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Modeling of Capacitor Impedance in Switching Converters

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***Abstract*—Switched capacitor (SC) converters are gaining acceptance as alternatives to traditional, inductor-based switching power converters. Proper design of SC converters requires an understanding of all loss sources and their impacts on circuit operation. In the present work, an equivalent resistance method is developed for analysis, and equivalent resistance formulae are presented for various modes of operation. Quasiresonant converters are explored and compared to standard SC converters. Comparisons to inductor-based switching power converters are made. A number of capacitor technologies are evaluated and compared for applications to both SC converters and inductor-based converters. The resulting model can be used to accurately predict and optimize converter performance in the design phase.**

***Index Terms*—Capacitor model, converter model, quasiresonant, switched capacitor.**

I. Introduction

SWITCHED capacitor (SC) converters have been gaining popularity recently as their power capabilities have increased [1] and as needs for simple low-power conversion functions continue to grow. In order to achieve maximum performance, an understanding of the basic efficiency mechanisms is crucial. To generalize, SC converters are characterized by a voltage transfer ratio, often an integer but always a rational number, that is determined by the topology. Additionally, they have an equivalent resistance determined by the operating frequency and the choice of components that captures the converter's ability to transport power [2]. The equivalent resistance of an SC converter can be large compared to effective impedances of inductor-based converters.

Two methods are used for output voltage control of SC converters. One is to vary the duty ratio. In an uncontrolled converter, two devices switch in complement. To control the output, one switch is fixed at or near 50% duty cycle, while the other switch has a variable duty cycle [3], [4]. This ultimately increases the equivalent resistance and alters the output in a manner akin to a linear power supply. A second, more advanced method based on current sources (controlled transistors) is required for higher power levels [5].

Proper design of an SC converter requires an understanding of the technology tradeoffs. Appropriate choice of capacitor technology is essential, as is proper switching element selection. Various technologies are listed in Table I. For example, aluminum electrolytic capacitors provide high capacitance per unit volume up to hundreds of volts at a low cost. Unfortunately, they exhibit self-resonance at a few kilohertz, and thus have limited application in SC circuits switching at 100 kHz or more. Better technology is essential for high-power converters, such as in [6]. An attractive alternative is to operate a converter in quasiresonance. In this case, a typical SC topology is used, but the capacitor is augmented with inductance to form a series LC tank. In [7], a quasiresonant converter is shown to achieve ideal efficiency in the absence of resistance, while an SC converter is always limited by capacitor impedance. This approach was explored in more depth in [8].

SC converters amount to capacitors excited by voltage sources. An inductor-based converter incorporates capacitors excited by current sources. Sufficient analysis is performed below to draw comparisons. A simple single-resistor model is used, rather than a more complex model such as given in [9], in order to mirror the SC work. The focus of the present work is power conversion. Previous work [10]–[12] has focused on analog filter applications or on small-signal dynamics.

II. SC Converter Model and Equivalent Resistance

A simplified SC converter is shown in Fig. 1. For most development, $L = 0$; for a quasiresonant converter, L has some nonzero value. More sophisticated converters are generally composed of multiple building blocks equivalent to this. The experimental results given below are for the voltage doubler shown in Fig. 2, in which each lumped switch of Fig. 1 is realized with two MOSFET's and V_1 and V_2 are connected in series to give an open-circuit voltage transfer ratio of 2. Moving the source and load terminals and ground can generate other SC converters, such as a voltage inverter. More complicated converters, such as a Cockroft–Walton bridge, are composed of many of these fundamental cells in series and/or parallel. Only resistance and capacitance are included in the SC model; inductance is considered below in the quasiresonant case. Also, all resistances are lumped, e.g., R_1 might represent the effective sum of multiple MOSFET on-state resistances, source resistance, and wiring or circuit board trace resistance. The capacitor voltage, V_c, is not measurable but is necessary for the modeling process. The switching frequency is $f = 1/T$.

The idealized capacitor C is charged during one half-cycle and discharged during the other half-cycle, following exponential curves. In periodic steady-state, V_c alternates between V_{c1} and V_{c2}, where V_{c1} is the voltage at the end of the charge half-

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TABLE I
CAPACITORS EVALUATED

Technology	*Value (μF)*	*ESR (mΩ)*	*τ (μs)*
Electrolytic	180	108	19.4
Os-Con	180	19.8	3.56
Ceramic	30	9.7	0.29
Polyester	2.2	79	0.17
Tantalum	68	224	15.2

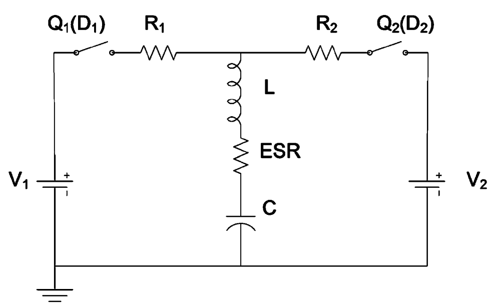

Fig. 1. Switched capacitor (when $L = 0$) or quasiresonant converter.

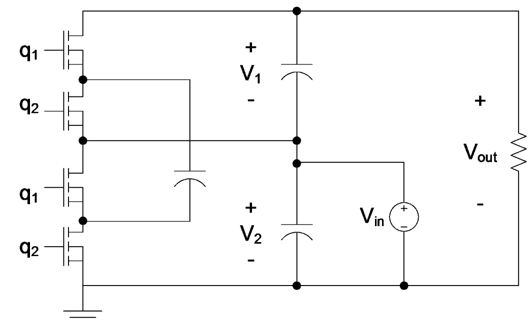

Fig. 2. Experimental voltage doubler.

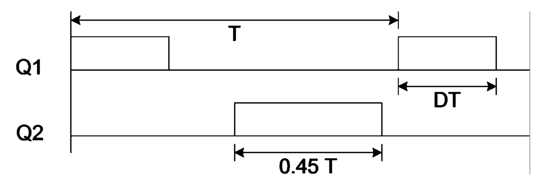

Fig. 3. Waveforms for duty-cycle-controlled SC converter.

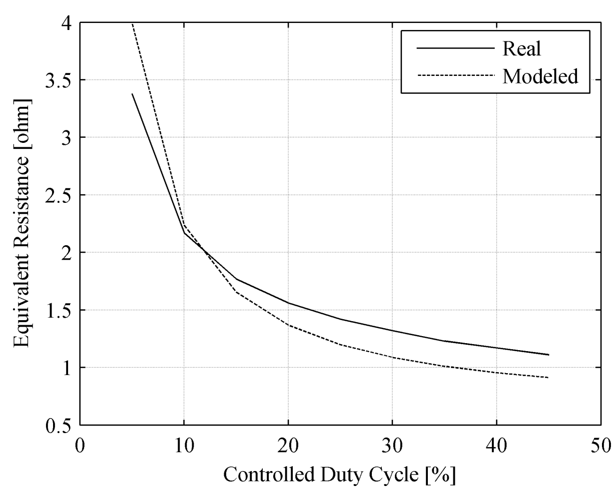

Fig. 4. Duty-cycle-controlled SC converter.

cycle and V_{c2} is the voltage at the end of the discharge half-cycle. The voltage difference becomes

$$V_{c1} - V_{c2} = (V_1 - V_2)\frac{\left(\exp\left(\frac{D_1T}{\tau_1}\right) - 1\right)\left(\exp\left(\frac{D_2T}{\tau_2}\right) - 1\right)}{\left(\exp\left(\frac{D_1T}{\tau_1} + \frac{D_2T}{\tau_2}\right) - 1\right)}$$
$$\tau_1 = (R_1 + \mathrm{ESR})\,C$$
$$\tau_2 = (R_2 + \mathrm{ESR})\,C. \tag{1}$$

The charge exchanged each cycle is

$$q = C(V_{c1} - V_{c2}) \tag{2}$$

and the current delivered is $i = fq$. Together, these expressions represent an *equivalent resistance*, given by

$$R_{eq} = \frac{V_1 - V_2}{i} = \frac{1}{fC}\frac{\left(\exp\left(\frac{D_1T}{\tau_1} + \frac{D_2T}{\tau_2}\right) - 1\right)}{\left(\exp\left(\frac{D_1T}{\tau_1}\right) - 1\right)\left(\exp\left(\frac{D_2T}{\tau_2}\right) - 1\right)}. \tag{3}$$

For duty cycle control, typically T is fixed, D_2 is fixed at or near 50%, capacitor and resistor values are fixed by construction, and D_1 is varied to control the output voltage. This result is similar to that of [13]. Typical duty-cycle-controlled switching waveforms are shown in Fig. 3.

Fig. 4 shows the effect of D_1 on the equivalent resistance of a typical converter. The converter is the voltage doubler of Fig. 2. The upper switches are IRF5210 p-channel MOSFET's, the lower switches are IRF520 n-channel MOSFET's, and the capacitors are each an organic semiconductor aluminum electrolytic (Os-Con), 180 μF, 20 V type. Experimental and theoretical results are overlayed. The converter is switching at 10 kHz, which is below self-resonance but above the point where an idealized capacitor model sufficiently describes the results.

In an unregulated SC converter, $D_1 = D_2 = D \approx 50\%$ (although always less than 50% to prevent shoot-through). Resistances are typically designed to be nearly equal, so that $\tau_1 \approx \tau_2 = \tau = RC$. Substituting and simplifying yields

$$R_{eq} = \frac{V_1 - V_2}{i} = \frac{1}{fC}\frac{1 + \exp\left(\frac{DT}{\tau}\right)}{1 - \exp\left(\frac{DT}{\tau}\right)}. \tag{4}$$

This form is useful for design purposes and for limit studies, as follows.

III. QUASI-RESONANT CONVERTERS

For some applications requiring a voltage doubler function, a quasiresonant converter might be appropriate. The analysis below assumes that the converter is symmetric and is operated open-loop but with the frequency adjusted to match resonance. Quasiresonance means that each half-cycle results in a complete half-sinusoid but that there are dead times between the two active half-cycles. The circuit to be analyzed is shown in Fig. 1, with $L > 0$.

Fourier analysis is applied to determine equivalent resistance. The applied voltage is a quasisquare wave with magnitude $(V_1 - V_2)/2$. The fundamental component is

$$b_1 = \frac{2}{\pi}\sin(\pi D)(V_1 - V_2). \tag{5}$$

If the quality factor $Q = (2\pi f L/R)$ is high, the fundamental current dominates. Quasiresonance requires operating the circuit at

$$f = \frac{\pi}{D}\sqrt{LC}. \tag{6}$$

Based on the charge delivered to the capacitor at this frequency, the equivalent resistance is found to be

$$R_{eqQR} = \frac{\pi^2 R}{2\sin(\pi D)}. \tag{7}$$

This form can be used for comparison to SC converters.

IV. Capacitors in Inductor-Based Converters

In inductor-based power converters, capacitor current takes one of two forms. In some situations, the capacitor simply acts as a filter, and the current is a small triangle corresponding to inductor current ripple. In more demanding situations, such as the output of a boost converter, the current in the capacitor is approximately a square wave. For these applications, equivalent resistance can be analyzed by representing the current in the capacitor as a positive value I_p for part of the cycle DT, then a negative value I_n for the remainder of the cycle $(1-D)T$. In steady state, a charge balance is required, and

$$I_n = I_p \frac{D}{D-1}. \tag{8}$$

The voltage on the ideal capacitor is proportional to the integral of the current and therefore is a triangle wave. The voltage across the equivalent series resistance (ESR) will be proportional to the current, a square wave known as ESR jump. The resulting voltage across the real capacitor will be the sum, a trapezoidal wave. In terms of I_p, the maximum positive voltage excursion is

$$V_{\max} = I_p\left(R + \frac{DT}{2C}\right). \tag{9}$$

The maximum negative voltage excursion is

$$V_{\min} = I_p\left(\frac{D}{D-1}R - \frac{DT}{2C}\right). \tag{10}$$

The current and voltage waveforms are shown in Fig. 5. The peak-to-peak voltage ripple is

$$V_{pp} = I_p\left(\frac{R}{1-D} + \frac{DT}{C}\right). \tag{11}$$

Equating (11) to the peak-to-peak voltage ripple of a resistor in the same circuit location gives the equivalent resistance of the real capacitor:

$$R_{eqL} = R + \frac{D(1-D)}{fC}. \tag{12}$$

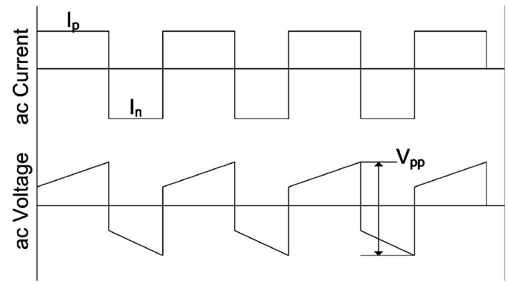

Fig. 5. Example waveforms for an inductor-based converter, such as a boost converter.

Comparison of (12) to (4) and (7) shows that a capacitor in an inductor-based converter is a much simpler device. Capacitive impedance and resistance effectively add, whereas SC and quasiresonant converters have transcendental relationships that involve all circuit parameters.

V. Limit Studies

A designer might consider the minimum equivalent resistance achievable for a given choice of components. In the limits of either high frequency or high capacitance, capacitive impedance goes to zero, so one expects resistance to dominate. The equivalent value becomes

$$R_{eq,\lim} = \lim_{f\to\infty} R_{eq} = \lim_{C\to\infty} R_{eq} = \frac{2R}{D}. \tag{13}$$

Since the maximum duty ratio is $D = 50\%$, the minimum equivalent resistance is 4R.

The limit in [7] for the equivalent resistance of a zero-parasitic-resistance SC converter was:

$$R_{\text{ideal}} = \frac{1}{fC}. \tag{14}$$

A design parameter may be the frequency at which $R_{eq} = kR_{\text{ideal}}$. This establishes the frequency at which parasitic resistances become more important than capacitance:

$$f = \frac{D}{\tau \ln\left(\frac{k+1}{k-1}\right)}. \tag{15}$$

For $k = 2$, $f = (D/1.1\tau)$. Clearly the required operating frequency is related to the circuit time constant, which is therefore important when choosing capacitor technology.

Depending on capacitor technology, the circuit may not operate properly at very high frequencies, so other frequency requirements should be considered. Although the model development assumes that series inductance is small for SC converters, many capacitor types, particularly electrolytic variants, have substantial inductance. ESR is also a nonlinear function of frequency. A reasonable design parameter is the frequency necessary to achieve some $R_{eq} = kR_{eq,\lim}$, where $k > 1$ is held relatively low. Given the transcendental nature of the problem, no analytical solution exists for generic k. For $k = 2$, a numerical solution is

$$f = \frac{3.83D}{\tau}. \tag{16}$$

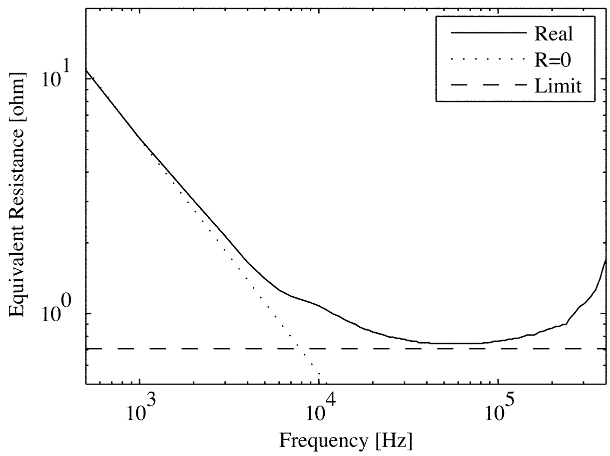

Fig. 6. Experimental, ideal, and limiting performance with Os-Con capacitors.

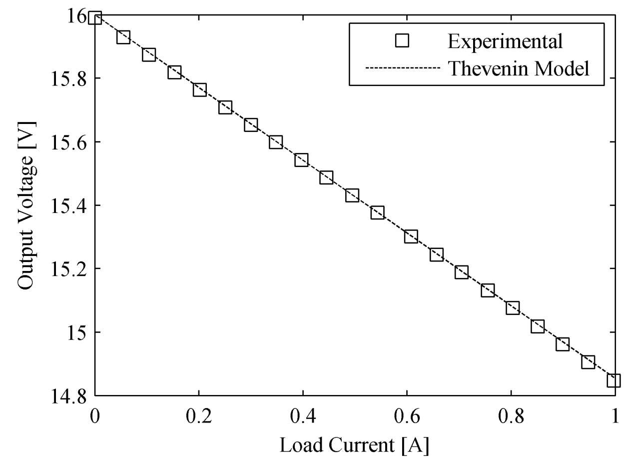

Fig. 7. Experimental and modeled load lines.

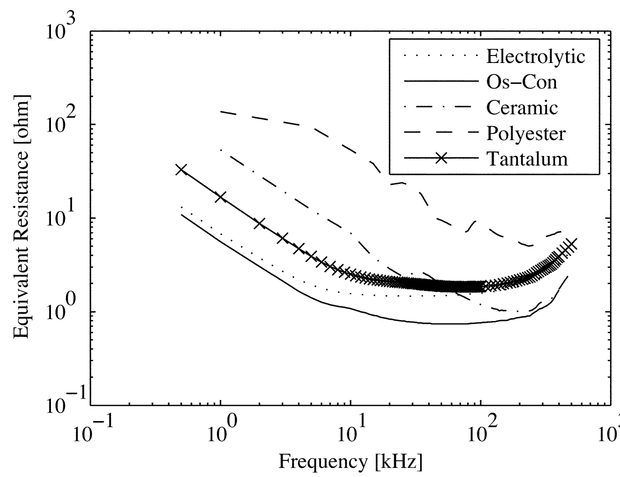

Fig. 8. Comparison of capacitor technologies.

Again, operating frequency is related to the circuit time constant. Gating power may also increase at higher frequencies. This would add an additional effective loss not represented in (4).

A typical SC converter may operate at 45% duty cycle to avoid shoot-through conditions. Its limiting equivalent resistance is then $4.44R$. A quasiresonant converter operated at 45% duty cycle and the proper frequency has an equivalent resistance of $4.996R$. Choosing between the two converter types is not obvious. On the one hand, a simple SC converter has greater potential for low equivalent resistance, particularly considering the series resistance introduced by the inductor in a quasiresonant converter. SC converters are simple to operate and require no frequency tracking to achieve the desired performance. On the other hand, the full potential of an SC converter requires high capacitance or high switching frequency. Considering practical circuit limitations, a properly-operated quasiresonant converter can be expected to more closely approach the predicted limit on performance.

VI. Experimental Results and Discussion

Experiments were performed to verify the model of (4). A voltage doubler was constructed using the 180 μF Os-Con capacitor described above, as in Fig. 4. Fig. 6 shows the experimental equivalent resistance, the ideal equivalent resistance as given by (14), and the limiting equivalent resistance given by (13). At low frequencies, the performance tracks the ideal case. As frequency increases, the limiting performance is approached. At high frequencies, performance degrades, most likely because of inductance effects that were not modeled. Inductance reduces the capacitor charge rate and increases equivalent resistance above the resonant frequency. In general, a designer should operate an SC converter somewhat above the intersection of the two limiting curves. At higher frequencies, the performance increase diminishes as the equivalent resistance approaches the asymptote of (13).

Variable duty cycle was explored using the same voltage doubler and (1). Resulting equivalent resistance is shown in Fig. 4. The agreement between the model and experimental data verifies that the duty cycle control methods adjust the equivalent resistance and rely on the load to reduce the output voltage. To be useful, the model should also predict large-signal characteristics, such as a load line. For the same voltage doubler, operating at fixed 45% duty cycle, load current was swept from 0 A to 1.0 A. The resulting load line is shown in Fig. 7. This approximates the implied Thevenin model, a controlled voltage source determined by the topology $(V_{th} = 2V_{in})$ in series with an equivalent resistance determined by component choices (1.15 Ω for the experimental circuit). The efficiency is approximately 92.8% at maximum load.

A designer may choose among a number of capacitor technologies. For this work, the capacitors in Table I were tested. All are approximately the same size. The resulting equivalent resistances in fixed duty cycle voltage doublers are shown in Fig. 8. The expected effects from varying resistance, capacitance, and circuit time constant are demonstrated. In this case, the organic semiconductor aluminum electrolytic capacitors performed best. MOSFET resistance (a total of 137 $\mathrm{m}\Omega$) was a significant contributor to total resistance when better capacitors are used, such as organic semiconductor electrolytic or ceramic. Polyester film capacitors displayed erratic behavior, likely due to the low capacitance available.

Inductor-based converters were evaluated to verify. For the same capacitor explored in SC converters, equivalent resistance

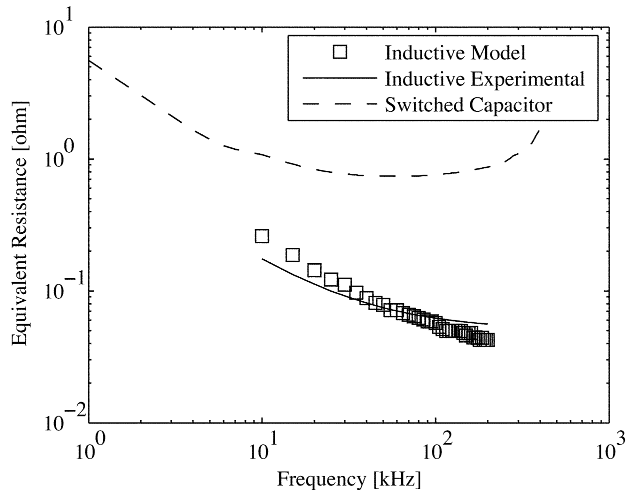

Fig. 9. SC and inductor-based converters.

is shown in Fig. 9, along with the model prediction. For comparison, the SC experimental data are also included. Notice the much lower equivalent resistance, about an order of magnitude, when a capacitor is driven by a lossless current source than when driven by a voltage source. SC converters are affected by all loss sources in a nonlinear fashion, while inductor-based converters are affected primarily by ESR in a linear way.

VII. Conclusion

Models for capacitors in three circuit types were developed and verified: switched capacitor, quasiresonant, and inductor-based power converters. Performance limits were explored as well as duty-cycle-based control of SC converters. Experimental data support the models and provide insights into cost/performance trade-offs. The experimental voltage doubler demonstrates that properly designed SC converters can reach low equivalent resistance at low switching frequency. If all loss sources are considered, converter performance can be predicted and optimized accurately in the design phase.

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