}|\left(\omega_i(d_j + L_{ijd}) + (v_i + 1)k_3 d_j + 1\right)}{1/k_2 + |e_{j2}|d_j} \tag{12}$$

The following mild assumptions are needed.

Assumption 1. Follower j is equipped with sensors capable of measuring the separation distance L_{ij} and bearing Ψ_{ij} and that both leader and follower are equipped with sensors to measure their linear and angular velocities as well as there orientations θ_j and θ_i.

Assumption 2. Wireless communication is available between follower j and leader i with communication delays being zero.

Assumption 3. Leader i communicates its linear and angular velocity v_i, ω_i as well as its orientation θ_i and control torque $\tau_i(t)$ to follower j.

Assumption 4. For the nonholonomic system of (2) and (3) with n generalized coordinates q, m independent constraints, and r actuators, the number of actuators is equal to the number of degrees of freedom ($r = n - m$).

Assumption 5. The reference linear and angular velocities measured from the leader i are bounded and $v_{jr}(t) \geq 0$ for all t.

Assumption 6. $K = [k_1 \quad k_2 \quad k_3]^T$ is a vector of positive constants.

Assumption 7. Let perfect velocity tracking hold such that $\dot{v}_j = \dot{v}_{jc}$ (this assumption is relaxed later).

Theorem 1[10]: Let a smooth velocity control input $v_{jc}(t)$ for follower j be given by (11). Then the origin $e_j = 0$ consisting of the position and orientation error for the follower is asymptotically stable.

Proof: Using the following Lyapunov function candidate

$$V_j = \frac{1}{2}(e_{j1}^2 + e_{j2}^2) + \frac{1 - \cos e_{j3}}{k_2}, \tag{13}$$

it is shown in [10] that the velocity control (11) provides asymptotic stability for the error system (8) and (10) and $e_j \to 0$ as $t \to \infty$.

Dynamical NN/RISE Controller Design: Now assume that the perfect velocity tracking assumption does not hold making *Assumption 7* invalid. A two-layer NN is considered here consisting of one layer of randomly assigned constant weights $V \in \Re^{axL}$ in the first layer and one layer of tunable weights $W \in \Re^{Lxb}$ in the second with a inputs, b outputs, and L hidden neurons. The *universal approximation property* for NN's [7] states that for any smooth function $f(x)$, there exists a NN such that $f(x) = W^T \sigma(V^T x) + \varepsilon$ where ε is the NN functional approximation error and $\sigma(\cdot): \Re^a \to \Re^L$ is the activation function in the hidden layers. The sigmoid activation function is considered here. For complete details of the NN and its properties, see [7].

Remark: $\|\cdot\|$ and $\|\cdot\|_F$ will be used interchangeably as the Frobenius vector and matrix norms [7].

Define velocity tracking and filtered tracking errors as

$$e_{jc} = v_{jc} - v_j \tag{14}$$

$$r_j = \dot{e}_{jc} + \alpha_j(t) e_{jc} \tag{15}$$

where $\alpha_j(t)$ is a real time varying function greater than zero defined as $\alpha_j(t) = \alpha_{j0} + \alpha_{j1}(t)$ where α_{j0} is a constant and $\alpha_{j1}(t)$ is a time varying term. Multiplying both sides of (15) by $\overline{M}_j$, substituting the robot dynamics (3) and adding and subtracting $\overline{V}_{m_j} v_{jc}$ and $\overline{F}_j(v_{jc})$ allows (15) to be rewritten as

$$\overline{M}_j r_j = f_{d_j} + T_j + \overline{\tau}_{d_j} - \overline{\tau}_j \tag{16}$$

where
$$f_{d_j} = \overline{M}_j \dot{v}_{jc} + \overline{V}_{m_j} v_{jc} + \overline{F}_j(v_{jc}) \tag{17}$$

and
$$T_j = e_{jc}(\alpha_j(t)\overline{M}_j - \overline{V}_{m_j}) + \overline{F}_j(v_j) - \overline{F}_j(v_{jc}) \tag{18}$$

Differentiating (16) then yields

$$\overline{M}_j \dot{r}_j = -\dot{\overline{M}}_j r_j + \dot{f}_{d_j} + \dot{T}_j + \dot{\overline{\tau}}_{d_j} - \dot{\overline{\tau}}_j. \tag{19}$$

Define the control torque as in [9] to be

$$\overline{\tau}_j = \hat{f}_{d_j} + \mu_j \tag{20}$$

where $\hat{f}_{d_j}$ is the estimate of f_{d_j} and μ_j is the RISE feedback term defined similarly to [8] and [9] as

$$\mu_j = (k_s + 1)e_{jc}(t) - (k_s + 1)e_{jc}(0) + \int_0^t [(k_s + 1)\alpha_j(s)e_{jc}(s) + (\beta_{j1}(s) + 1)\operatorname{sgn}(e_{jc}(s))]ds \tag{21}$$

such that
$$\dot{\mu}_j = (k_s + 1)r_j + (\beta_{j1}(t) + 1)\operatorname{sgn}(e_{jc}) \tag{22}$$

where k_s is a real positive constant, $\beta_{j1}(t)$ is a real, positive, time varying gain function, and *sgn* is the signum function.

Remark: In [8] and [9], $\beta_{j1}(t)$ and $\alpha_j(t)$ are considered to be positive constants. We choose time varying functions here to facilitate in defining the upper bounds necessary for the RISE aspects of the NN/RISE controller which will be discussed in the proceeding development and in the Appendix.

Substituting the derivative of (20) and adding and subtracting e_{jc} into (19) as well as defining

$$\dot{f}_{d_j} = W_j^T \sigma(V^T x_{dj}) + \varepsilon_j \text{ and } \dot{\hat{f}}_{d_j} = \hat{W}_j^T \sigma(V^T x_{dj}) \tag{23}$$

yields

$$\overline{M}_j \dot{r}_j = -\frac{1}{2}\dot{\overline{M}}_j r_j + \tilde{N}_j + N_{Bj1} + N_{Bj2} - e_{jc} - (k_s + 1)r_j - \beta_{j1}(t)\operatorname{sgn}(e_{jc}) \tag{24}$$

where

$$\tilde{N}_j = -\frac{1}{2}\dot{\overline{M}}_j r_j + \dot{T}_j + e_{jc} \tag{25}$$

$$N_{Bj1} = \varepsilon_j + \dot{\overline{\tau}}_{d_j}, \quad N_{Bj2} = \tilde{W}_j \sigma(\overline{x}_{d_j}) \tag{26}$$

and $\hat{W}_j^T$ is the NN estimate of the ideal weight matrix, $\sigma = \sigma(\overline{x}_{d_j})$, $\overline{x}_{d_j} = V^T x_{d_j}$, $x_{d_j} = [1 \quad v_{jc} \quad \dot{v}_{jc} \quad \ddot{v}_{jc} \quad \theta_j]^T$ and $\tilde{W}_j = W_j - \hat{W}_j$. An upper bound for $\tilde{N}_j$ can be obtained using the Mean Value Theorem as [8]

$$\|\tilde{N}_j\| \leq \rho(\|z_j\|)\|z_j\| \tag{27}$$

where $z_j = [e_{jc}^T \quad r_j^T]^T$ and $\rho(\|z_j\|)$ is a positive, globally invertible, non-decreasing function. Before proceeding, the following mild assumptions are needed.

Assumption 8. On any compact subset of $\Re^n$, the ideal NN weights are bounded by known positive values for all followers $j=1,2,\ldots N$ such that $\|W_j\|_F \leq W_M$ [7].

Assumption 9. Let the NN approximation property hold for the function (23) with accuracy ε_N for all followers j and for all x_{dj} in the compact set S[7] such that $\|\varepsilon_j\| < \varepsilon_N$, $j=1,2,\ldots N$. Furthermore, let $\|\dot{\varepsilon}_j\| < \varepsilon'_N$ and the disturbances and their derivatives be bounded such that $\|\overline{\tau}_{dj}\| \leq d_M$ and $\|[\dot{\overline{\tau}}_{dj} \quad \ddot{\overline{\tau}}_{dj}]\| \leq d'_M$ [1][9].

Using the above assumptions along with (26) and $\tilde{W}_j = W_j - \hat{W}_j$, the following inequalities can be defined.

$$\left\|N_{Bj1}\right\| \le \varepsilon_N + d'_M \equiv \varsigma_{j1} \quad \left\|\dot{N}_{Bj1}\right\| \le \varepsilon'_N + d'_M \equiv \varsigma'_{j1} \quad (28)$$

$$\left\|N_{Bj2}\right\| \le W_M + \left\|\hat{W}_j\right\|_F \equiv \varsigma_{j2}(t) \quad (29)$$

$$\left\|\dot{N}_{Bj2}\right\| \le c_1\left\|e_{jc}\right\| + c_2(t)(W_M + \left\|\hat{W}_j\right\|_F) \equiv \varsigma'_{j2}(t) \quad (30)$$

where c_1 is a known positive constant and $c_2(t)$ is a positive time varying function based on $\left\|\dot{x}_{dj}\right\|$. Proof of these bounds can be found in the Appendix.

It should be noted at this point that

$$\dot{v}_{jc} = f_j(\dot{v}_i, \dot{\omega}_i, v_i, \omega_i, e_j, \dot{e}_j) \quad (31)$$

The leader i's dynamics in the form of (3) can be rewritten as

$$\dot{v}_i = \overline{M}_i^{-1}(q_i)[\overline{B}_i(q_i)\tau_i - \overline{V}_{m_i}(q_i, \dot{q}_i)v_i - \overline{F}_i(v_i) - \overline{\tau}_{d_i}] \quad (32)$$

Substituting (10) and (32) into (31) results in the error dynamics of follower j and the dynamics of leader i to become apart of $\dot{v}_{jc}$ as

$$\dot{v}_{jc} = f_j(v_i, \omega_i, \theta_i, \tau_i, e_j). \quad (33)$$

It is assumed that the leader and follower robots' dynamics are sufficiently smooth such that $\ddot{v}_i$, v_{jc}, and $\dot{v}_{jc}$ are also smooth functions. Under these assumptions $\ddot{v}_{jc}$ can be approximated with relatively small error by the standard second order backwards difference equation for a small sample period Δt as

$$\ddot{\hat{v}}_{jc} = v_{jc}(t) - 2v_{jc}(t-\Delta t) + v_{jc}(t-2\Delta t) \quad (34)$$

Using (34) and forming $\dot{v}_{jc}$ under the assumption that $\dot{v}_i = 0$ and including the terms of the function defined in (33), x_{d_j} takes the form of

$$x_{d_j} = [1 \;\; v_{jc} \;\; \dot{v}_{jc}\big|_{\dot{v}_i=0} \;\; \ddot{\hat{v}}_{jc} \;\; \theta_j \;\; v_i \;\; w_i \;\; \tau_i \;\; \theta_i \;\; e_j]^T \quad (35)$$

so that the dynamics of the leader i can be estimated by the NN, and the terms of $\dot{v}_{jc}$ omitted by assuming $\dot{v}_i = 0$ can be accounted for.

Theorem 2: Let Assumptions *1-6* and *8-9* hold, and let k_s be sufficiently large positive constant. Let a smooth velocity control input $v_{jc}(t)$ for follower j be defined by (11), and let the torque control for follower j given by (20) be applied to (3). Let the weight tuning law be given as

$$\dot{\hat{W}}_j = F\sigma_j e_{jc}^T \quad (36)$$

where $F = F^T > 0$. Then the position, orientation, and velocity tracking errors e_j and e_{jc} are asymptotically stable, and the neural network weight estimate errors $\widetilde{W}$ are bounded for follower j provided that β_{j1} and β_{j2} are selected as

$$\beta_j \ge \varsigma_1 + \varsigma_2(t) + \frac{1}{\alpha_{j0}}\left(\varsigma'_1 + c_2(t)(W_M + \left\|\hat{W}\right\|_F)\right), \;\; \beta_{j2} \ge F \quad (37)$$

Proof: Consider the following Lyapunov candidate

$$V'_j = V_j + V_{jNN} \quad (38)$$

where V_j is defined as (13) and

$$V_{jNN} = e_{jc}^T e_{jc} + \frac{1}{2} r_j^T \overline{M}_j r_j + P_j + Q_j \quad (39)$$

where $$P_j = \beta_{j1}(0)\left\|e_{jc}(0)\right\| - e_{jc}(0)^T N_{Bj3}(0) - \int_0^t L_j(s)ds \quad (40)$$

$$L_j = r_j^T(N_{Bj1} - \beta_{j1}\,\mathrm{sgn}(e_{jc})) + \dot{e}_{jc}^T N_{Bj2} - \beta_{j2}\left\|e_{jc}\right\|^2 - \dot{\beta}_{j1}\left\|e_{jc}\right\| \quad (41)$$

and $$Q_j = \frac{\alpha}{2} tr\{\widetilde{W}_j^T F^{-1} \widetilde{W}_j\} \quad (42)$$

If β_{j1} and β_{j2} are chosen according to (37), the following inequality can be obtained (this claim is proved in the Appendix)

$$\int_0^t L_j(s)ds \le \beta_{j1}(0)\left\|e_{jc}(0)\right\| - e_{jc}(0)^T N_{Bj3}(0) \quad (43)$$

Therefore, it can be concluded that $P \ge 0$ and noted that $\dot{P} = -L$. Taking the time derivative of (38) yields $\dot{V}'_j = \dot{V}_j + \dot{V}_{jNN}$, and it was stated in *Theorem 1* and proved in [10] that $\dot{V}_j < 0$, so our efforts will focus on V_{jNN}. Before proceeding, it is important to note there exists $U_1(y_j)$ and $U_2(y_j)$ such that

$$U_1(y_j) \le V_{jNN} \le U_2(y_j) \quad (44)$$

where $y_j = [z_j^T \;\; \sqrt{P_j} \;\; \sqrt{Q}_j] \in \Re^{2r+2}$. $U_1(y)$ and $U_2(y)$ are defined in [9] to be $U_1(y) = \lambda_1\|y\|^2$ and $U_2(y) = \lambda_2(q)\|y\|^2$ where $\lambda_1 = \frac{1}{2}\min\{1, m_1\}$, $\lambda_2 = \max\{\frac{1}{2}\overline{m}(q), 1\}$ and m_1 and $\overline{m}(q)$ are defined as a known positive constant and a known positive function respectively in reference to the bounds on the mass matrix $M(q)$ in (3). See [9] for further details. Differentiating (39), making use of (15), (24), and the derivatives of (40) and (42) yields

$$\begin{aligned}\dot{V}_{jNN} &= e_{jc}^T r_j - 2\alpha_j(t)\left\|e_{jc}\right\|^2 + r_j^T \widetilde{N}_j - r_j^T(k_s+1)r_j - \dot{e}_{jc}^T\,\mathrm{sgn}(e_{jc}) \\ &\quad + \alpha_j(t)(e_{jc}^T \widetilde{W}\sigma + tr\{\widetilde{W}F^{-1}\dot{\widetilde{W}}\}) + \beta_{j2}(t)\left\|e_{jc}\right\|^2 + \left\|e_{jc}\right\|(\dot{\beta}_{j1} - \alpha_j(t))\end{aligned} \quad (45)$$

Noting $e_{jc}^T r_j \le \frac{1}{2}\left\|e_{jc}\right\|^2 + \frac{1}{2}\left\|r_j\right\|^2 \le \left\|e_{jc}\right\|^2 + \left\|r_j\right\|^2$ and $\left\|\dot{e}_{jc}\right\| - a_{j0}\left\|e_{jc}\right\| \le \left\|r_j\right\|$ and substitution of (36) results in

$$\dot{V}_{jNN} \le -(2\alpha_{j0} - 1 - \beta_{j2})\left\|e_{cj}\right\|^2 + \left\|r_j\right\|(\widetilde{N}_j + 1) - k_s\left\|r_j\right\|^2 \quad (46)$$

after recalling $\alpha_j(t) = \alpha_{j0} + \alpha_{j1}(t)$ and selecting $\alpha_{j1}(t) = \left|\dot{\beta}_{j1}(t)\right|$. Based on (27), it is reasonable to assume that a second bounding function can be defined as

$$\left\|\widetilde{N}_j\right\| + 1 \le \rho'(\left\|z_j\right\|)\left\|z_j\right\| \quad (47)$$

where $\rho'(\left\|z_j\right\|)$ is a positive, globally invertible, non-decreasing function. Utilizing (47) and completing the square with respect to $\left\|r_j\right\|$ yields

$$\dot{V}_{jNN} \le -\lambda_j\left\|z_j\right\|^2 - \frac{k_s}{2}\left(\left\|r_j\right\| - \frac{\rho'(\left\|z_j\right\|)\left\|z_j\right\|}{k_s}\right)^2 + \frac{\rho'(\left\|z_j\right\|)^2\left\|z_j\right\|^2}{2k_s} \quad (48)$$

where $\lambda_j = \min\left\{2\alpha_{j0} - 1 - \beta_{j2}, \frac{k_s}{2}\right\}$, and select $\alpha_{j0} \geq (\beta_{j2}+1)/2$. The second term is always less than or equal to zero, so considering the first and third terms, a continuous positive-semi-definite function $U(y_j) = c\|z_j\|^2$, for some real positive constant c can be defined on the domain D such that

$$\dot{V}_{jNN} \leq -U(y_j) \text{ for } D = \{y_j \in \Re^{2r+2} \mid \|y_j\| \leq \rho'^{-1}(\sqrt{2\lambda_j k_s})\}. \quad (49)$$

The inequalities in (44) and (49) can be used to show that $V_{jNN} < \infty$ and bounded in D, and therefore e_{jc}, r_j, P_j, and Q_j are also bounded in D. Continuing this way by observing the boundedness of e_{jc} and r_j in D, standard linear analysis methods can be used to prove that all of the quantities in (14), (15), (21), (22), and (24) are also bounded in D. Therefore, the definitions for $U(y_j)$ and $z_j(t)$ can be used to prove that $U(y_j)$ is uniformly continuous. For complete details of the steps to draw this conclusion, see [9].

Let $S \subset D$ denote a region of attraction such that

$$S = \left\{y_j(t) \subset D \mid U(y_j(t)) < \lambda_1(\rho'^{-1}(\sqrt{2\lambda_j k_s}))^2\right\}. \quad (50)$$

Applying Theorem 8.4 of [11], it can be concluded $c\|z_j\|^2 \to 0$ as $t \to \infty \quad \forall \quad y_j(0) \in S$. From the definition of $z_j(t)$, it is clear that $\|e_{jc}\| \to 0$ as $t \to \infty$ for all $y_j(0) \in S$ thus illustrating the asymptotic stability of the tracking error and the boundedness of the neural network weight estimates.

Remark: The region of attraction (50) can be made arbitrarily large to include a larger set of initial conditions by increasing the gain k_s. Also, the boundedness $\hat{W}_j$ does not guarantee that the estimates converge to the ideal W unless certain signals are persistently excited [7].

Leader Control Structure: In every formation, there is at least a leader i such that under the following assumptions:

Assumption 10. The formation leader follows the virtual leader described in [1], and the virtual leader's velocity is defined by a time varying function that is twice differentiable.

Assumption 11. The leader is capable of measuring its absolute position.

The kinematics and dynamics of the formation leader i are defined similarly to (2) and (3) respectively. From [1], the leader tracks a virtual reference robot with the kinematic constraints of (5), and the control velocity $v_{ic}(t)$ can be defined as

$$v_{ic} = \begin{bmatrix} v_{ir}\cos e_{i3} + k_{i1}e_{i1} \\ \omega_{ir} + k_{i2}v_{ir}e_{i2} + k_{i3}v_{ir}\sin e_{i3} \end{bmatrix} \quad (51)$$

Using the same steps and justifications to form (14)-(19), the leader's error dynamics can be written similarly to the follower's shown in (19), and the torque $\bar{\tau}_i$ is defined as

$$\bar{\tau}_i = \hat{f}_{d_i} + \mu_i \quad (52)$$

where $\hat{f}_{d_i}$ is the estimate of f_{d_j} and μ_j is the RISE feedback term defined similarly to the follower's controllers in (21). Using the same steps and justifications used to form (24)-(26), a NN/RISE controller can be derived for the lead robot i with the only deviation from the follower's controller being a slight modification to the NN input vector, x_{d_i}. For leader i, we define $x_{d_i} = [1 \;\; v_{ic} \;\; \dot{v}_{ic} \;\; \ddot{v}_{ic}|_{\dot{v}_i=0} \;\; \theta_i \;\; v_i \;\; \omega_i \;\; \tau_i]^T$. The NN weight updates for the leader i are defined similarly to follower j's in (36).

Theorem 3: Let *Assumptions 1-6* and *8-11* hold for leader i, and let $K_i=[k_{i1}\ k_{i2}\ k_{i3}]^T$ be a vector of positive constants. Let there be a smooth velocity control input $v_{ic}(t)$ for the leader i given by (51), and let the torque control input for the lead robot i defined by (52) be applied to the mobile robot system in the form of (3). Then leader's position, orientation, and velocity tracking errors are asymptotically stable and the NN weight estimates are bounded.

Proof: Due to page limitations, the proof of *Theorem 3* is not included. However, the theorem can be proved by selecting the Lyapunov candidate $V_i' = V_i + V_{iNN}$

where
$$V_i = \frac{1}{2}(e_{i1}^2 + e_{i2}^2) + \frac{1-\cos e_{i3}}{k_{i2}} \quad (53)$$

and
$$V_{iNN} = e_{ic}^T e_{ic} + \frac{1}{2} r_i^T \overline{M}_i r_i + P_i + Q_i. \quad (54)$$

and noting the similarities between *Theorem 2* and *Theorem 3*.

*Remark: T*he stability of the entire formation consisting of 1 leader and N followers can be proved as well as the stability of the formation for the case when follower j becomes a leader to follower $j+1$. Proofs of these claims are not presented here due to length constraints, but they follow as a result of *Theorems 2* and *3*.

III. Simulation Results

A wedge formation of five identical nonholonomic mobile robots is considered where the leader's trajectory is the desired formation trajectory, and simulations are carried out in MATLAB under two scenarios. First, the NN controller of our previous work which is proven to be *Uniformly Ultimately Bounded* (UUB) in [10] is considered, and then the NN/RISE controller which has been shown to be *asymptotically stable* (AS) in this paper is tested. The torque controller developed in [10] takes the form of

$$\bar{\tau}_j = \hat{W}_j^T \sigma(\bar{x}_j) + (k_s + 1)e_{jc} = \hat{f}_j + (k_s + 1)e_{jc}$$

which is identical the torque control of (20), but without the extra RISE terms added in (21). In both cases, the NN learning parameter is set as $F=10$, and the following gains were utilized.

Leader	$k_{is}=daig\{50\}$	$K_{i1}=10$	$K_{i2}=5$	$K_{i3}=4$
Follower j	$k_s=diag\{50\}$	$k_1=7$	$K_2=20$	$k_3=.01$

The following gain parameters are selected for the NN/RISE controller: $\beta_{i,j1} = 2.5 + (5 + \|\hat{W}_j\|_F)(2.5 + 1.5\|e_j\|)$ and

$$\alpha_{i,j} = F + F\|e_{jc}\|(2.5 + 1.5\|e_j\|) + (5 + \|\hat{W}_j\|_F)(3 + 2.5\|e_j\|).$$

Also, the following robotic parameters are considered for the leader and its followers in both scenarios: m=5 kg, I = 3 kg^2, R=.175 m, r = 0.08 m, and d=0.45 m. In both scenario's, the

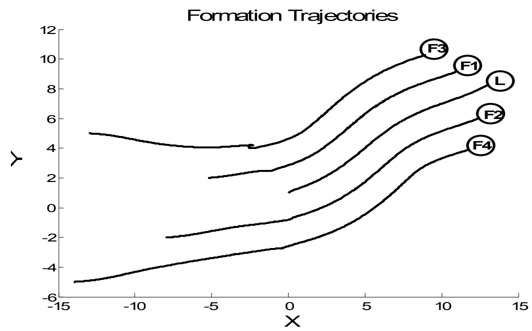

Figure 1: Formation Trajectories

leader follows a virtual robot traveling at a constant linear velocity and a time varying angular velocity of

$$\omega_{ir} = .1\cos(.75t)$$

The formation trajectories are similar under both scenarios and can be seen in Figure 1. It is apparent both controllers are able to learn the full dynamics of the robots; however, the advantages of the NN/RISE controller over the NN controller of [10] become clear when examining the formation errors during steady state conditions, where the strength of AS over UUB is revealed.

Figures 2 and 3 show the formation errors for Followers 2 and 4 after 3 seconds. In each case, the NN/RISE control performs better than the NN controller alone and achieves smaller formation errors. The NN controller alone experiences fluctuations in the region near the origin. Furthermore, the formation errors for follower 4 are larger than the formation errors for follower 2 for the NN only controller, which is evidence of error propagation. This trend is similar for followers 1 and 3 although not shown. The formation errors for the NN/RISE controller on the other hand, do not appear to suffer from this problem. In a formation where follower j is the leader to follower $j+1$, it is important that fluctuations like these are not present, so that formation errors do not propagate throughout the entire formation. These fluctuations can also be troublesome when the formation is maneuvering around tight spaces.

IV. Conclusions

An asymptotically stable NN tracking controller for leader-follower based formation control was presented that considers the dynamics of the leader and the follower using backstepping with RISE feedback. The feedback control scheme is valid even when the dynamics of the followers and their leader are unknown since the NN learns them all online. Numerical results were presented and the asymptotic stability of the system was verified. Simulation results verify the theoretical conjecture and reveal the strength of asymptotic stability over the common result of most NN literature, UUB.

V. References

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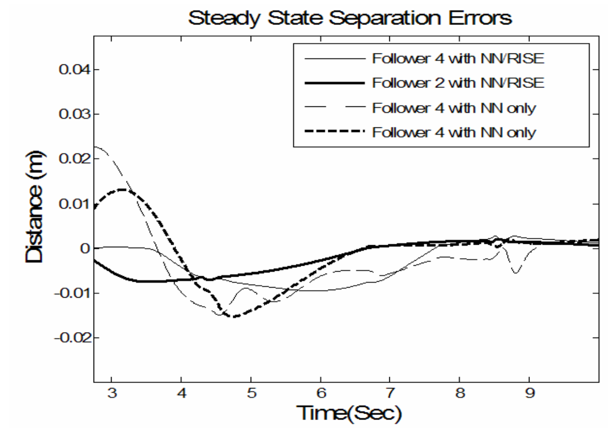

Figure 2: Followers 2 and 4 Separation Errors

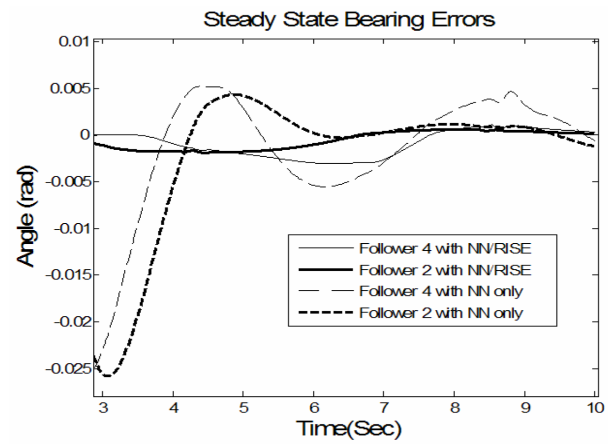

Figure 3: Followers 2 and 4 Bearing Errors

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VI. Appendix

Due to length constraints, the Appendix has been posted on: (URL: http://www.umr.edu/~tad5x4/NNRISE_APP.pdf)