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Estimation of Heart-Surface Potentials using Regularized Multipole Sources

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Abstract—Direct inference of heart-surface potentials from body-surface potentials has been the goal of most recent work on electrocardiographic inverse solutions. We developed and tested indirect methods for inferring heart-surface potentials based on estimation of regularized multipole sources. Regularization was done using Tikhonov, constrained-least-squares, and multipole-truncation techniques. These multipole-equivalent methods (MEMs) were compared to the conventional mixed boundary-value method (BVM) in a realistic torso model with up to 20% noise added to body-surface potentials and ± 1 cm error in heart position and size. Optimal regularization was used for all inverse solutions. The relative error of inferred heart-surface potentials of the MEM was significantly less ($p < 0.05$) than that of the BVM using zeroth-order Tikhonov regularization in 10 of the 12 cases tested. These improvements occurred with a fourth-degree (24 coefficients) or smaller multipole moment. From these multipole coefficients, heart-surface potentials can be found at an unlimited number of heart-surface locations. Our indirect methods for estimating heart-surface potentials based on multipole inference appear to offer significant improvement over the conventional direct approach.

Index Terms—Constrained least squares, inverse electrocardiology, multipole expansion, Tikhonov regularization.

I. INTRODUCTION

THE objective of the electrocardiographic inverse problem is to estimate the electrical activity of the heart from measurements on the body surface. For example, equivalent sources, such as the familiar single dipole of vectorcardiography [1], multiple dipole sources [2], or multipole expansions [3], can successfully represent cardiac activity [4]. These equivalents, however, can be difficult to interpret. In contrast, heart-surface potentials are easy to interpret and can be readily inferred. Consequently, their direct inference, with either boundary- or finite-element methods, has been the goal of most recent work on inverse solutions [5].

Compared to body-surface potentials, heart-surface potentials are not strongly affected by torso shape and are more indicative of cardiac sources [6]. Heart-surface potentials have been used to find the location and extent of myocardial infarction and ischemia, the accessory pathway

in Wolf-Parkinson-White (WPW) syndrome, reentry sites in ventricular arrhythmias, prearrhythmic events, and conduction abnormalities (see [7]). Unfortunately, errors in heart-surface potentials estimated with current techniques may limit their widespread clinical application. The ill-posed nature of the inverse problem causes small errors in measured body-surface potentials or heart–torso geometry to produce large errors in estimated heart-surface potentials. Regularization is almost always required for usable results.

Here, we propose methods to estimate heart-surface potentials from a cardiac-equivalent multipole source [7], [8]. Possible advantages of using an equivalent multipole source have been suggested by others [9]. In this study, we develop regularization techniques for a multipole source used to estimate heart-surface potentials and evaluate these techniques in a realistic torso model. Direct estimation of heart-surface potentials with the conventional boundary-value method (BVM) of Barr and coworkers [10] using zeroth-order Tikhonov regularization was compared to indirect estimation with the multipole-equivalent method (MEM). Comparisons were made with noise added to body-surface potentials and for errors in heart geometry.

II. MULTIPOLE-EQUIVALENT METHOD

The multipole expansion is an infinite series representation suitable for describing sources in the heart. The first term is the familiar cardiac dipole. Multipole coefficients, a_{nm} and b_{nm}, can be found either from an integral of the cardiac sources $\mathbf{J}^i$ within volume V or from an integral of potentials Φ on the surface S of volume V with conductivity σ [11]. Using complex notation for convenience, we have

$$a_{nm} + jb_{nm} = \int_V \mathbf{J}^i \cdot \nabla \Psi_{nm} dV = \int_S \sigma \Phi \nabla \Psi_{nm} \cdot d\mathbf{S} \quad (1)$$

where

$$\Psi_{nm} = \left(2 - \delta_m^0\right) \frac{(n-m)!}{(n+m)!} r^n P_n^m (\cos \theta) e^{jm\phi} \quad (2)$$

and δ_m^0 is the Kronecker delta ($\delta_m^0 = 0, m \neq 0; \delta_0^0 = 1$), $P_n^m(\cdot)$ is the associated Legendre polynomial, and (r, θ, ϕ) is a point location in spherical coordinates.

In practice, multipole coefficients $\mathbf{M}$ are found from body-surface potentials, $\mathbf{\Phi}_B$, using a transfer coefficient matrix $\mathbf{T}_B$. $\mathbf{T}_B$ is based on forward-problem solutions for body-surface potentials in an appropriate torso model. The least-squares-error estimate for the multipole coefficients is

$$\hat{\mathbf{M}} = (\mathbf{T}_B^* \mathbf{T}_B)^{-1} \mathbf{T}_B^* \mathbf{\Phi}_B \quad (3)$$

where $\mathbf{T}_B^*$ is the Hermitian of $\mathbf{T}_B$. $\mathbf{M}$ and $\mathbf{\Phi}_B$ may either be column vectors representing conditions at a particular instant or may be matrices representing a sequence of conditions over time.

Once $\hat{\mathbf{M}}$ is known, then heart-surface potentials $\mathbf{\Phi}_H$ can be estimated using transfer coefficients $\mathbf{T}_H$, which are based on forward-problem solutions on the heart-surface

$$\hat{\mathbf{\Phi}}_H = \mathbf{T}_H \hat{\mathbf{M}}. \tag{4}$$

Because the inverse problem is ill-posed, the least-squares estimate may be a pathological solution. To overcome this limitation, the inverse solution should be regularized.

III. REGULARIZATION OF THE MULTIPOLE

Techniques for regularizing boundary-element methods include the use of truncated singular value decomposition [12], [13], the generalized eigensystem approach [14], constrained least-squares (CLS) methods [12], [15], and Tikhonov regularization [16]. Tikhonov regularization is the most common approach used in the electrocardiographic inverse problem.

We developed three regularization techniques for multipole estimation. Previously, we found that useful estimates of the multipole coefficients could be found without regularization using (3) [3]. This approach is the basis for regularization via truncation of the multipole expansion, which we examine in more detail here. CLS methods have been applied by others to regularize multipole coefficients [9]. Here we formalize and extend their use. In addition, we derive Tikhonov regularization techniques for zeroth-order estimation of multipole coefficients [7], [8].

A. Multipole Truncation

Previously, we found that lower degree multipole estimates were more consistent when higher degree terms were included in (3) [3]. Specifically, the dipole $(n = 1)$, which is independent of location, had less variation at two different origins when the quadrupole $(n = 2)$ was simultaneously estimated. Here this technique was applied to the estimation of heart-surface potentials. A higher degree multipole was estimated using (3) than was used to calculate heart-surface potentials using (4). Experimental results will be presented later to show optimal sizes for the multiple expansion.

The multipole truncation approach is similar to truncated singular value decomposition and the generalized eigensystem approach where the solution is separated into orthogonal components from which heart-surface potentials are estimated using an expansion based only on the most significant few values. These methods differ, however, in the expansion technique.

B. Constrained Least-Squares Method

CLS regularization is based on the statistical characteristics of the signal and the noise. If $\mathbf{N}$ is a covariance matrix for noise and $\mathbf{S}$ is the covariance matrix for multipole coefficients, then the best statistical estimate of $\mathbf{M}$ is [15]

$$\hat{\mathbf{M}} = \left(\mathbf{T}_B^* \mathbf{N}^{-1} \mathbf{T}_B + \mathbf{S}^{-1}\right)^{-1} \mathbf{T}_B^* \mathbf{N}^{-1} \mathbf{\Phi}_B. \tag{5}$$

If $\mathbf{N} = \sigma_n^2 \mathbf{I}$ and $\mathbf{S} = \sigma_s^2 \mathbf{I}$, where $\mathbf{I}$ is the identity matrix, then

$$\hat{\mathbf{M}} = \left(\mathbf{T}_B^* \mathbf{T}_B + \tau \mathbf{I}\right)^{-1} \mathbf{T}_B^* \mathbf{\Phi}_B \tag{6}$$

where $\tau = \sigma_n^2 / \sigma_s^2$. This equation is similar to Tikhonov zeroth-order regularization for the BVM, which can also be shown to be a statistically constrained solution given the correct assumptions.

Statistical characteristics of the signal and noise are difficult to obtain, particularly for a theoretical construct like a multipole expansion. A basic version of CLS regularization was developed for the MEM based on the following simplifying assumptions [7]:

- noise n is uncorrelated with zero mean and variance σ_n^2 and is uniformly distributed over a sphere of radius r surrounding the heart;
- potentials on that sphere are generated from a multipole source in the infinite medium, have zero mean, variance σ_s^2, and are uniformly distributed spatially;
- each multipole moment contributes equally to the power of the potentials.

Using these assumptions, the expected value of signal power of potentials on a sphere of radius r in the infinite medium is

$$\sigma_s^2 = \left(\frac{1}{4\pi\sigma}\right)^2 \sum_{n=1}^{\infty} \sum_{m=0}^{n} \frac{1}{r^{2(n+1)}} \\ \times \left(\sigma_{a_{nm}}^2 \cos^2 m\phi + \sigma_{b_{nm}}^2 \sin^2 m\phi\right) P_n^m(\cos\theta)^2 \tag{7}$$

where $\sigma_{a_{nm}}^2$ and $\sigma_{b_{nm}}^2$ are the variance of coefficients a_{nm} and b_{nm}, respectively. If each component is independent of polar angle θ and azimuthal angle ϕ, then

$$\sigma_s^2 = \left(\frac{1}{4\pi\sigma r^2}\right)^2 \left(\frac{1}{r^2}\sigma_{\text{dipole}}^2 + \frac{1}{r^4}\sigma_{\text{quad}}^2 + \cdots\right). \tag{8}$$

Further, if each pole contributes equally to the expected power in heart-surface potentials, then

$$\frac{1}{N}\sigma_s^2 (4\pi\sigma r^3)^2 = \sigma_{\text{dipole}}^2 = \frac{1}{r^2}\sigma_{\text{quad}}^2 = \cdots \tag{9}$$

where N is the total number of multipole coefficients. If the noise-to-signal power ratio of heart-surface potentials is given by σ_n^2 / σ_s^2, then the noise-to-signal power ratio matrix for the multipole is proportional to

$$\mathbf{H}_m = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \frac{1}{r^2} & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \frac{3}{r^2} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & 0 & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & \frac{c_{nm}}{r^{2n}} \end{bmatrix} \tag{10}$$

where c_{nm} is the proportionality constant between coefficients that allows signal power on the surface of the sphere to be uniformly distributed, as given in (9). $\mathbf{H}_m$ is proportional to the inverse covariance matrix between individual multipole coefficients, as indicated by $\mathbf{S}^{-1}$ in (5). Using $\mathbf{H}_m$ in the CLS approach given in (5), multipole coefficients may be calculated as

$$\hat{\mathbf{M}} = \left(\mathbf{T}_B^* \mathbf{T}_B + \tau \mathbf{H}_m\right)^{-1} \mathbf{T}_B^* \mathbf{\Phi}_B \tag{11}$$

where τ is a regularization constant.

Pilkington and Morrow were able to reduce the error in inferred heart-surface potentials using CLS regularization of the MEM [9]. They empirically chose $\sigma_n^2/\sigma_{\text{pole}}^2 = 0.001$ for the dipole and $\sigma_n^2/\sigma_{\text{pole}}^2 = 1$ for the quadrupole. Our formalization of a regularization technique for the MEM based on CLS may provide an explanation for why these particular values improved performance. For a heart radius of 3.2 cm, $\sigma_n^2/\sigma_{\text{dipole}}^2 \approx 0.001\sigma_n^2/\sigma_{\text{quad}}^2$ using MKS units in (9).

C. Tikhonov Regularization

Conventional BVM estimation of heart-surface potentials using Tikhonov regularization yields

$$\hat{\boldsymbol{\Phi}}_H = (\mathbf{Z}_{BH}^*\mathbf{Z}_{BH} + \tau\mathbf{R}^*\mathbf{R})^{-1}\mathbf{Z}_{BH}^*\boldsymbol{\Phi}_B \qquad (12)$$

where $\mathbf{Z}_{BH}$ is a transfer-coefficient matrix relating heart- to body-surface potentials, τ is a regularization constant, and $\mathbf{R}$ is a regularization matrix. $\mathbf{R}$ is either $\mathbf{I}$, the identity matrix, $\mathbf{L}$, the Laplacian operator, or $\mathbf{G}$, the gradient operator. The case in which $\mathbf{R} = \mathbf{I}$ is known as zeroth-order Tikhonov regularization and is commonly used in the literature as a basis for comparison of regularization techniques.

Tikhonov regularization is based on minimizing the cost function $J_\tau(\boldsymbol{\Phi_H})$ [17] as

$$J_\tau(\boldsymbol{\Phi_H}) = \tau\|\mathbf{R}\boldsymbol{\Phi_H}\|^2 + \|\mathbf{Z_{BH}}\boldsymbol{\Phi_H} - \boldsymbol{\Phi_B}\|^2 \qquad (13)$$

where τ is a regularization constant and $\mathbf{R}$ is a regularization matrix. The cost function is a weighted sum of the norm of regularized inferred heart-surface potentials $\|\mathbf{R}\boldsymbol{\Phi}_H\|^2$ and the body-surface residual $\|\mathbf{Z}_{BH}\boldsymbol{\Phi}_H - \boldsymbol{\Phi}_B\|^2$. Setting the derivative of the cost function to zero yields (12).

For the MEM, the regularized squared norm of heart-surface potentials is $\|\mathbf{R}\mathbf{T}_H\mathbf{M}\|^2$. The body-surface residual is $\|\mathbf{T}_B\mathbf{M} - \boldsymbol{\Phi}_B\|^2$. The cost function for Tikhonov regularization of the MEM is, therefore

$$J_\tau(\mathbf{M}) = \tau\|\mathbf{R}\mathbf{T}_H\mathbf{M}\|^2 + \|\mathbf{T}_B\mathbf{M} - \boldsymbol{\Phi}_B\|^2 \qquad (14)$$

where $\mathbf{T}_H$ is the regularizing operator. Minimizing with respect to $\mathbf{M}$, the regularized estimate is

$$\hat{\mathbf{M}} = (\mathbf{T}_B^*\mathbf{T}_B + \tau\mathbf{T}_H^*\mathbf{R}^*\mathbf{R}\mathbf{T}_H)^{-1}\mathbf{T}_B^*\boldsymbol{\Phi}_B. \qquad (15)$$

For zeroth-order Tikhonov regularization where $\mathbf{R} = \mathbf{I}$, we have

$$\hat{\mathbf{M}} = (\mathbf{T}_B^*\mathbf{T}_B + \tau\mathbf{T}_H^*\mathbf{T}_H)^{-1}\mathbf{T}_B^*\boldsymbol{\Phi}_B. \qquad (16)$$

Although the BVM and MEM expressions for $\hat{\boldsymbol{\Phi}}_H$, (12) and (4) [using (16)], may yield the same results under some conditions, in general they will not. For example, the two will differ for a simple heart–torso model where the only inhomogeneity is the heart. In our case, the BVM model is homogeneous because the heart is excluded from the volume conductor, but the MEM model is inhomogeneous because it includes the cardiac blood mass. In addition, the MEM and BVM may give different results because the implicit constraints (the number of multipole terms) placed on the MEM limit its ability to reconstruct the heart-surface potentials. For example, under ideal conditions, the BVM may be able to represent heart-surface potentials exactly but heart-surface potentials calculated with a dipole source may have significant error. These implicit constraints may be an
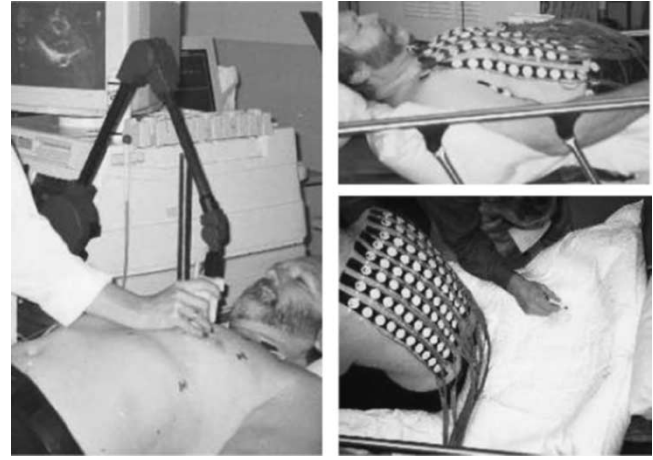

Fig. 1. Geometric measurements. The torso surface and electrode locations were measured with the IPD shown in the left panel. The position of electrodes is shown in the pictures in the right panel. Heart size and location were estimated from ultrasonic images registered to the surface measurements. Torso measurements were performed on R. Martin Arthur, an author of this study.

advantage when calculating inverse solutions in the presence of errors in transfer coefficients or body-surface potentials [18], which is the thrust of this study.

Similarities occur between CLS and zeroth-order Tikhonov regularization. If the regularizing operator $\mathbf{T}_H$ is constructed from uniform samples on a spherical heart then, due to the orthogonality properties of the Legendre polynomial, off-diagonal components of $\mathbf{T}_H^*\mathbf{T}_H$ will be zero. Diagonal components will be proportional to $1/r_H^{2n}$, where n is the degree of the associated multipole term. For a uniformly sampled spherical heart in the infinite medium, the CLS formulation assumes energy is contributed equally by each multipole moment to heart-surface potentials, whereas the Tikhonov expression assumes energy is contributed equally by each coefficient.

IV. METHODS

Inference techniques were tested using a human torso model and known heart-surface potentials. The torso model was created from measurements of the heart and torso geometry of an adult male. Known heart-surface potentials were taken from sock-electrode measurements. Effects of electrical noise in surface potentials and geometric errors in the torso model were compared to assess the BVM and MEM estimates of heart potentials.

A. Heart–Torso Model

Torso shape and the location of 175 body-surface electrodes were measured on an adult male using an Immersion Personal Digitizer (IPD) as shown in Fig. 1. The torso was approximated with a tenth-degree spherical harmonic, which fit measured locations with an rms error of less than 0.5 cm and provided caps to close the torso-model surface as shown in Fig. 2.

The heart was approximated as a sphere. The 91-node sphere had a radius of 5.28 cm and a position within the torso based on size and orientation measurements made on 10 adult male subjects. Ultrasound measurements were taken from images registered to the body surface by coupling an ultrasound probe to the
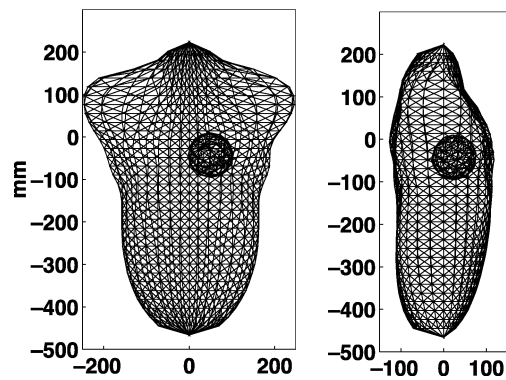
Fig. 2. Heart–torso model. The spherical heart surface contained 91 nodes; the spherical-harmonic torso model 1026 nodes.

IPD. A spherical model was used because we could determine the center and radius of a sphere, which circumscribed the heart, from the registered ultrasound images in each of the 10 subjects. Torso and heart conductivities were set to 0.21 and 0.67 S/m, respectively [19]. Additional details of data acquisition and torso approximation are reported in [7] and [20].

B. Forward-Problem Solutions

Forward-problem solutions relate sources in the heart to potentials they produce on the body surface. The mixed BVM directly relates heart-surface and body-surface potentials. The transfer coefficients $\mathbf{Z}_{BH}$ from heart H to body B are given by Barr and coworkers as [10]

$$\mathbf{Z}_{BH} = -\left(\mathbf{P}_{BB} - \mathbf{G}_{BH}\mathbf{G}_{HH}^{-1}\mathbf{P}_{HB}\right)^{-1} \times \left(\mathbf{P}_{BH} - \mathbf{G}_{BH}\mathbf{G}_{HH}^{-1}\mathbf{P}_{HH}\right) \quad (17)$$

where $\mathbf{P}$ is a matrix of solid angles and $\mathbf{G}$ is a matrix of gradient integral coefficients. Transfer coefficients were calculated using seven-point Radon numerical integration to approximate gradient integral terms [21].

MEM transfer coefficients $\mathbf{T}_B$ and $\mathbf{T}_H$ were calculated from the integral solution to the potential distribution on the surface of an inhomogeneous conductor as [22]

$$\begin{bmatrix} \mathbf{T}_B \\ \mathbf{T}_H \end{bmatrix} = \begin{bmatrix} \mathbf{P}_{BB} & \frac{\sigma_D}{\sigma_B}\mathbf{P}_{BH} \\ \frac{\sigma_B}{\sigma_S}\mathbf{P}_{HB} & \frac{\sigma_D}{\sigma_S}\mathbf{P}_{HH} \end{bmatrix} \begin{bmatrix} \mathbf{T}_B \\ \mathbf{T}_H \end{bmatrix} + 2 \begin{bmatrix} \frac{\mathbf{\Psi}_B}{\sigma_B} \\ \frac{\mathbf{\Psi}_H}{\sigma_S} \end{bmatrix} \quad (18)$$

where $\mathbf{P}$ is a solid-angle matrix, $\mathbf{\Psi}$ is the infinite medium potential for unit multipole sources, σ is conductivity in the heart H, the body B, the difference between heart and body D, or the sum of heart and body S.

The accuracy of BVM transfer coefficients was verified using concentric and eccentric spheres models by comparing our results with values found in the literature or values calculated analytically [7], [10]. Potentials for unit multipole sources were verified by determining their multipole content via surface integral [3]. The multipole transfer coefficients were also found by spherical harmonic approximation, which matched the results of (18) in the limit [22].

C. Heart- and Body-Surface Potentials

Epicardial potentials were measured with a 90-electrode sock on the heart of a human adult male with recent and remote myocardial infarction. We estimated potentials on a static model

surrounding the heart. Such a model is more indicative of a pericardial surface, but epicardial potentials were more easily obtained and are similar to pericardial potentials [23].

The 90-electrode sock was aligned with the left anterior descending artery so that 45 electrodes covered the left side of the heart and 45 covered the right, from apex to base. Unipolar potentials were recorded with a gain of 500 for 1 s during normal sinus rhythm. Bandwidth was 0.5–500 Hz. Potentials were digitized over a range of ± 20 mV at 1000 samples per second with 12 bits of precision.

Measurements from 11 electrodes were discarded due to artifact. Others were corrected for effects of baseline shift assuming an isoelectric T-P interval. Locations of the 79 usable signals were made nodes of the heart model. The surface was completed in the apex and base regions with the addition of 12 nodes for a total of 91. Potentials at the additional nodes were approximated using linear interpolation. These 91 signals formed the known heart-surface potentials $\mathbf{\Phi}_H$.

Body-surface potentials $\mathbf{\Phi}_B$ were found at all electrode locations from the BVM transfer coefficients $\mathbf{Z}_{BH}$ and the known heart potentials $\mathbf{\Phi}_H$

$$\mathbf{\Phi}_B = \mathbf{Z}_{BH}\mathbf{\Phi}_H. \quad (19)$$

Inspection of the calculated potentials showed they were consistent with those measured on patients. These calculated body surface potentials were used to test and to compare inverse solutions.

D. Inverse-Problem Solutions

Multipole inference via the truncation of least-squares-error estimates (3), CLS solutions (11), and Tikhonov techniques (16) were compared to the conventional BVM using zeroth-order Tikhonov regularization (12). Each inverse was optimized to minimize relative error (RE) at each instant of the QRS complex as follows:

$$\mathrm{RE}(t_i) = \sqrt{\frac{\left(\mathbf{\Phi}_H^c(t_i) - \mathbf{\Phi}_H^i(t_i)\right) * \left(\mathbf{\Phi}_H^c(t_i) - \mathbf{\Phi}_H^i(t_i)\right)}{\mathbf{\Phi}_H^c(t_i) * \mathbf{\Phi}_H^c(t_i)}} \quad (20)$$

where $\mathrm{RE}(t_i)$ is the relative error calculated at time t_i, $\mathbf{\Phi}_H^c(t_i)$ is a column vector representing the correct (known) potentials on the heart surface at time t_i, and $\mathbf{\Phi}_H^i(t_i)$ is a vector representing the inferred potentials. Solutions were calculated with noise added to body-surface potentials and with errors in heart size and location.

Gaussian white noise was added to body-surface potentials, with power ranging from 1% to 20% of total signal power in the QRS complex [16], [20], [24]. Each result was an average over 25 trials. We also shifted the heart position left (-1 cm) and right ($+1$ cm) and under- and over-estimated heart size by 1 cm [25].

E. Data Analysis

The performance of the BVM and MEM techniques was assessed by comparing relative errors in inferred heart-surface potentials. Average REs over the QRS complex were compared

using an unpaired t-test. REs were considered significantly different if the probability from the t-test was < 0.05.

V. RESULTS

Heart-surface potentials in an adult male torso model were estimated directly using the conventional mixed BVM method and indirectly using the MEM. Zeroth-order Tikhonov regularization was used with the BVM. Three forms of regularization were tested with the MEM: Tikhonov (TIK), constrained-least-squared (CLS), and truncation (TRN). Relative error (RE) was used to evaluate the quality of estimated heart-surface potentials.

Before evaluating the regularization techniques, experiments were performed to find the appropriate sizes of the multipole expansion to use in inverse calculations with the MEM. Based on those findings an analysis of the multipole truncation method was performed.

A. Multipole Size

The theoretical constructs for TIK and CLS regularization of the multipole were described in Section III. In contrast to the conventional BVM, useful estimates of heart-surface potentials can be based on least-squares-error estimation using the MEM, provided that the size of the expansion is limited. For example, Table I shows the RE for multipole estimates found using (3), i.e., least-squares estimates, for 5% noise added to body-surface potentials then using those estimates to find heart-surface potentials with (4). Minimum RE occurred for an octopole ($n = 3$) estimate.

B. Multipole Truncation

The REs of least-squares multipole estimates can be reduced by truncating the number of terms in the multipole used to find heart-surface potentials. Truncation was tested by calculating least-squares-error multipoles of degree n using (3), then using only degree $n-1$ terms to find heart-surface potentials with (4). Table II shows REs in inferred heart-surface potentials averaged over the QRS complex for 1%–20% additive noise. Table III depicts relative errors in inferred heart-surface potentials averaged over the QRS complex with changes in heart position and size of ± 1 cm.

Best performance (emphasized in bold face) occurred either when truncating from $n = 4$ to $n = 3$ or from $n = 3$ to $n = 2$. The best choice for a single strategy, based on the results in Tables II and III, was to truncate from $n = 3$ to $n = 2$. This strategy increased the average RE by 2.5 percentage points compared to picking the optimal truncation result. Optimal results, however, were used in the comparisons in the following section.

To further test the truncation method, heart-surface potentials were determined from truncated least-squares-error multipoles of degree $n - i$, where $i = 1$ to $n - 1$. Calculations were performed for 1%–20% noise added to body-surface potentials and for ± 1 cm changes in heart size and position. Table IV shows the result for a 1-cm shift left $(S - 1)$ in heart position. The bottom diagonal is the same as the first line of Table III.

TABLE I
RE OVER QRS OF LEAST-SQUARES MULTIPOLE WITH 5% NOISE ADDED TO BODY-SURFACE POTENTIALS

Pole	$n=2$	$n=3$	$n=4$	$n=5$
RE	0.54±0.11	**0.50±0.07**	0.82±0.17	2.18±0.49

TABLE II
RE OVER QRS OF TRUNCATED LEAST-SQUARES MULTIPOLES OF DEGREE N WITH ADDITIVE NOISE

%	$n=2$->1	$n=3$->2	$n=4$->3	$n=5$->4
1	0.73±0.11	0.54±0.11	**0.49±0.07**	0.98±0.17
5	0.73±0.11	0.54±0.11	**0.49±0.08**	1.02±0.14
10	0.73±0.11	0.54±0.11	**0.51±0.09**	1.14±0.17
20	0.73±0.11	**0.55±0.11**	0.57±0.15	1.49±0.44

TABLE III
RE OVER QRS OF TRUNCATED LEAST-SQUARES MULTIPOLES OF DEGREE N WITH HEART SHIFT (S) AND RADIUS (R)

cm	$n=2$->1	$n=3$->2	$n=4$->3	$n=5$->4
S-1	0.73±0.11	**0.58±0.10**	0.65±0.06	1.28±0.21
S+1	0.73±0.11	0.56±0.10	**0.50±0.07**	0.88±0.14
R-1	0.79±0.09	**0.76±0.06**	0.99±0.17	2.71±0.64
R+1	0.75±0.10	0.60±0.09	**0.55±0.07**	0.66±0.06

TABLE IV
RE OVER QRS WITH A -1-cm HEART SHIFT $(S - 1)$ FOR LEAST-SQUARES MULTIPOLE OF DEGREE N TRUNCATED TO LOWER DEGREES

n	1	2	3	4
5	0.73±0.11	**0.56±0.11**	0.57±0.08	1.28±0.21
4	0.73±0.11	**0.58±0.10**	0.65±0.06	
3	0.73±0.11	**0.58±0.10**		
2	0.73±0.11			

The minimum RE of 0.56 occurred when a fifth-degree least-squares-error multipole was truncated to second degree for calculation of heart-surface potentials. In this case, the REs using an $n = 2$ multipole (dipole plus quadrupole) to calculate heart-surface potentials varied by only two percentage points. In general, the primary determinant of performance was the degree of multipole used to calculate heart-surface potentials, not the degree of the estimated multipole. Although the amount of the optimal truncation varied somewhat, results for other error conditions were similar.

C. Conventional BVM Versus Regularized Multipoles

Optimal regularization results using zeroth-order Tikhonov (TIK), CLS, and truncation (TRN) of the multipole were compared to optimal zeroth-order regularization of the conventional BVM. A fourth-degree $(n = 4)$ multipole was found for the TIK and CLS solutions. The TRN solutions were optimized as shown in Tables II–IV. Comparisons were made for 1%–20% additive noise in body-surface potentials, shift in heart location
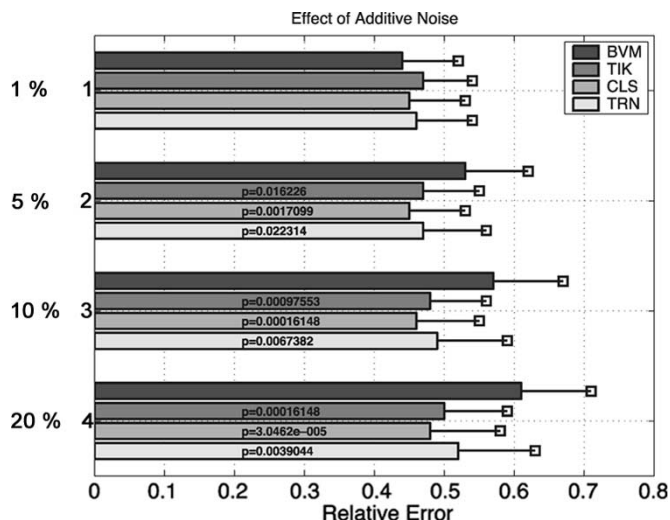
Fig. 3. Effect of 1%–20% additive noise. REs with optimal regularization using the conventional BVM and the MEM with TIK, CLS, and TRN regularization. The MEM was significantly better than the BVM where p values are shown.

Fig. 5. Effect of combined 10% noise, heart shift (S), and heart radius (R) change of ± 1 cm. REs with optimal regularization using the conventional BVM and the MEM with TIK, CLS, and TRN regularization. The MEM was significantly better than the BVM where p values are shown.
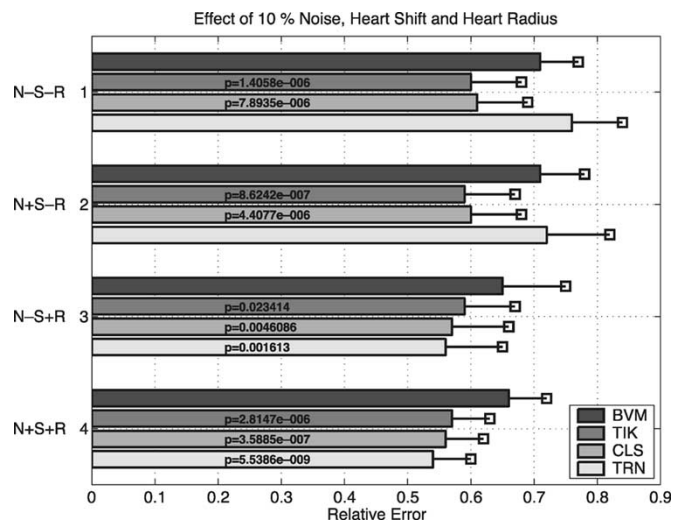

Fig. 4. Effect of heart shift (S) and change in heart radius (R) by ± 1 cm. REs with optimal regularization using the conventional BVM and the MEM with TIK, CLS, and TRN regularization. The MEM was significantly better than the BVM where p values are shown.
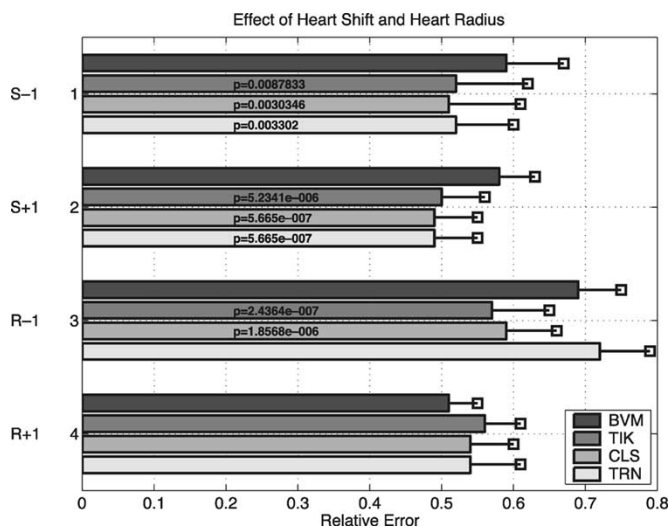
of ± 1 cm, change in heart radius of ± 1 cm, and combinations of 10% additive noise with changes in heart location and size. These results are shown in Figs. 3–5.

The REs of the multipole solutions were compared to the BVM solution for each case using an unpaired t-test. For cases in which the p value was <0.05, that value is shown on the bar indicating the RE. With additive noise, the MEM REs were significantly less than those for the BVM for TIK, CLS, and TRN regularization at 5%, 10%, and 20%. For combinations of noise and geometric errors with increased heart radius, the MEM REs were significantly less for TRN regularization than those for the BVM. For all combinations of noise and geometric errors, the MEM REs using TIK and CLS regularization were significantly less than those for the BVM.

VI. DISCUSSION

Three methods to regularize estimation of the multipole moments of the body-surface distribution for estimating heart-surface potentials were described and tested. They were Tikhonov, CLS, and truncation regularization. Their performance was evaluated by finding the relative error in heart-surface potentials calculated from multipole estimates with body-surface noise and geometric errors in the heart model. REs were significantly lower than those of the conventional mixed VM in 10 of the 12 cases tested.

It is difficult to ascribe the improved performance of the MEM over the conventional BVM to any one factor. One possible reason is that the number of multipole coefficients to be inferred may be much smaller than the number of heart-surface potentials of interest. In general, the fewer unknowns for a given measurement set, the more accurate the estimate of those unknowns [26], [27]. The performance of the BVM might have been improved by reducing the number of nodes representing the heart surface, thus reducing the number of unknowns, but reducing the number of nodes would also reduce the spatial resolution of the inferred potentials. The number of unknowns and the spatial-frequency content of MEM solutions is determined by the degree of the multipole, not the number of nodes in the heart model, which is arbitrary for the MEM.

We estimated a fourth-degree multipole with both Tikhonov and CLS regularization and either a third-, fourth-, or fifth-degree multipole with regularization via truncation. A forth-degree multipole contains 24 coefficients, i.e., 24 unknowns. The number of unknowns (heart nodes) for the BVM in this study at 91 was nearly 4 times as large. A lower limit for the number of BVM unknowns is probably around the number used in the original study of the BVM by Barr and coworkers [10]. Their value of 58 is still more than twice the number of MEM unknowns routinely used here.

Another possible reason the MEM outperformed the BVM is that inference of multipole terms can easily be made dependent on spatial frequency. The higher the spatial frequency of the inferred source, the greater the error because higher spatial frequencies in heart-potential maps are more attenuated on the body-surface than lower spatial frequencies. Our results suggest that spatial frequencies beyond those described by a fourth-degree multipole may be lost in the noise on the body surface (see Table I). This limit on recoverable spatial frequency determines the number of unknowns for the MEM. The reduced number of unknowns needed for the MEM to outperform the BVM may give the MEM a significant advantage over the BVM in inferring heart-surface potentials. Furthermore, although the same number of nodes was used for both the BVM and MEM studies to provide a consistent basis for comparison, the MEM can be applied to a heart surface with any number of nodes without affecting the number of unknowns in the inverse problem.

Inverse solutions using the BVM were constrained in this study using zero-order Tikhonov regularization because zeroth-order regularization is often used as a baseline for performance. Other regularization techniques for the BVM may perform better under some conditions. A rough comparison of the MEM approach can be made to the BVM with other regularization techniques based on performance reported in the literature. Considering published results under similar conditions as were used here, studies with noise in body-surface potentials show anywhere from no improvement to 18% improvement in RE's compared to zeroth-order Tikhonov regularization when using truncated singular value decomposition [13], first-order Tikhonov regularization [28], second-order Tikhonov regularization [13], the generalized eigensystem approach (GES), or the modified GES (tGES) [14]. While the relative performance of the MEM approach was not better than most of these approaches, its performance was equivalent. However, these results also suggest that better results can be expected using the MEM with first- or second-order Tikhonov regularization [13], [28]. Similar studies of geometric errors found that truncated singular value decomposition [14], first-order Tikhonov regularization [29], GES [14], and tGES [14] return relative errors from 12% to 14% lower than zeroth-order Tikhonov regularization with the BVM. In our study, the MEM outperformed standard zeroth-order Tikhonov regularization of the BVM by more than any of these approaches. Care should be taken when comparing our results to these studies, however, as subtle differences in geometry or potentials can skew results. A more thorough evaluation of the MEM with high-order regularization matrices and a comparison to other regularization methods are topics for further study.

The MEM may not perform as well as it performed here when using a heart–torso model that includes inhomogeneities like the lungs. Convergence criteria for the multipole require that a sphere can be created which surrounds the cardiac sources and does not intersect any torso inhomogeneities [30]. Such a sphere may not exist in the presence of the lungs; however, studies indicate that multipoles of degree five to eight or less may yield acceptable results even when lungs are included [30], [31]. Our study indicates that multipoles of degree five or less are adequate

for estimation of heart-surface potentials under noise conditions that are typically encountered. Thus, the results presented here may be valid for torso models with lungs. Nevertheless, the effect of torso inhomogeneities on the MEM should be studied in the future.

Poor estimates of regularization parameters produce large errors in estimated potentials when using the BVM with zeroth-order Tikhonov regularization. In contrast, the MEM performed well without any regularization (see Table I). The stability of the MEM is likely to be useful when regularization parameters are estimated from *a posteriori* information. Because of this stability, the MEM is likely to be more resilient to errors in the choice of the regularization parameter than the BVM. Determining the performance of the MEM with *a posteriori* regularization parameters for the Tikhonov and CLS techniques, as well as development of an algorithm for *a posteriori* application of the truncation method, remains for future work.

VII. CONCLUSION

Estimating heart-surface potentials indirectly in an adult male using the multipole-equivalent method generally led to lower relative errors compared to direct estimation based on zeroth-order Tikhonov regularization with the conventional mixed-boundary-value approach. Multipole estimates were regularized with zeroth-order Tikhonov, CLS, and truncation techniques. The CLS technique was best in noise and with most geometric errors. Results for combinations of noise and geometric error were mixed with Tikhonov the best in two cases and truncation in the remaining two. Which one will prevail will likely depend on which is best at using *a posteriori* information to estimate the regularization parameter or truncation degree. Because the multipole estimates without regularization were comparable to the results using regularization, we expect the advantage of the MEM to be even greater compared to the BVM when using *a posteriori* information than it was in this study using optimal regularization.

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