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# A Powerful and Computationally Efficient Algorithm for Transmission Loss Calculation

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**Abstract**-This paper deals with the formulation of transmission loss  $P_L$  of a power system through a set of new coefficients (henceforth called A coefficients) which are extremely efficient, exact and robust and suitable for real time application. Results on a few IEEE test systems are exciting and encouraging. They demonstrate that these A coefficients faithfully represent the system loss, are extremely robust and need not be re-evaluated for changes in the system loading conditions either for evaluation of system loss or cost of generation for economic load dispatch.

## INTRODUCTION

The calculation of transmission loss  $P_L$  in a power system through B coefficients or through similar loss formulations is well known [1]. Such formulations, however, are computationally involved and therefore time consuming to calculate. This makes them unsuitable for real time application. The use of B coefficients for loss formulation is still popular with many utilities for economic load dispatch (ELD). It is well known that B coefficients are inexact and hence cannot provide the best estimates to minimize the cost of generation using ELD. These B coefficients are far from robust and must be re-evaluated with changes in the operating conditions of the system in order to achieve better economy in the system.

The main thrust in this paper is to explore the feasibility of formulating the transmission losses through a set of new coefficients which are called A coefficients. These coefficients are more exact and robust and can be efficiently realized with little computational effort so as to be amenable for real time ELD and achieve maximum benefit in the cost of generation.

## MODEL

The total transmission losses,  $P_L$ , can be expressed as a quadratic function of plant generation as:

$$P_L = \left[ \sum_{i=1}^n A_i P_i \right]^2 \quad (1)$$

where  $n$ =total number of generating plants in the system and  $P_i$  = generation at plant  $i$ .

The  $A_i$  coefficients can be evaluated in an innovative manner with little computational effort from already existing load flow information by taking advantage of the load flow Sensitivity Matrix and using a perturbation technique. This is explained briefly below.

Let  $N$  be the total number of buses in the system and bus 1 be the slack bus. From the base load flow (BLF) solution, the total transmission losses,  $P_L^{(0)}$ , and the real power generation at plant  $i$ ,  $P_i^{(0)}$  ( $i=1,2,\dots,n$ ), are all known. Thus from (1) we can write:

$$\sqrt{P_L^{(0)}} = A_1 P_1^{(0)} + A_2 P_2^{(0)} + \dots + A_r P_r^{(0)} + A_n P_n^{(0)} \quad (2)$$

Subscript (0) refers to the base load flow conditions. To evaluate all the  $A_i$  ( $i=1, \dots, n$ ) loss coefficients, we need another  $(n-1)$  number of equations similar to equation (2). These additional equations are obtained through a perturbation technique from the available BLF information as follows.

For a known small perturbation  $\Delta P^{(r)}$  at the  $r^{\text{th}}$  plant (perturbation considered one at a time for  $r=2, \dots, n$ ) and keeping P, Q conditions fixed at all other buses except at the slack bus 1, let the change in system loss be  $\Delta P_L^{(r)}$ , the change in slack bus active and reactive power be  $\Delta P_1^{(r)}$ ,  $\Delta Q_1^{(r)}$  and the change in the vector of voltage profile  $[\Delta V^{(r)}]$ . Thus, we have  $\Delta P_L^{(r)} = \Delta P_1^{(r)} + \Delta P^{(r)}$  and  $P_L^{(r)} = P_L^{(0)} + \Delta P_L^{(r)}$

$$\sqrt{P_L^{(r)}} = A_1 (P_1^{(0)} + \Delta P_1^{(r)}) + A_2 P_2^{(0)} + \dots + A_r (P_r^{(0)} + \Delta P_r^{(r)}) + \dots + A_n P_n^{(0)} \quad (3)$$

$(r=2, \dots, n)$

From equations (2) and (3) we have:

$$\sqrt{P_L^{(0)} + \Delta P_1^{(r)} + \Delta P^{(r)}} - \sqrt{P_L^{(0)}} = A_1 \Delta P_1^{(r)} + A_r \Delta P^{(r)} \quad (4)$$

$(r=2, \dots, n)$

Examining equation (4) it is seen that since  $\Delta P^{(r)}$  ( $r=2, \dots, n$ ) is a known small perturbation, any  $A_r$  ( $r=2, \dots, n$ ) can be expressed in terms of  $A_1$  only, if the change in the slack bus power  $\Delta P_1^{(r)}$  ( $r=2, \dots, n$ ) can be determined for corresponding perturbations at the plant buses ( $r=2, \dots, n$ ), considered one at a time. Once all the  $A_r$  ( $r=2, \dots, n$ ) terms are expressed in terms of  $A_1$ , these are substituted in (2) to evaluate  $A_1$  and hence all  $A_r$  are evaluated. It shall be demonstrated that the evaluation of change in slack bus power  $\Delta P_1^{(r)}$  due to a small known perturbation of  $\Delta P^{(r)}$  at the  $r^{\text{th}}$  plant bus requires little computational effort and hence generation of all  $A_r$  coefficients is extremely fast. A brief explanation for the evaluation of  $\Delta P_1^{(r)}$  is provided below.

In the last iteration of the BLF solution using the Newton-Raphson (N-R) technique in rectangular coordinates [3], the two column vectors approach zero.

For a small known perturbation  $\Delta P^{(r)}$  at the  $r^{\text{th}}$  plant bus, keeping P and Q conditions same at all buses except at the slack bus, we can write:

$$\begin{aligned} [\Delta P_1 \ 0 \dots \Delta P^{(r)} \dots \Delta Q_1 \ 0 \dots 0]^T = \\ (2N \times 1) \\ [J^*][\Delta l_1^{(r)} \dots \Delta l_r^{(r)} \dots \Delta l_N^{(r)} \Delta f_1^{(r)} \dots \Delta f_r^{(r)} \dots \Delta f_N^{(r)}] \\ (2N \times 2N) \quad (2N \times 1) \end{aligned} \quad (5)$$

where the modified Jacobian  $[J^*]$  can be generated from the BLF data. The real and imaginary parts of the voltage update vector, voltage are  $\Delta l^r$  and  $\Delta f^r$  respectively. Since bus 1 is the slack bus, (5) can be written as:

$$\begin{aligned} [0 \dots \Delta P^{(r)} \dots 0]^T = [J][\Delta l_2^{(r)} \dots \Delta l_N^{(r)} \Delta f_2^{(r)} \dots \Delta f_N^{(r)}]^T \\ (2N-2) \times 1 \quad (2N-2) \times 1 \\ (r=2, \dots, n) \end{aligned} \quad (6)$$

The matrix  $[J]$ , of dimension  $(2N-2) \times (2N-2)$ , is the usual Jacobian encountered in the N-R load flow. Equation (6) can be expressed in compact form as:

$$[\Delta V^{(r)}] = [J]^{-1}[\Delta S] \quad (7)$$

where  $[\Delta V^{(r)}] = [\Delta l_2^{(r)} \dots \Delta l_N^{(r)} \Delta f_2^{(r)} \dots \Delta f_N^{(r)}]$  is the change in the voltage profile due to a change of  $\Delta P^{(r)}$  at the  $r^{\text{th}}$  bus and  $[\Delta S] = [0 \dots \Delta P_1^{(r)} \dots 0]$ . Computation of  $[\Delta V^{(r)}]$  from (7) is straightforward, as  $[J]^{-1}$  is already available from the BLF solution and there is only one non-zero element in  $[\Delta S]$  corresponding to the known perturbation  $\Delta P^{(r)}$  at the  $r^{\text{th}}$  plant bus and hence only elements of the corresponding column of  $[J]^{-1}$  need be multiplied by  $\Delta P^{(r)}$  to obtain the elements of  $[\Delta V^{(r)}]$ . Knowing  $[\Delta V^{(r)}]$ , the change in slack bus power can be easily computed from:

$$\Delta P_1^{(r)} = \frac{\partial P_1}{\partial l_2} \Delta l_2^{(r)} + \dots + \frac{\partial P_1}{\partial l_N} \Delta l_N^{(r)} + \frac{\partial P_1}{\partial f_2} \Delta f_2^{(r)} + \dots + \frac{\partial P_1}{\partial f_N} \Delta f_N^{(r)} \quad (8)$$

$(r=2, \dots, n)$

(Note that the derivatives of the form  $\frac{\partial P}{\partial l}$  and  $\frac{\partial P}{\partial f}$  in (8) are all known from the elements of  $J^*$  of the available BLF solution). No matrix inversion at any stage is encountered in evaluating the A coefficients.

### SYSTEM STUDIES

System studies were carried out on the IEEE 14, 30, 57 and 118 bus systems [1]. The 14 and 30 bus systems have two generating plants while the 57 and 118 bus systems have four and nineteen plants respectively. Convergence criteria of .0001pu on power mismatch was used.

The transmission losses were calculated using the A coefficients, determined by the technique discussed in the model, and using B coefficients, determined by the Stevenson-Hill method. These estimated losses were then compared with the losses obtained from the load flow solution for the different cases. The results are listed in the following tables for the IEEE 14, 30, 57 and 118 bus systems. Clearly, the A coefficients predict losses that are much closer to that computed from the load flows than the B coefficient method.

In computer controlled power systems, when the system loading changes, there is always a corresponding load flow solution and hence as explained earlier in the model, the corresponding A coefficients can be evaluated in real time with trivial computational burden from the information of the available load flow solution. It was noted that the A coefficients evaluated for different operating conditions were close to the values obtained at the nominal load condition. Thus, the losses calculated by the nominal A coefficients were quite close to the losses obtained from the load flows for a very wide change in system loading. (The results for the different test cases are shown in Fig. 1, Fig. 2, Fig. 3, and Fig. 4). The numerical values are listed in tables I, II, III and IV. Thus, the nominal A coefficients prove to be quite robust in representing system losses and need not be reevaluated for different operating conditions.

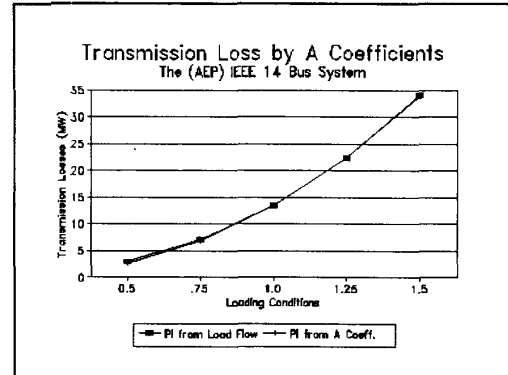


Figure 1

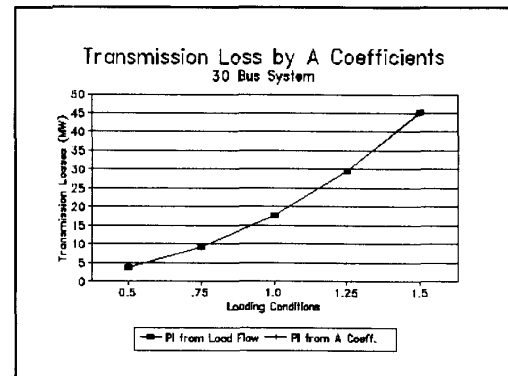


Figure 2

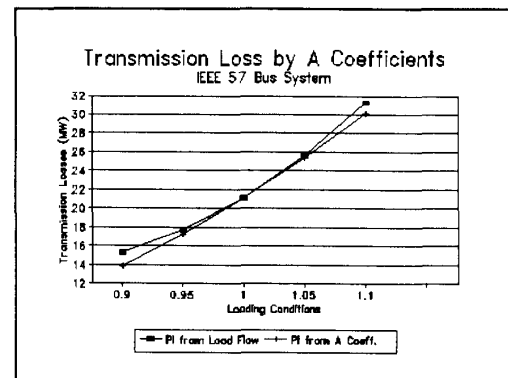


Figure 3

In comparison, the B coefficients, which were computed using the Stevenson-Hill method [3], were not as accurate and extremely involved. This computation required 'n' load flows in addition to the base load flow and involved  $n^2 + n$  multiplications

**Table I** The AEP (IEEE) 14 Bus System

Loading	$P_L$ from Load Flow (MW)	$P_L$ from B Coefficients (MW)	% Error	$P_L$ from A Coefficients (MW)	% Error
.5(P+jQ)	2.88	2.66	7.64	2.69	6.6
.75(P+jQ)	6.98	6.81	2.43	6.93	.72
1.0(P+jQ)	13.39	13.18	1.57	13.39	0
1.25(P+jQ)	22.3	22	1.34	22.29	.08
1.5(P+jQ)	33.99	33.55	1.3	33.91	.23

**Table II** The IEEE 30 Bus System

Loading	$P_L$ from Load Flow (MW)	$P_L$ from B Coefficients (MW)	% Error	$P_L$ from A Coefficients (MW)	% Error
.5(P+jQ)	3.78	3.52	6.8	3.6	4.76
.75(P+jQ)	9.18	8.95	2.51	9.15	0.32
1.0(P+jQ)	17.64	17.31	1.8	17.64	0
1.25(P+jQ)	29.51	28.96	1.86	29.45	0.2
1.5(P+jQ)	45.23	44.37	1.9	45.03	0.44

**Table III** The IEEE 57 Bus System

Loading	$P_L$ from Load Flow (MW)	$P_L$ from B Coefficients (MW)	% Error	$P_L$ from A Coefficients (MW)	% Error
.9(P+jQ)	15.34	10.78	29.72	13.90	9.5
.95(P+jQ)	17.68	13.02	26.35	17.28	2.26
1.0(P+jQ)	21.08	16.26	22.86	21.08	0
1.05(P+jQ)	25.59	20.6	19.49	25.34	.976
1.1(P+jQ)	31.28	26.06	16.6	30.07	3.87

**Table IV** The IEEE 118 Bus System

Loading	$P_L$ from Load Flow (MW)	$P_L$ from B Coefficients (MW)	% Error	$P_L$ from A Coefficients (MW)	% Error
.95(P+jQ)	114.09	117.47	-2.96	105.93	7.15
.98(P+jQ)	122.94	130.32	-6.00	121.53	1.15
1.0(P+jQ)	133.14	143.43	-7.73	133.14	0
1.02(P+jQ)	147.14	160.21	-8.88	145.84	.88
1.05(P+jQ)	176.04	194.41	-10.44	167.23	5.00

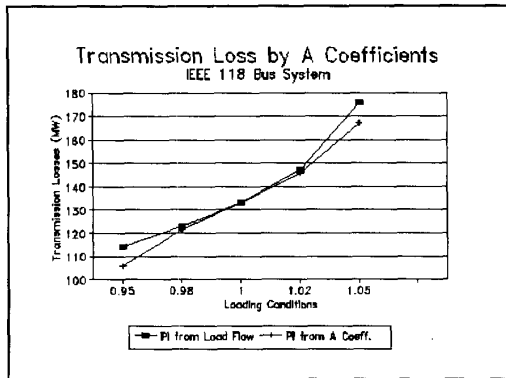


Figure 4

(where  $n$  is the total number of generating plants) to finally obtain the transmission losses. It should be appreciated that this would prove most burdensome for larger systems. Moreover, the B coefficients change with changes in operating conditions and need to be reevaluated. Since the calculation of the A coefficients use the Jacobian from the base load flow case only, they require no additional load flows. Transmission losses are predicted with only  $n + 1$  multiplications.

## CONCLUSIONS

As long as the system structure remains constant, a set of A coefficients, equal to the number of generating plants in the system can be generated extremely elegantly and efficiently from one available base load flow solution with little computational effort so as to accurately express the total transmission losses of the system. These A coefficients are extremely robust and need not be re-evaluated even for very wide changes in the loading pattern of the system, either to evaluate system transmission loss or cost of generation for ELD.

It is strongly believed that these powerful A coefficients shall greatly appeal to the utilities for their rich potential for practical application. Utilities using several sets of B coefficients for their system for loss formulation and ELD can conveniently switch over to one set of robust A coefficients to derive better efficiency and economy in their system.

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