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### Recommended Citation

Z. Yang et al., "An Improved StatCom Model for Power Flow Analysis," *Proceedings of the IEEE Power Engineering Society Summer Meeting, 2000 (2000, Seattle, WA)*, vol. 2, pp. 1121-1126, Institute of Electrical and Electronics Engineers (IEEE), Jul 2000.

The definitive version is available at <https://doi.org/10.1109/PSS.2000.867536>

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# AN IMPROVED STATCOM MODEL FOR POWER FLOW ANALYSIS

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**Abstract:** The StatCom is traditionally modeled for power flow analysis as a PV or PQ bus depending on its primary application. The active power is either set to zero (neglecting the StatCom losses) or calculated iteratively. The StatCom voltage and reactive power compensation are usually related through the magnetics of the StatCom. This traditional power flow model of the StatCom neglects the impact of the high-frequency effects and the switching characteristics of the power electronics on the active power losses and the reactive power injection (absorption). In this paper, the authors propose a new StatCom model appropriate for power flow analysis derived directly from the dynamic model of the StatCom. The proposed model can therefore account for the high-frequency effects and power electronic losses, and more accurately predict the active and reactive power outputs of the StatCom.

**Keywords:** StatCom, FACTS, load flow, power systems

## I. INTRODUCTION

The STATIC synchronous COMPensator (StatCom) is a main member of the FACTS family of power electronic-based controllers. It has been studied for many years, and is probably the most widely used FACTS device in today's power systems. Many papers have discussed its operating principles, static and dynamic models, control theories and applications [1-5]. Few papers however, address the issue of how to model StatComs for load flow calculations. The StatCom is traditionally modeled for power flow analysis as a PV or PQ bus depending on its primary application. The active power is either set to zero (neglecting the StatCom losses) or calculated iteratively. The StatCom voltage and reactive power compensation are usually related through the magnetics of the StatCom. This traditional power flow model of the StatCom neglects the impact of the high-frequency effects and the switching characteristics of the power electronics on the active power losses and the reactive power injection (absorption).

In a load flow calculation, a StatCom is typically treated as a shunt reactive power controller assuming that the StatCom can adjust its injected reactive power to control the voltage magnitude at the StatCom terminal bus. Fig.1

depicts a StatCom and the traditional simple model used for load flow calculations. Note that specified reactive power load at bus  $i$ ,  $jQ_i$ , is combined with the StatCom reactive power output  $jQ_c$  - and therefore the reactive power varies as  $|V_i|$  varies. This model is essentially a PV bus with the StatCom's active power output set to zero [6]. The primary difficulty with this model is that inaccuracies occur when the device losses (including the losses of the connection transformer and StatCom) are neglected.

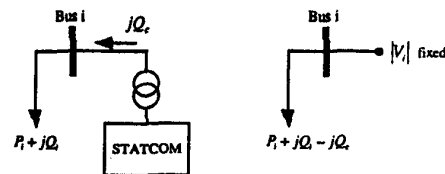


Fig.1 Simple model of a StatCom in load flow calculation

In order to consider the loss of the connection transformer, a modified model is presented (as shown in Fig.2). Note that a new PV bus, bus  $j$ , is added to represent the StatCom's output terminal, while the connection transformer is replaced by its leakage reactance and resistance -  $R_T + jX_T$ . The losses on the transformer are then calculated iteratively within the standard load flow.

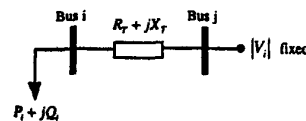


Fig.2 Modified StatCom model in loadflow calculation

Because the losses of the connection transformer of a StatCom have been included, the accuracy of load flow calculation can be improved by using the modified StatCom model. However, inaccuracies are still present in this model present due to the power losses caused by the StatCom's Voltage Source Inverter (VSI), which are neglected.

In this paper, a new StatCom model is proposed that is appropriate for power flow analysis that can account for the high-frequency effects and power electronic losses, and more accurately predict the active and reactive power outputs of the StatCom.

## II. AN IMPROVED STATCOM MODEL

An accurate load flow analysis should accurately forecast the steady-state losses of a StatCom, including both transformer and inverter losses. The losses caused by the VSI include main three parts: the harmonic losses, the switching losses, and the conduction losses of the power electronic elements. The percentage of each loss component relates to the conduction mode of the StatCom's VSI and the steady state operating point.

### A. Harmonic losses

Generally speaking, a StatCom output voltage always contains harmonics, due to the switching behavior of the VSI. These voltage harmonics will generate harmonic currents and further cause power losses in the system network. If the impedance of the lines that connect a StatCom to the power system is neglected, the harmonic losses are primarily apparent on the connection transformer. The effect of these losses in the transformer can be analyzed by considering an expansion of the transformer impedance.

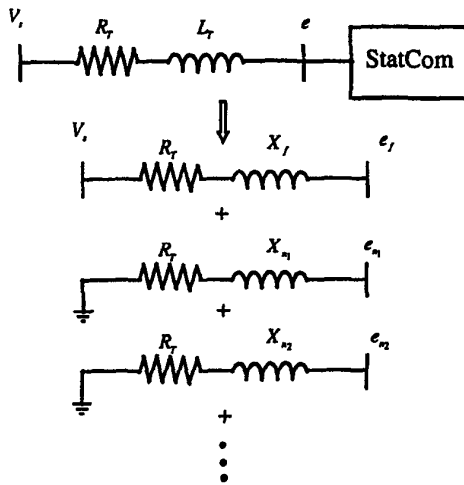


Fig.3 Modified StatCom model in the load flow calculation

Fig.3 shows the circuit of a StatCom connected to a power system by a connection transformer, where  $V_s$  and  $e$  represent the system RMS voltage and the StatCom's RMS output potential respectively, and  $R_T$  and  $L_T$  denote the resistance and leakage reactance of the connection transformer. Assuming that there are not any harmonics in the system voltage  $V_s$ , the StatCom's output voltage  $e$  consists of fundamental and high-order harmonics, and may be represented as:

$$\begin{aligned} e &= e_f + e_{n_1} + e_{n_2} + \dots \\ &= e_f + \sum_{i=n_1, n_2, \dots} e_i \end{aligned} \quad (1)$$

where  $e_f$  is the RMS value of the fundamental harmonic,

$e_i$  represents the RMS values of high-order harmonics, and  $n_1, n_2, \dots$  are the harmonic indices. Thus, the first diagram of Fig.3 can be represented as the sum of the other harmonic diagrams (where  $X_f, X_{n_1}, X_{n_2}, \dots$  denote the transformer's inductance under different harmonic frequencies).

The harmonic losses on the connection transformer can be expressed as:

$$\begin{aligned} P_{\text{loss}} &= P_{\text{fundamental}} + P_{\text{harmonics}} \\ &= P_{\text{fundamental}} + \sum_{i=n_1, n_2, \dots} \frac{e_i^2 * R}{R^2 + X_i^2} \\ &= P_{\text{fundamental}} + \sum_{i=n_1, n_2, \dots} \frac{e_i^2 * R}{R^2 + i^2 X_f^2} \end{aligned} \quad (2)$$

Usually, the magnitude of a StatCom's output voltage relates to the StatCom's DC side voltage and the conduction mode of the StatCom's VSI. For example, if the VSI applies the square wave conduction mode, the output voltage magnitude is a function of the DC side voltage and the firing angles of the VSI. If the PWM mode is used, the output voltage magnitude is a function of the DC side voltage and the duty cycle ratio of the PWM. In the following parts of this paper all derivations will be based on PWM assumption. Therefore using PWM, the output voltage magnitude of the StatCom can be expressed as

$$e_i = f_i(V_{dc}, K) \quad i = n_1, n_2, \dots \quad (3)$$

where  $K$  is the duty cycle ratio. Since,  $e_i$  is directly proportional to the DC side voltage  $V_{dc}$ , equation (3) can be simplified as

$$e_i = V_{dc} * f_i(K) \quad i = n_1, n_2, \dots \quad (4)$$

Substituting equation (4) into equation (2), the losses caused by the high order harmonics can be expressed as

$$\begin{aligned} P_{\text{harmonic}} &= V_{dc}^2 \sum_{i=n_1, n_2, \dots} \frac{f_i^2(K) * R_T^2}{R_T^2 + i^2 X_f^2} \\ &= \frac{V_{dc}^2}{R_h} \end{aligned} \quad (5)$$

where

$$\frac{1}{R_h} = \sum_{i=n_1, n_2, \dots} \frac{f_i^2(K) * R_T^2}{R_T^2 + i^2 X_f^2} \quad (6)$$

From equation (6), it can be seen that the high order harmonic losses relate to the StatCom's operating point and vary with the duty cycle ratio. Typically, when a StatCom is in steady-state operation, the duty cycle ratio does not change or changes in a very limit range. The StatCom's output reactive power is regulated through firing angle change. Then  $R_h$  can be treated as a constant. Equation (5) also implies that the high order harmonic losses can be equivalently represented as the active power losses caused by a DC side shunt resistor.

## B. Switching and conduction losses

The switching losses are introduced when the power electronic switches of a StatCom are in their turn-on and turn-off transients. Because of the strong non-linear characteristics of the switching behavior of power electronic switches, it is difficult to precisely model the switching losses of a StatCom. The conduction losses of a StatCom are caused by the voltage drops across the electronic power switches when they are in the on-state. In this section, the switching and conduction losses of a StatCom will be estimated.

Fig.4 shows the collector-emitter voltage  $V_{ce}$  and current  $i_c$  of a power electronic switch (such as an IGBT) in a typical turn-on and turn-off process [7].

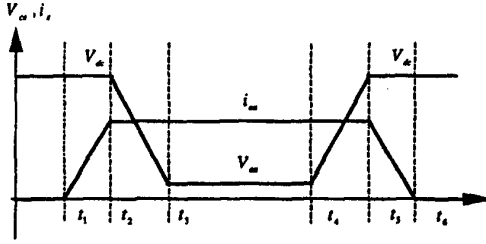


Fig.4 A typical turn-on and turn-off procedure of a power switch

Assuming that no losses are incurred when the switch is off, then all the switching and conduction losses are introduced in the period from  $t_1 \sim t_6$ . If this period is divided into five intervals, the losses can be estimated segment by segment as follows

$$\begin{aligned} t_1 - t_2: w_{12} &= \frac{1}{2} V_m * i_m * (t_2 - t_1) \\ t_2 - t_3: w_{23} &= \frac{1}{2} (V_m + V_m) * i_m * (t_3 - t_2) \\ t_3 - t_4: w_{34} &= V_m * i_m * (t_4 - t_3) \\ t_4 - t_5: w_{45} &= \frac{1}{2} (V_m + V_m) * i_m * (t_5 - t_4) \\ t_5 - t_6: w_{56} &= \frac{1}{2} V_m * i_m * (t_6 - t_5) \end{aligned} \quad (7)$$

Suppose  $t_2 - t_1 = t_6 - t_5$  and  $t_3 - t_2 = t_5 - t_4$ , then by combining the above equations, it is possible to get the switching and conduction losses of a switch in one phase leg and in one switching cycle:

$$w = V_m * i_m * (t_3 - t_1) + V_m * i_m * (t_6 - t_2) \quad (8)$$

If the switching frequency  $f_{sw}$  of a VSI is constant, then the average switching and conduction power losses  $P_{switch}$  of the VSI can be approximately expressed as:

$$P_{switch} = m * f_{sw} * [V_m * (t_3 - t_1) + V_m * (t_6 - t_2)] * i_m \quad (9)$$

where  $m$  is a coefficient which relates to the VSI's topology. Further, the following relationships hold:

$$V_{on} \approx V_0 + k_v * i_{on} \quad (10)$$

$$t_3 - t_1 \approx t_r + k_t * i_{on} \quad (11)$$

$$t_4 - t_2 = const. = t_{const} \quad (12)$$

where  $V_0$  represents the constant voltage drop across a power electronic switch in its on-state, and  $t_r, t_{const}, k_v, k_t$  are constant coefficient. Therefore, equation (9) can be rewritten as:

$$P_{switch} = m * f_{sw} * [(t_r * V_m + V_0 * t_{const}) * i_m + (k_v * V_m + k_t * t_{const}) * i_m^2] \quad (13)$$

Equation (13) indicates that the switching and conduction losses of a StatCom relate to the current passing through its VSI into the AC side system. When the StatCom is operating at high current levels, the second term on the right of equation (13) dominates the switching and conduction losses of the StatCom.

If the effect of the first term on the right of equation (13) is neglected, then the use of a series resistor in the AC side system of a StatCom can approximately represent the StatCom power electronic losses.

## C. An improved model of a StatCom

By shunting a resistor in the DC side of a StatCom and putting a resistor in series with the AC line, the approximate losses of the StatCom can be taken into account.

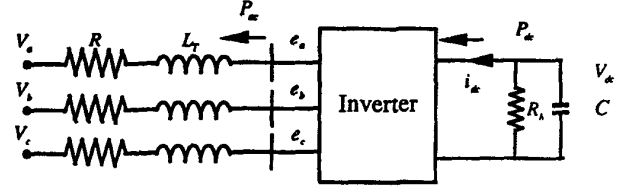


Fig.5 Schematic of a StatCom connected to a power system

Fig. 5 shows the proposed improved StatCom model, where  $R$  is the combination of the series resistor and the connection transformer's resistance. To derive the new model, let

$$\begin{aligned} v_a &= \sqrt{2} * V_s * \cos(\omega * t) \\ v_b &= \sqrt{2} * V_s * \cos(\omega * t - \frac{2\pi}{3}) \\ v_c &= \sqrt{2} * V_s * \cos(\omega * t + \frac{2\pi}{3}) \\ e_a &= \frac{K}{2} * V_s * \cos(\omega * t + \delta) \\ e_b &= \frac{K}{2} * V_s * \cos(\omega * t + \delta - \frac{2\pi}{3}) \\ e_c &= \frac{K}{2} * V_s * \cos(\omega * t + \delta + \frac{2\pi}{3}) \end{aligned} \quad (15)$$

where  $\delta$  is the firing angle of the StatCom's VSI, and  $V_s$  is the RMS value of the system line-to-neutral voltage. Because the losses of the VSI have already been represented by two equivalent resistors, the VSI can be

then assumed to be lossless. This yields the following power balance equation:

$$P_x = V_x i_x = P_x = (e_x i_x + e_y i_y + e_z i_z) \quad (16)$$

The state-space equations of the StatCom are:

$$\frac{d}{dt} \begin{bmatrix} i_x \\ i_y \\ i_z \\ V_x \end{bmatrix} = A \begin{bmatrix} i_x \\ i_y \\ i_z \\ V_x \end{bmatrix} + u \quad (17)$$

where

$$A = \begin{bmatrix} \frac{R}{L_T} & 0 & 0 & \frac{K}{2L_T} \cos(\omega t + \delta) \\ 0 & \frac{R}{L_T} & 0 & \frac{K}{2L_T} \cos(\omega t + \delta - \frac{2\pi}{3}) \\ 0 & 0 & \frac{R}{L_T} & \frac{K}{2L_T} \cos(\omega t + \delta + \frac{2\pi}{3}) \\ \frac{K}{2C} \cos(\omega t + \delta) & \frac{K}{2C} \cos(\omega t + \delta - \frac{2\pi}{3}) & \frac{K}{2C} \cos(\omega t + \delta + \frac{2\pi}{3}) & \frac{1}{C} \frac{1}{R_s} \end{bmatrix}$$

$$u = \begin{bmatrix} \frac{\sqrt{2}V_s \cos(\omega t)}{L_T} \\ \frac{\sqrt{2}V_s \cos(\omega t - \frac{2\pi}{3})}{L_T} \\ \frac{\sqrt{2}V_s \cos(\omega t + \frac{2\pi}{3})}{L_T} \\ 0 \end{bmatrix}$$

In order to validate the accuracy of the proposed model, a device level simulation and a state-space simulation are carried out in Matlab. In the device level simulation, the full power switches' characteristics are specified. In the state-space simulation, two cases are considered. In the first case, the simple model of a StatCom is used in which the losses of the VSI are neglected (by substituting  $R$  with  $R_T$ , and letting  $R_h \rightarrow \infty$  in equation (17)). In the second case, the improved model of the StatCom expressed by equation (17) is used. Fig.6 and Fig.7 show the start-up dynamics of a StatCom's AC side current and DC side voltage. The device level simulation is shown with the solid line, the simple model with a dashed line, and the proposed model results with a dotted line. The dotted line is coincident with the center of the solid line so it is difficult to differentiate.

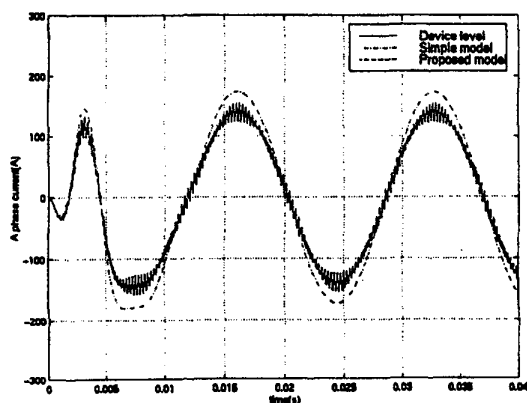


Fig.6 AC side current of a StatCom in start process

From the simulation results, it is apparent that the improved model can accurately capture the StatCom's dynamic behavior, whereas the traditional simple model produces some errors. The simulation results demonstrate that the proposed model is more accurate in representing a StatCom response.

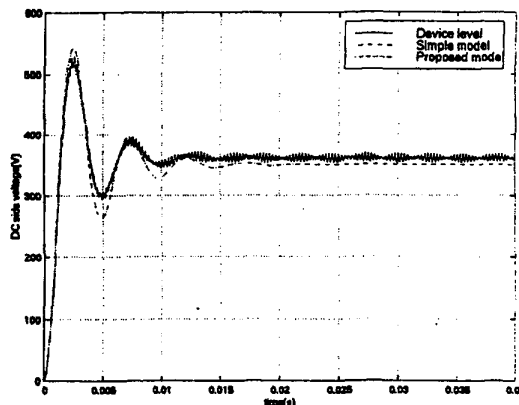


Fig.7 DC side voltage of a StatCom in start process

### III. AN IMPROVED STATCOM MODEL FOR LOADFLOW ANALYSIS

Based on the analysis presented in the previous section on the improved modeling of StatCom losses, an improved StatCom model for load flow calculations is presented in this section.

#### A. An improved StatCom model for load flow calculations

To better reflect the effect of a StatCom on line power flow, the StatComs' power losses should be considered in the load flow calculation. As discussed in the last section, the switching and conduction losses can be represented by an AC side series resistor. This resistor can be added to the connection transformer's resistance. Although the harmonic losses of a StatCom can be roughly reflected by a DC side shunt resistor, in a load flow calculation the shunt resistor must be manipulated so that it can take part in the load flow calculation.

The harmonic losses are given as:

$$P_{\text{harmonic}} = \frac{V_h^2}{R_h}$$

Therefore, when PWM mode is applied the voltage becomes

$$V_h = \frac{K}{2\sqrt{2}} V_x$$

leading to

$$P_{\text{StatCom}} = \frac{V_j^2}{R_h}, \quad R_h = \frac{K^2}{8} R_s \quad (18)$$

This implies that the DC side resistor can be moved to the AC side so long as a scaling coefficient is added.

The proposed improved load flow StatCom model given in equation (18) is shown in Fig.8.

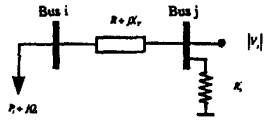


Fig.8 Improved model of a StatCom in LF calculation

In Fig.8, bus  $j$  represents the StatCom's VSI output terminal. It is treated as a PV bus  $j$  in the load flow calculation. The injected power of bus is set to zero. The StatCom's reactive power compensation holds bus  $j$  voltage magnitude constant. The resistance  $R$  includes the VSI switching and conduction losses and the connection transformer's resistance. The harmonic losses are embodied in  $R_h$ .

### B. Loadflow calculation examples

A simple two-area power system shown in Fig.9 is used to illustrate the new StatCom model. The system consists of two similar areas. Each area consists of two coupled units, each having a rating of 900MVA and 20kV. The transmission system nominal voltage is 230kV. The per unit system power and voltage bases are chosen as: 900MVA and 20kV/230kV respectively. A StatCom is connected to bus 8. The compensated reactive power of the StatCom maintains the voltage magnitude of bus 8 at 1.0pu. All the other information about the sample system can be found in reference [9].

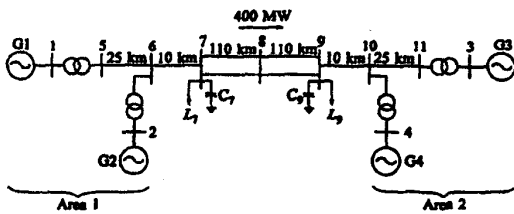


Fig.9 A simple two-area system

Both the improved model (shown in Fig.8) and the model shown in Fig.2 are used in load flow calculation. The results are compared to demonstrate the impact of the StatCom's power losses to the accuracy of the load flow calculation.

The values of shunt and series resistors are determined in the following way:

1. Neglect the StatCom's losses;

2. Calculate the StatCom's output reactive power which is needed to maintain bus 8's voltage magnitude at 1.0pu;
3. Assume the effectiveness of the StatCom is 90%. Switching and conduction losses occupy half of the total losses and the harmonic losses share the other half.
4. According to the StatCom's output reactive power and its effectiveness determine the parameters of the shunt and series resistors

Table 1 gives the maximal errors in the load flow calculations when different StatCom models are used.

Table 1. Maximal errors in loadflow calculation results

	Voltage magnitude	Phase angle	Active power on transmission lines	Reactive power on transmission lines
errors	0.02%	0.4'	1.2%	3.6%

Table 2 shows the maximal errors in the load flow calculation when the entire loading of the power system increases by 50% and the generators increase their output correspondingly to fulfill the energy demand.

Table 2. Maximal errors in LF calculation results

	Voltage magnitude	Phase angle	Active power on transmission lines	Reactive power on transmission lines
errors	0.08%	0.75'	1.2%	13.2%

From the above comparison, it can be noted that the bus voltage magnitudes do not change much regardless of whether the StatCom's losses are considered or not. The StatCom's losses will have a noticeable impact the accuracy of the phase angles and active power on transmission lines. But the most significant impact of the StatCom's losses is on the accuracy of the reactive power flow on the transmission lines, especially when the power system is heavily loaded.

## IV. CONCLUSION

Although the power losses of a StatCom are small compared to its capacity rate, the losses play a significant role in the StatCom's mathematical model and the accuracy of the corresponding simulation or calculation results. This paper analyzes the power losses of a StatCom that are caused by the switching behaviors of the StatCom's VSI and, according to the analysis results, present an improved model of the StatCom that take into account the power losses. The model is validated by device level simulation. Consideration of the StatCom's losses during load flow calculations is also addressed. The effects of the StatCom's losses on the load flow calculation accuracy are also

demonstrated by several examples.

## V. ACKNOWLEDGEMENTS

The authors gratefully acknowledge the support of the National Science Foundation under grants EEC-9527345 and ECS-9257208 and Sandia National Laboratories under contract BD-0071-D

## VI. ACKNOWLEDGEMENTS

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