

01 Jan 2007

## Channel Estimation and Phase-Correction for Robust Underwater Acoustic Communications

Y. Rosa Zheng

Missouri University of Science and Technology, zhengyr@mst.edu

Follow this and additional works at: [https://scholarsmine.mst.edu/ele\\_comeng\\_facwork](https://scholarsmine.mst.edu/ele_comeng_facwork)



Part of the [Electrical and Computer Engineering Commons](#)

---

### Recommended Citation

Y. R. Zheng, "Channel Estimation and Phase-Correction for Robust Underwater Acoustic Communications," *Proceedings of the IEEE Military Communications Conference, 2007. MILCOM 2007*, Institute of Electrical and Electronics Engineers (IEEE), Jan 2007.

The definitive version is available at <https://doi.org/10.1109/MILCOM.2007.4454811>

This Article - Conference proceedings is brought to you for free and open access by Scholars' Mine. It has been accepted for inclusion in Electrical and Computer Engineering Faculty Research & Creative Works by an authorized administrator of Scholars' Mine. This work is protected by U. S. Copyright Law. Unauthorized use including reproduction for redistribution requires the permission of the copyright holder. For more information, please contact [scholarsmine@mst.edu](mailto:scholarsmine@mst.edu).

# CHANNEL ESTIMATION AND PHASE-CORRECTION FOR ROBUST UNDERWATER ACOUSTIC COMMUNICATIONS

Yahong Rosa Zheng

Dept. of ECE, University of Missouri-Rolla,  
MO 65409, USA, Email:zhengyr@umr.edu

*Abstract*—This paper presents a new channel estimation, equalization, and phase correction scheme to combat the convergence and stability problem encountered by time-domain adaptive equalizers in underwater acoustic communication systems. Large Doppler spread and symbol scaling in underwater channels have been challenging problems causing significant phase drift and performance degradation. Our new method targets this problem by first allowing phase errors in the estimation of the fading channel coefficients and then perform group-wise (rather than symbol-wise) phase estimation and correction after equalization and multiple channel combining. Single transmitter and multiple receiver data obtained through ocean experiments have been processed using the proposed method and the results show that the new methods can achieve Bit Error Rate (BER) on the order of  $10^{-4}$  with very high stability.

## I. INTRODUCTION

Shallow water horizontal communication channels are extremely hostile for high data rate underwater communications [1], [2]. There are two major obstacles: one is the excessive multipath delay spread in a medium range shallow water channel which is usually on the order of 10–50 ms and causes the intersymbol interference (ISI) to extend over 20–300 symbols at a data rate of 2–10 kilosymbols per second. Another obstacle is the severe Doppler shift and Doppler spread due to relative motion between the source (transducers) and receiver (hydrophones), dynamic motion of water mass, and varying sound speed, etc. A few Hz of Doppler in underwater channels can be very significant because the velocity of acoustic propagation in water is only about 1500 m/s in comparison to  $3 \times 10^8$  m/s for Radio Frequencies (RF) in air. The ratio of Doppler to carrier frequency in underwater channels is in the order of  $10^{-3}$  to  $10^{-4}$ ; while the ratio in RF

wireless channels is in the order of  $10^{-7}$  to  $10^{-9}$ . The significant Doppler shift and Doppler spread cause not only rapid fluctuation in the fading channel response but also compression or dilation of signal waveforms. These two obstacles make the coherent receiver of underwater communication systems much more complex [3]–[12] than those in RF systems.

It has been successfully demonstrated in [3] that coherent detection of underwater acoustic communications can be realized by joint decision feedback equalization (DFE) and phase synchronization that employs a phase-locked loop (PLL) or delay-locked loop (DLL). However, the DFE and PLL/DLL interact in a nonlinear fashion and in a symbol-by-symbol basis, therefore it requires careful selection of the number of equalizer taps and tuning of the equalizer and PLL/DLL coefficients. Stable and robust operation of this time-domain equalizer is sometimes difficult to obtain in different channel conditions. Recent improvement on robust time-domain DFE has been reported in [12] using a fixed set of parameters at a cost of slightly degraded bit error performance. An alternate method is the passive phase conjugate equalizer [4], [11], which also employs symbol-by-symbol phase correction. The robustness, stability and bit error rate (BER) of passive phase conjugate equalizers are similar to the time-domain DFE with PLL/DLL phase compensation.

To combat the phase drift and rotation problem, we propose a new time-domain channel estimation, equalization, and phase correction scheme for single-input multiple-output (SIMO) systems where one transmit transducer and multiple receive hydrophones are used. The new scheme estimates the acoustic fading channel without separating the phase drift and phase rotation for each symbol. Then the SIMO receive signals are

equalized and combined. Finally the phase drift/rotation of symbols is corrected per group of symbols using estimated average phase drift/rotation. This scheme differs from existing schemes that perform phase compensation symbol-by-symbol. Results obtained by processing ocean experiment data show that the new scheme provides better robustness against channel and phase estimation errors. It achieves BER on the order of  $10^{-4}$  which is much better than existing methods.

## II. THE SYSTEM MODEL

Consider an underwater acoustic communications system using a single transducer source and  $M$  hydrophone receivers. The baseband equivalent signal received at the  $m$ -th hydrophone can be expressed in the discrete-time domain form as

$$y_m(k) = \sum_{l=1}^L h_m(l, k) x(k+1-l) e^{j(2\pi f_{m,0} kT + 2\pi f_{m,k} kT + \theta_{m,0})} + v_m(k) \quad (1)$$

where  $k$  is the time index,  $T$  is the data symbol interval,  $x(k)$  is the transmitted data symbol or pilot symbol,  $h_m(l, k)$  is the impulse response of the frequency-selective, time-varying fading channel with length  $L$  in terms of  $T$ ,  $f_{m,0}$  and  $f_{m,k}$  are the carrier frequency offset (CFO) or the average Doppler shift and the instantaneous Doppler spread, respectively. The phase  $\theta_{m,0}$  denotes the phase error after coarse synchronization and  $v_m(k)$  is the white Gaussian noise.

If the Doppler shift  $f_{m,0}$  is significant, then it causes the received signal  $y_m(k)$  to be time-scaled (compressed or dilated) [7]. In this case, re-scaling and re-sampling are required before equalization takes place to cancel ISI. If the relative motion between the source and receiver is insignificant, then the Doppler shift  $f_{m,0}$  is close to zero, but the instantaneous Doppler spread  $f_{m,k}$  is a zero-mean time-varying random variable which causes significant phase drift or rotation of the received symbols. It is found that the fading channel coefficients  $h_m(l, k)$  usually change much slower than the instantaneous phase  $2\pi f_{m,k} kT$  in many practical underwater acoustic channels [3], [4]. Therefore, it is the phase that causes major problem in channel equalization and coherent detection.

## III. CHANNEL ESTIMATION AND EQUALIZATION

Channel estimation is achieved in the training mode by using pilot symbols. If the multipath channel length

is  $L$ , then at least  $2L - 1$  pilot symbols are needed for accurate channel estimation [13]. When the channel length is less than 40 taps and the channel impulse response time duration is much less than the channel coherence time, the phase drift is approximately a constant over the block of pilot symbols and the channel impulse response is assumed time-invariant within the block. The channel impulse response is then estimated by

$$\tilde{\mathbf{h}} = e^{-j\phi_m} \hat{\mathbf{h}}_m = \mathbf{P}^\dagger \mathbf{y}_m + \mathbf{v}_m \quad (2)$$

where  $\phi_m$  is an unknown phase drift,  $\mathbf{v}_m$  is the noise vector at the  $m$ -th hydrophone, and  $\mathbf{P}$  is the pilot symbol matrix

$$\mathbf{P} = \begin{bmatrix} p(L) & \cdots & p(2) & p(1) \\ p(L+1) & \cdots & p(3) & p(2) \\ \vdots & \ddots & \ddots & \vdots \\ p(2L-1) & \cdots & p(L+1) & p(L) \end{bmatrix}. \quad (3)$$

$$\hat{\mathbf{h}}_m = \begin{bmatrix} \hat{h}_m(1,1) \\ \hat{h}_m(2,1) \\ \vdots \\ \hat{h}_m(L,1) \end{bmatrix}, \quad \mathbf{y}_m = \begin{bmatrix} y_m(L) \\ y_m(L+1) \\ \vdots \\ y_m(2L-1) \end{bmatrix} \quad (4)$$

With the estimated channel coefficient vector  $\tilde{\mathbf{h}}_m$ , a finite-length linear equalizer for the  $m$ -th hydrophone is usually designed by the minimum mean square error (MMSE) criterion, as detailed in Section 10-2 of [14]. However, the underwater channel coefficient vector  $\tilde{\mathbf{h}}_m$  contains an unknown phase drift  $\phi_m$  which is very large and causes instability of the equalizer. Instead of using PLL/DLL with symbol-by-symbol phase compensation [3] or passive phase conjugate [4], we equalize each channel first, then combine the SIMO data outputs, and then compensate the phase drift at last. The equalized  $k$ -th symbol at the  $m$ -th channel is given by

$$\tilde{x}(k) = \alpha_m e^{j(2\pi f_{m,k} kT + \theta_{m,0} + \phi_m)} x(k) + \tilde{v}_m(k) \quad (5)$$

The equalized  $M$  hydrophone outputs are combined to yield

$$\begin{aligned} \hat{x}(k) &= \left[ \sum_{m=1}^M \alpha_m e^{j(2\pi f_{m,k} kT + \theta_{m,0} + \phi_m)} \right] x(k) + \hat{v}(k) \\ &= |\beta_k| e^{j\angle\beta_k} x(k) + \hat{v}(k) \end{aligned} \quad (6)$$

where  $\beta_k = \sum_{m=1}^M \alpha_m e^{j(2\pi f_{m,k} kT + \theta_{m,0} + \phi_m)}$ .

From (6), we can conclude that the complex-valued symbol-wise scaling factor  $\beta_k$  is actually a diversity combining factor determined by the  $M$  channel transfer functions, time-varying Doppler spreads and timing-error phases. In other words, the equalized data symbol  $\hat{x}(k)$  is an amplitude-scaled and phase-rotated version of the transmitted data symbol  $x(k)$ . The rotating phase  $\angle\beta_k$  is a collection of all the contributions from the instantaneous Doppler spreads  $f_{m,k}$  and timing-error phases  $\theta_{m,0}$  of all the  $M$  fading channels. For each individual fading channel, the rotating phase  $\angle\beta_k = 2\pi f_{m,k}kT + \theta_{m,0} + \phi_m$ , which represents the  $m$ -th channel's Doppler-driven shifting phase, timing-error phase, and the channel transfer function effect. This is certainly a clear physical interpretation for the time-domain equalized data.

If  $x(k)$  is phase shift keying (PSK) modulated data, then the time-varying rotating phase  $\angle\beta_k$  must be compensated at the receiver after the linear equalizer and before demodulation and detection. This is discussed in detail in the next section.

#### IV. PHASE-COHERENT DETECTION

In this section, we present a new algorithm for estimating the phase  $\angle\beta_k$ , which is crucial for successful data detection of PSK modulated symbols. The challenge of this phase estimation is that we have  $M$  sub-channels each having different timing-error phase and time-varying Doppler spread. the rotating phase  $\angle\beta_k$  represents a nonlinearly composed effect of these random (or time-varying) factors of all the  $M$  fading channels. Therefore, directly estimating these Doppler spreads and timing-error phases will be very costly if at all possible.

In the literature of underwater acoustic communications, phase tracking is commonly carried out by utilizing first-order or second-order phase-locked loop or delay-locked loop [3],[4], and it is often jointly done with decision feedback equalizers. However, this approach is sensitive to channel conditions [12], [11] and background noises.

We know from the nature of ocean waters that the instantaneous Doppler spreads  $f_{m,k}$  changes gradually from time to time, rather than changing arbitrarily. Therefore, the rotating phase  $\angle\beta_k$  is also changing gradually with time. We treat  $\angle\beta_k$  to be a constant for a small number of  $N_s$  consecutive symbols and adjust the phase compensation for every group of  $N_s$  symbols.

Let  $\psi_p$  denote the estimated phase for the  $p$ -th group of  $\{\angle\beta_{(p-1)N_s+1}, \angle\beta_{(p-1)N_s+2}, \dots, \angle\beta_{(p-1)N_s+N_s}\}$ , with  $p = 1, 2, \dots, N_g$ , where  $N_g$  is the total number of groups. Let  $\psi_0$  denote the initial phase,  $\Delta\psi_p$  the phase difference  $\psi_p - \psi_{p-1}$ . Hence

$$\psi_p = \psi_{p-1} + \Delta\psi_p, \quad p = 1, 2, \dots, N_g. \quad (7)$$

For M-ary PSK (MPSK) modulation, we define a phase quantization function  $\mathbb{Q}[\phi]$  as follows

$$\mathbb{Q}[\phi] = \frac{i2\pi - \pi}{M}, \quad \frac{(i-1)2\pi}{M} \leq \phi < \frac{i2\pi}{M} \quad (8)$$

$$i = 1, 2, \dots, M.$$

Our proposed group-wise phase estimation and compensation algorithm is presented in several steps:

**Step 1.** Designate the first  $N_{ts}$  symbols  $\{x(k)\}_{k=1}^{N_{ts}}$  in each transmitted block data  $\mathbf{x}$  as the training symbols for phase reference and determine the initial phase  $\psi_0$  by

$$\psi_0 = \frac{1}{N_{ts}} \sum_{k=1}^{N_{ts}} \angle\hat{x}(k) - \angle x(k). \quad (9)$$

Set  $p = 1$  for the next step.

**Step 2.** Compensate the phase of the  $p$ -th group data by  $e^{-j\psi_{p-1}}$ , yielding

$$\tilde{x}_p(k) = \hat{x}((p-1)N_s + k)e^{-j\psi_{p-1}}, \quad k = 1, 2, \dots, N_s. \quad (10)$$

**Step 3.** Calculate the individual phase deviation from its nominal phase of each symbol in the  $p$ -th group

$$\varphi_{p,k} = \angle\tilde{x}_p(k) - \mathbb{Q}[\angle\tilde{x}_p(k)], \quad k = 1, 2, \dots, N_s. \quad (11)$$

**Step 4.** Calculate the average phase deviation and estimate the rotating phase for the  $p$ -th group as below

$$\Delta\psi_p = \frac{1}{N_s} \sum_{k=1}^{N_s} \varphi_{p,k} \quad (12)$$

$$\psi_p = \psi_{p-1} + \Delta\psi_p. \quad (13)$$

**Step 5.** Increment  $p$ , repeat Step 2 – 4 till  $p = N_g$ . Note that the  $N_g$ -th group may have less than  $N_s$  symbols. If this is the case, then the calculation needs to be carried out based on the actual number of symbols in the  $N_g$ -th group.

After estimating the  $N_g$  group phases, we can compensate the phase rotation of the equalized data  $\hat{x}$  on group basis:

$$\check{x}_p(k) = \hat{x}((p-1)N_s + k)e^{-j\psi_p}, \quad \begin{matrix} k = 1, 2, \dots, N_s \\ p = 1, 2, \dots, N_g \end{matrix} \quad (14)$$

Finally, the binary information data of the block can be obtained via standard MPSK demodulation procedure on the phase-compensated signal  $\check{x}_p(k)$  of the block.

It is worth noting that the choice of  $N_s$  symbols in a group needs to satisfy the condition:  $2\pi |f_d| N_s T < \frac{\pi}{M}$ , to ensure that the maximum rotating phase does not exceed a decision region of MPSK, where  $|f_d|$  is the absolute value of the maximum possible Doppler spread. Besides, the equivalent  $M$ -channel composite Doppler spread for the  $p$ -th group can be approximated by  $f_{d,p} = \frac{\Delta\psi_p}{2\pi N_s T}$ .

The advantage of the group-wise estimation of the rotating phase is its insensitivity to noise perturbations because of the averaging operation (12), which is an implicit low-pass filter.

## V. EXPERIMENTAL RESULTS

The proposed time-domain channel estimation and equalization are evaluated using experimental data. The data were collected at Saint Margaret's Bay, Nova Scotia, Canada, in May 2006. Eight hydrophones were arranged unequally spaced over 1.86 meters on a vertical linear array. The array was deployed in water of 30 m depth. The transducer was deployed in water at 21 m depth and 44 m above the bottom. The source and receivers were suspended in the shallow water, and the source-receiver range is 3.06 km. Quadrature Phase Shift Keying (QPSK) signals with a bandwidth of 2 kHz were transmitted on a carrier frequency of 17 kHz.

The QPSK signals were partitioned in packets. Each packet consisted of a probe signal followed by a gap, then the data package followed by another gap. The data package begins with four training symbols for phase reference, followed by the message data of 18910 symbols (9.455 s in duration). The gap after the probe signal is long enough so that the channel impulse response can be estimated from the probe signal. The gap after the message data is sufficiently long for avoiding inter packet interference. The symbol synchronization is carried out by the probe signal, which is an  $m$ -sequence of 511 bits.

The initial channel estimation was also achieved using the probe signal by the proposed time-domain method detailed in Section III. As an example, the time-domain amplitude responses of the eight channels for the first packet is depicted in Fig. 1. Most of the channel energy is concentrated within 15 ms which corresponds

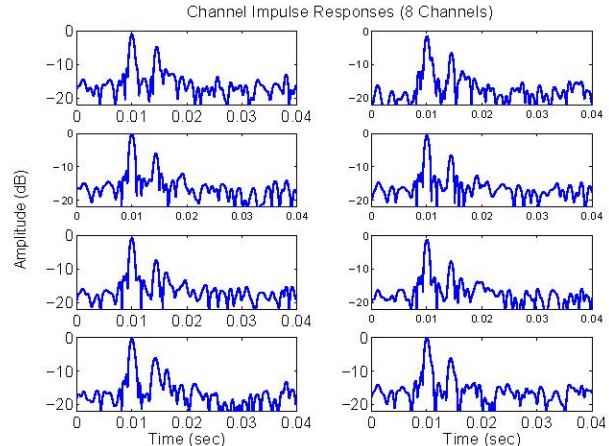


Fig. 1. Channel impulse responses of the eight channels estimated by the probe signal (511-symbol  $m$ -sequence) of the first packet.

to a channel length of  $L = 30$  in terms of symbol-interval.

The equalized data and the four training symbols were then used to estimate the time-varying rotating phases  $\psi_p$  and correct for the phase shift. The equalized and phase-corrected data were demodulated to obtain the binary information bits. Since the channel coherence time is about one second, the channel impulse responses were re-estimated every 0.8 seconds using the detected message data. The scatter plots in Figs. 2 – 4 are the original received baseband signals, the equalized signals (without phase correction), and the phase corrected QPSK signals, respectively. It is clear that the equalizer converted the most of the signals to a correct amplitude but was unable to correct all phase shifts. This results in the “doughnut” shape in the scatter plot due to the phase drift of each symbol. After phase correction, the QPSK signals are well separated into four little “nuggets”.

The average phase drift was estimated from the four training symbols and the equalized signals in the packet. The estimated phase drifts are shown in Fig. 5 for 6 packets. Based on the phase drifts, we can find the time-varying Doppler spreads shown in Fig. 6. It is noted that the isolated spikes are due to the re-estimation of the channel parameters, which leads to a phase jump between currently estimated channel parameters and the previously estimated channel parameters. Although the Doppler spread varies between  $-2.5$  Hz and  $+2$  Hz, its average value in each packet is almost zero. This time-varying Doppler spread will lead to individual

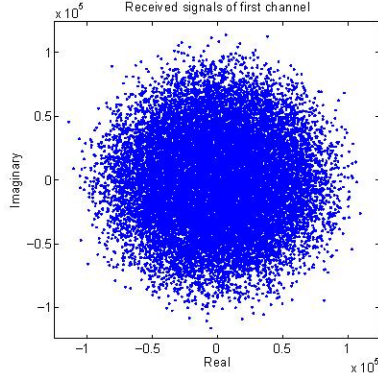


Fig. 2. Scatter plot of received QPSK signals of the first channel in the first packet.

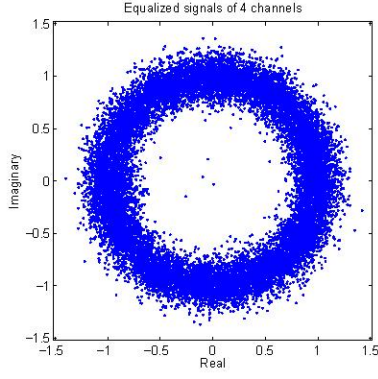


Fig. 3. Scatter plot of equalized QPSK signals using four-channel LE.

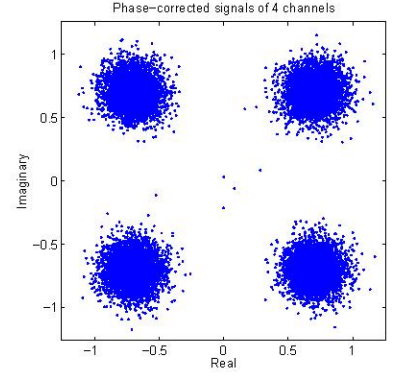


Fig. 4. Scatter plot of equalized and phase-corrected QPSK signals using 4-channel LE.

data symbols to be compressed or dilated. However, the entire packet data does not have visible compression or dilation because the Doppler spread has a zero mean over a packet.

The bit error rate was also evaluated using the demodulated and detected data bits. The scatter plot in Fig. 4 indicates that most of the symbols are properly classified except a few. The calculated numbers of bit errors and the bit error rates (BER) without coding are listed in Table 1 – 3 for SISO,  $1 \times 2$  SIMO, and  $1 \times 4$  SIMO equalizers, respectively. The BERs of the SISO channels vary wildly due to different channel conditions. For stable channels such as channel # 2–5 and 8, the BER is achieved on the order of  $10^{-3}$ . But for the high Doppler channels such as #1, 6, and 7, the BER is as high as 6%. When multiple channel combining is used, the BER performance is improved significantly. Both the  $1 \times 2$  and  $1 \times 4$  SIMO equalizers provide similar performance which is on the order of  $10^{-4}$ . If the phase correction is done less often, then the BER performance degrades accordingly.

## VI. CONCLUSION

A new channel estimation, equalization, and phase correction scheme has been presented for underwater acoustic communication systems. The new scheme estimates the acoustic fading channel without separating the phase drift and phase rotation for each symbol. Then the SIMO receive signals are equalized and combined. Finally the phase drift/rotation of symbols is corrected per group of symbols using estimated average phase drift/rotation. This scheme differs from existing schemes that perform phase compensation symbol-by-

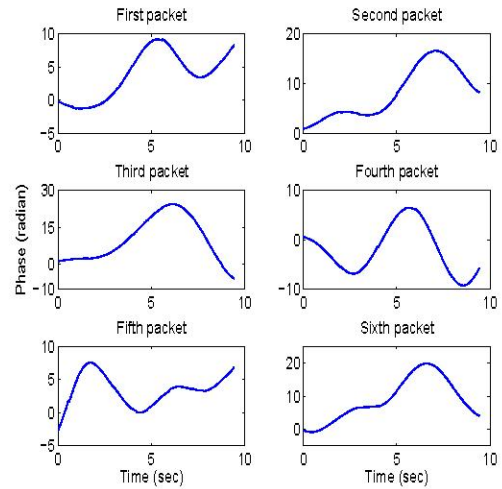


Fig. 5. Estimated phase drift.

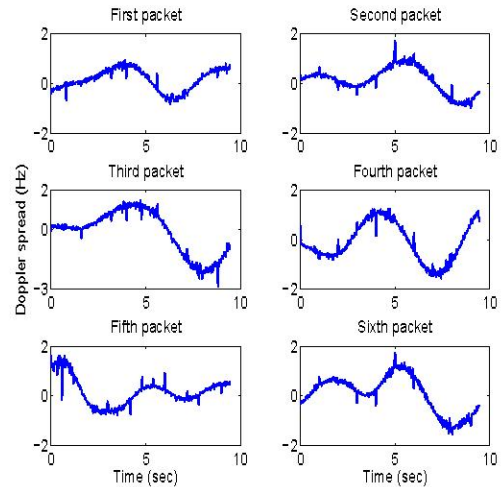


Fig. 6. Estimated Doppler spread.

symbol. Results obtained by processing ocean experiment data show that the new scheme provide better robustness against channel and phase estimation errors. It achieves BER on the order of  $10^{-4}$  which is much better than existing methods.

Table 1: BER of a single channel for the first packet

Channel number	number of message bits	number of bit errors	bit error rate
1	37820	2318	0.0613
2	37820	27	$7.14 \times 10^{-4}$
3	37820	16	$4.23 \times 10^{-4}$
4	37820	76	0.002
5	37820	49	0.0013
6	37820	1554	0.0411
7	37820	2160	0.0571
8	37820	11	$2.91 \times 10^{-4}$

Table 2: BER of two-channel combining in the first packet

Channel numbers	number of message bits	number of bit errors	bit error rate
1, 2	37820	3	$7.93 \times 10^{-5}$
3, 4	37820	6	$1.59 \times 10^{-4}$
5, 6	37820	8	$2.11 \times 10^{-4}$
7, 8	37820	5	$1.32 \times 10^{-4}$

Table 3: BER of 6 packets with four-channel combining

Packet number	number of message bits	number of bit errors	bit error rate
1	37820	4	$1.06 \times 10^{-4}$
2	37820	3	$7.93 \times 10^{-5}$
3	37820	4	$1.06 \times 10^{-4}$
4	37820	5	$1.32 \times 10^{-4}$
5	37820	4	$1.06 \times 10^{-4}$
6	37820	5	$1.32 \times 10^{-4}$

#### ACKNOWLEDGMENTS

The authors thank Dr. T.C. Yang and Dr. Wen-bin Yang of Naval Research Laboratory for providing the ocean experiment data.

This work was supported in part by the Office of Naval Research under Grant N00014-07-1-0219 and in part by University of Missouri Research Board fund.

#### REFERENCES

- [1] D.B. Kilfoyle, and A.B. Baggeroer, "The state of the art in underwater acoustic telemetry," *IEEE J. Oceanic Eng.*, vol.25, pp.4-27, Jan. 2000.
- [2] M. Siderius, M.B. Porter, P. Hursky, and V.K. McDonald, "Measurements and modeling comparisons of underwater communications performance at three shallow-water sites," in *Proc. OCEANS'03*, vol.4, pp.1773-1779, Sept. 2003.
- [3] M. Stojanovic, J. Catipovic, and J. Proakis, "Phase-coherent digital communicaitons for underwater acoustic channels," *IEEE J. Ocean Eng.*, vol.19, pp.100-111, Jan. 1994.
- [4] T. C. Yang, "Differences between passive-phase conjugation and decision-feedback equalizer for underwater acoustic communications," *IEEE J. Ocean Eng.*, vol.29, pp.472-487, April 2004.
- [5] T. C. Yang, "Correlation-based decision-feedback equalizer for underwater acoustic communications," *IEEE J. Ocean Eng.*, vol.30, pp.865-880, Oct. 2005.
- [6] M. Stojanovic, "Low complexity OFDM detector for underwater acoustic channels," in *Proc. Oceans'06*, 2006.
- [7] B. S. Sharif, J. Neasham, O. R. Hinton, and A. E. Adams, "A computationally efficient Doppler compensation system for underwater acoustic communications," *IEEE J. Ocean Eng.*, vol.25, pp.52-61, Jan. 2000.
- [8] T. H. Eggen, A. B. Baggeroer, and J. C. Preisig, "Communication over Doppler spread channels – Part I: channel and receiver presentation," *IEEE J. Ocean Eng.*, vol.25, pp.62-71, Jan. 2000.
- [9] Baosheng Li, Shengli Zhou, M. Stojanovic, and L. Freitag, "Pilot-tone based ZP-OFDM demodulation for an underwater acoustic channel," in *Proc. Oceans'06*, 2006.
- [10] Y. R. Zheng and Chengshan Xiao, "Frequency-domain channel estimation and equalization for broadband wireless communications," *Proc. IEEE Int. Conf. Commun.*, Glasgow, Scotland, June 2007.
- [11] D. Rouseff, D. R. Jackson, W. L. J. Fox, C. D. Jones, J. A. Ritcey, and D. R. Dowling, "Underwater acoustic communication by passive-phase conjugation: theory and experimental results," *IEEE J. Oceanic Eng.*, vol.26, pp.821-831, Oct. 2001.
- [12] T. C. Yang, "Correlation-based decision-feedback equalizer for underwater acoustic communications," *IEEE J. Ocean Eng.*, vol.30, pp.865-880, Oct. 2005.
- [13] J. Wu, C. Xiao, and J. C. Olivier, "Time-varying and frequency-selective channel estimation with unequally spaced pilot symbols," *Int. J. Wireless Information Networks*, vol.11, no.2, pp.93-104, April 2004.
- [14] J. G. Proakis, *Digital Communications*, 3rd Ed., McGraw Hill, 1995.