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CORRELATIVE TRACKING OF PSEUDO-NOISE CODES USING A PHASE SHIFTED REFERENCE

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ABSTRACT

This work investigates a first-order correlation loop for tracking pseudo-noise (PN) codes. The local reference is produced by a PN sequence generator in series with a Hilbert transform filter. The tracking performance of this non-linear loop is compared with conventional early-late delay-lock loops (ELDLL). Using a conservative definition of bandwidth, the new loop was found to be superior at moderate to low signal-to-noise ratios (SNR). Another advantage of the new structure is that it has a very large (arguably infinite) lock range.

I. INTRODUCTION

Correlation loops such as phase-locked loops (PLL) and delay-locked loops are widely used in synchronization subsystems. The general structure for these devices is illustrated in Figure 1. The tracking performance of this device is a function of the SNR, loop filter, loop gain and the cross-correlation function between the transmitted $S_x(t)$ and the reference signal $S_L(t)$, $R_{XL}(\tau)$. The optimization of $R_{XL}(\tau)$ has been studied [1,2], but these results are not widely used. This is due in part to the difficulty of producing an arbitrary waveshape with the local reference generator. Following the approach of [3], we restrict our attention to waveform generators that are a replica of the transmitted waveform generator followed by a linear time-invariant filter, $H(f)$, as shown in Figure 2. The cross-correlation function $R_{XL}(\tau)$ can be controlled by altering $H(f)$, which is called the VCC filter. In a conventional ELDLL this filter approximates a differentiator. A differentiator is also used in a PLL, but it is usually combined with the voltage controlled oscillator (VCO). Differentiators can be viewed as linear filters with a 90 degree phase characteristic and an amplitude response that is proportional to frequency.

While the 90 degree phase shift appears to be optimal [2,3], there are reasons to believe that other amplitude characteristics may result in superior loop performance [4]. A simple example is a Hilbert transform filter, which Cabot [5] suggested using for time delay estimation loops.

This work compares the mean-square tracking error of a first-order correlation loop with a Hilbert transform filter to conventional ELDLL. This comparison is difficult to make because the new loop is non-linear. It is necessary to develop a definition of bandwidth that will encompass both

loops before the comparison can be made. A mathematical derivation of the cross-correlation function is presented in Section II. The definition of bandwidth used in the comparison and simulation results are summarized in Section III.

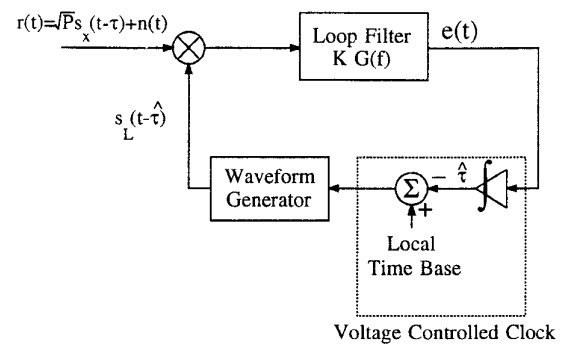


Figure 1 Generalized Correlation Loop

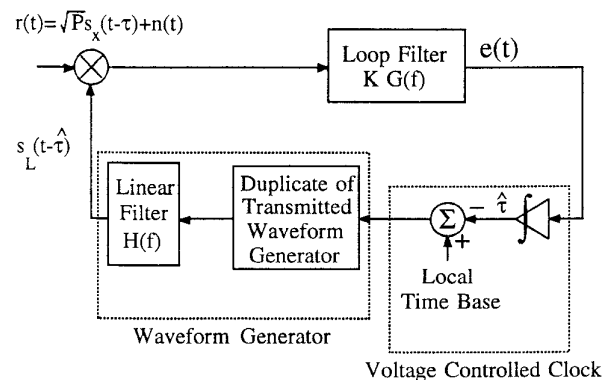


Figure 2 Modified Correlation Loop

II. ANALYSIS

In the first-order correlation loop (FOCL) shown in Figure 2, the received signal $r(t) = \sqrt{P}s_x(t-\tau) + n(t)$ is a delayed version of the originally transmitted signal $s_x(t)$ that has been corrupted by additive white Gaussian noise (AWGN). Signal $s_x(t)$ is assumed to be unity power and $n(t)$ has a double-sided power-spectral density of $N/2$ W/Hz.

The correlation loop generates an estimate, $\hat{\tau}$ for the channel delay τ . The signals $s_x(t)$ and $s_L(t)$ are assumed to be periodic with period T . It can be shown that the control function, $e(t)$, can be expressed as [4]

$$e(t) = K\sqrt{P}R_{XL}(\tau - \hat{\tau}) + Kn'(t) \quad (1)$$

where

$$n'(t) \triangleq n(t)s_L(t - \hat{\tau}(t)). \quad (2)$$

Since $s_x(t)$ and $s_L(t)$ are periodic, the cross-correlation function, $R_{XL}(\tau)$ is defined as

$$R_{XL}(\tau) \triangleq \frac{1}{T} \int_0^T s_L(t)s_x(t - \tau) dt. \quad (3)$$

This means that the baseband equivalent model for Figure 2 is the non-linear loop shown in Figure 3.

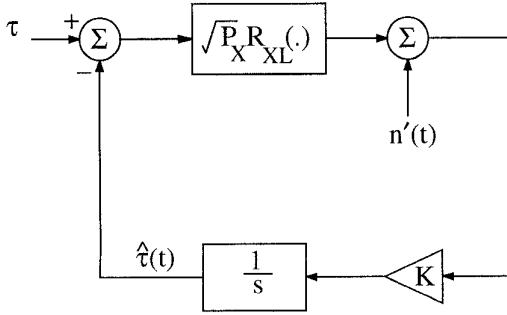


Figure 3 Non-Linear Equivalent Model of First-Order Correlation Loop

It is possible to show [4] that the cross-correlation function $R_{XL}(\tau)$ is the Hilbert transform of the auto-correlation function of a PN sequence. For a PN sequence of length N , the auto-correlation function is given by [6]

$$R_s(\tau) = \begin{cases} 1 - A|\tau| & |\tau| < T_c \\ -\frac{1}{N} & \text{otherwise} \end{cases} \quad (4)$$

where $A = (N+1)/T_c N$ and T_c is the chip time.

Therefore, $R_{XL}(\tau)$ is given by

$$R_{XL}(t) = \int_{-\infty}^{\infty} \frac{1}{\pi\tau} R_s(t - \tau) d\tau \quad (5)$$

Let

$$f_1(t) = -\frac{1}{\pi N} \ln \left[\frac{(N-1)T_c + t}{t + T_c} \right] \quad (6)$$

$$f_2(t) = -\frac{1}{\pi N} \ln \left[\frac{t - T_c}{t - (N-1)T_c} \right] \quad (7)$$

$$f_3(t) = \frac{(-At+1)}{\pi} \ln \left[\frac{t}{t - T_c} \right] \quad (8)$$

$$f_4(t) = \frac{(At+1)}{\pi} \ln \left[\frac{t + T_c}{t} \right] \quad (9)$$

and

$$f_m(t) = -\sum_{m=1}^{\infty} \frac{1}{\pi N} \ln \left[\frac{t - (mN+1)T_c}{t - (m+1)N-1)T_c} \right] - \sum_{m=1}^{\infty} \frac{1}{\pi N} \ln \left[\frac{t + ((m+1)N-1)T_c}{t + (mN+1)T_c} \right] + \sum_{m=1}^{\infty} \frac{1}{\pi} (-A(t - mNT_c) + 1) \ln \left[\frac{t - mNT_c}{t - (mN+1)T_c} \right] + \sum_{m=1}^{\infty} \frac{1}{\pi} (A(t - mNT_c) + 1) \ln \left[\frac{t - (mN-1)T_c}{t - mNT_c} \right] + \sum_{m=1}^{\infty} \frac{1}{\pi} (-A(t + mNT_c) + 1) \ln \left[\frac{t + mNT_c}{t + (mN-1)T_c} \right] + \sum_{m=1}^{\infty} \frac{1}{\pi} (A(t + mNT_c) + 1) \ln \left[\frac{t + (mN+1)T_c}{t + mNT_c} \right] \quad (10)$$

Then, $R_{XL}(\tau)$ for a single period is given by

for $-NT_c/2 < \tau \leq -T_c$

$$R_{XL}(\tau) = -\frac{1}{\pi N} \ln \left[\frac{(N-1)T_c + \tau}{-(\tau + T_c)} \right] + f_2(\tau) + f_3(\tau) + f_4(\tau) + f_m(\tau) \quad (11)$$

for $-T_c < \tau < 0$

$$R_{XL}(\tau) = f_1(\tau) + f_2(\tau) + f_3(\tau) + \frac{(A\tau+1)}{\pi} \ln \left[\frac{\tau + T_c}{-\tau} \right] + f_m(\tau) \quad (12)$$

For $0 < \tau < T_c$

$$R_{XL}(\tau) = f_1(\tau) + f_2(\tau) + \frac{(-A\tau+1)}{\pi} \ln \left[\frac{\tau}{-(\tau - T_c)} \right] + f_4(\tau) + f_m(\tau) \quad (13)$$

For $T_c < \tau < NT_c/2$

$$R_{XL}(\tau) = f_1(\tau) - \frac{1}{\pi N} \ln \left[\frac{-(\tau - T_c)}{\tau - (N-1)T_c} \right] + f_3(\tau) + f_4(\tau) + f_m(\tau) \quad (14)$$

The resulting cross-correlation function is shown in Figure 4.

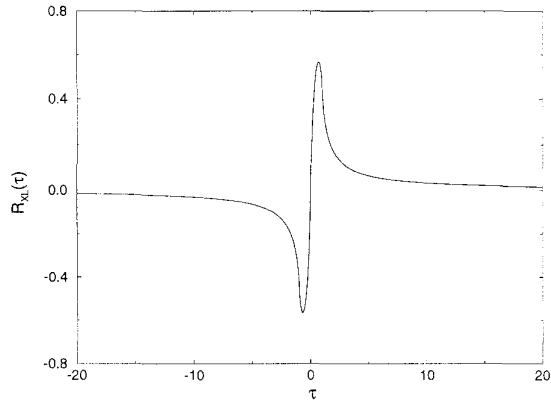


Figure 4 Cross-Correlation Function, $R_{XL}(\tau)$

III. TRACKING PERFORMANCE

The tracking performance of the new loop was studied by examining its mean-square tracking error (MSTE). If the code self-noise terms are ignored the MSTE can be computed using the Fokker-Plank technique [8]. The self-noise appears to be Gaussian [9], but it is difficult to analytically determine its level. Computer simulations have been used to establish this value.

The MSTE of the new loop will be compared to more conventional loops, such as the ELDLL. To make this comparison meaningful, both loops must have the same "bandwidth". As shown in Figure 4, the characteristic function of the new loop has an infinite slope at the origin [4] which makes it difficult to determine a linearized model. This in turn makes it difficult to describe the bandwidth of the loop. When it was not possible to determine the bandwidth using mathematical analysis, computer simulations were used to estimate a reasonable value.

Some conventional ELDLL have a broad region where the S-curve [7] is linear. In this case it is straightforward to calculate the bandwidth of the loop. If the linear region of the S-curve is small, or non-existent, a different technique must be used. We measured the bandwidth of these devices by using a sinusoidal signal for the delay

$$r(t) = \sqrt{P} s_x(t - A \sin(\omega_m t)) + n(t) \quad (15)$$

The amplitude of the modulating term, A , was stepped over a wide range of values. At each amplitude the time delay estimate $\hat{\tau}(t)$ was decomposed as

$$\hat{\tau}(t) = \hat{A} \sin(\omega_m t + \theta) + \zeta(t) \quad (16)$$

where $\zeta(t)$ is a noise term due to non-linearities and self noise. By plotting \hat{A}/A as a function of ω_m it is possible to determine a "frequency response" for each value of A . A sample of the curves generated by this algorithm are shown in Figures 5 and 6. For conventional ELDLL it is possible to develop a linearized model, and calculate the 3dB bandwidth of this model. As A increases, it will reach a point, A_o , where the ELDLL performance begins to deviate from the linear model. We assume that for normal tracking applications $A < A_o$. For the purpose of comparison, the new loop was adjusted to exhibit equal or higher bandwidths for the input amplitudes that are within the linear region of the corresponding ELDLL (i.e. for all $A < A_o$). Figures 5 and 6 illustrate the frequency response curves obtained for a 1-Chip ELDLL and the new loop adjusted to the same bandwidth.

Using this definition of bandwidth, the MSTE of the new loop is compared to a conventional ELDLL and the results summarized in Figures 7 and 8. For the MSTE test the delay is held constant and AWGN corrupts the received signal, $r(t) = \sqrt{P} s_x(t - \tau) + n(t)$.

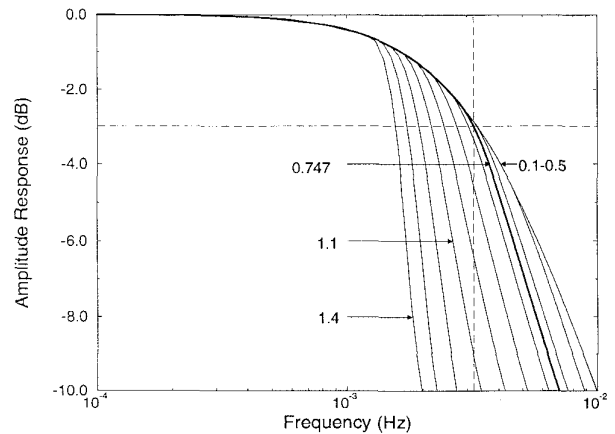


Figure 5 Frequency Response of 1.0-Chip ELDLL for Different Input Amplitudes

At high SNR, the self noise of the loop dominates the Gaussian noise term, $n(t)$. Since the new correlation loop has higher self noise than a conventional ELDLL, it will have inferior performance at high SNR. For moderate SNR the new loop is superior to a conventional ELDLL. Figure 7 implies that at very low SNR the new loop has inferior performance again. This is somewhat misleading since the ELDLL will have a very short mean time to loss of lock (MTLL) at low SNR. The new loop has a much wider

(essentially infinite) lock range, which substantially increases the MTLL.

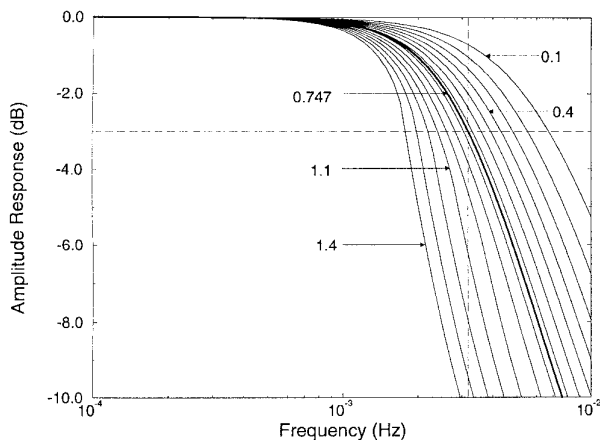


Figure 6 Frequency Response of the New Loop with the Equivalent Bandwidth of 1.0-Chip ELDLL

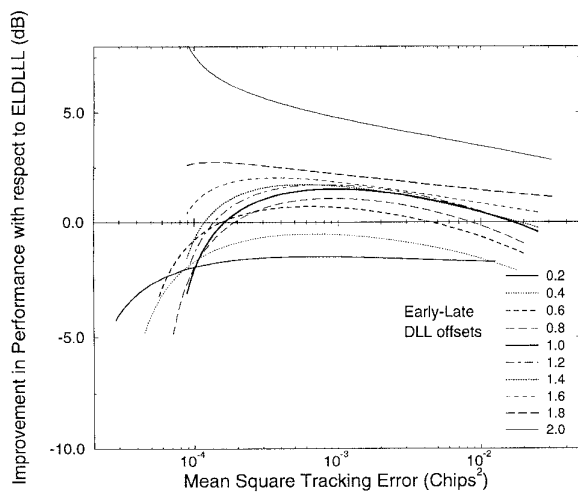


Figure 7 Noise Measurement Results

IV. CONCLUSIONS

The correlation loop with the Hilbert transformer displays superior performance over the conventional ELDLL at moderate SNR. The bandwidth definition used in the comparison should allow the new loop to track the delay dynamics at least as well as a conventional ELDLL, if not better. A feature of the new loop that was not explored in this paper is the lock range. A conventional ELDLL will have a lock range of only a few chip times. One could argue that the new loop never loses lock since it has an S-curve that remains non-zero for nearly all delay offsets. This means the new loop may be more useful than conventional loops at low SNR and during acquisition.

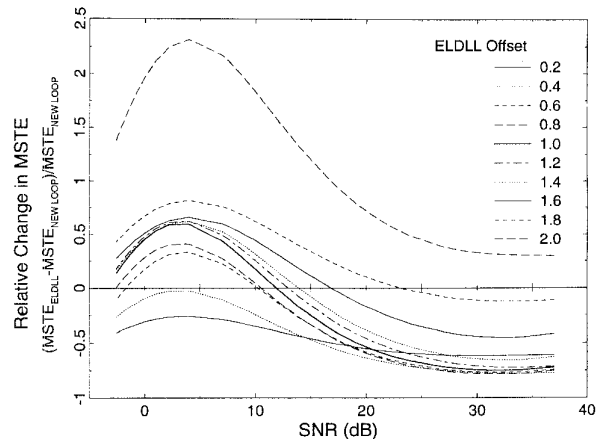


Figure 8 Normalized MSTE difference

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