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Differential Evolution Approach for Reactive Power Optimization of Nigerian Grid System

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Abstract - The goal of reactive power dispatch is to minimize the system losses and improve the system voltage profiles at all times. This is achieved by adjusting various generating units' excitation systems continuously, discrete tap positions of on-load tap changers of transformers as well as switching of correct doses of inductors or capacitors. This is a mixed integer non-linear optimization problem. In this paper, the differential evolution (DE), a novel evolutionary computation technique which was originally designed for continuous problems is applied to solve this problem. DE appears to ally qualities of established computational intelligence (CI) techniques with a more striking computational performance, thus suggesting the possibility of having the potential for on line applications in the control center; comparison work with other techniques is presently conducted. The developed tool was demonstrated on the Nigerian power system grid for three case scenarios preset on the power world simulator which was linked with DE for power flow calculation (fitness check of solutions). The results achieved revealed that DE procured a significant reduction of real power losses while simultaneously keeping the voltage profiles within the acceptable limits.

Index Terms - Reactive power control, differential evolution, mixed integer non-linear optimization.

I. INTRODUCTION

ONE of the principal tasks of a system operator is to guarantee that network parameters such as voltage and line loads are kept within predefined limits for high quality of services to the consumer load point and power system stability. However, changes in network topology and/or loading conditions often cause corresponding variation in voltage profiles of present day power systems. This problem can be addressed through re-distribution of reactive power sources with concomitant decrease in transmission losses. The reactive power dispatch has a two-fold goal thus: to improve system voltage profiles and minimize system losses at all times. Reactive power flow can be controlled by suitably adjusting the following facilities: tap changing under load transformers, generating units'

reactive power capability variation, switching of inductors, switching of unloaded or unused lines and flexible AC transmission system (FACTS) devices. It is therefore clear that reactive power and voltage control is a constrained, non-linear problem of considerable complexity.

Many useful studies based on classical techniques for solving the reactive power dispatch problem have been carried out [1, 2]. This includes nonlinear programming (NLP), successive linear programming, mixed integer programming, Newton and quadratic techniques. Most of these approaches can be broadly categorized as constrained optimization techniques. Notwithstanding that these techniques have been successfully employed in some sample power systems, there are several issues to be addressed with regard to real power systems. The reactive power control problem is, by nature, a global optimization with several local minima. In an attempt to circumvent the extant computational complexity and other limiting mathematical assumptions several search techniques have been proposed. They are expert system (ES), genetic algorithm (GA), tabu search, simulated annealing (SA), particle swarm optimization (PSO), etc. [3-8].

In this paper, differential evolution (DE) is explored as an optimization tool for controlling the reactive power for the improvement of the voltage profiles and reduction of system losses. The reactive power control devices such as generators, tap positions of on-load tap changers of transformers, and switching of inductors were considered in this work. Differential evolution is an improved version of GA for faster optimization. It was initially presented by Storn and Price as in [9-11] as heuristic optimization method which can be used to minimize nonlinear and non-differentiable continuous space functions with real-valued parameters. This has been extended to handle mixed integer discrete continuous optimization problems [10]. The main advantages of differential evolution are its simple structure, ease of use, robustness and its effectiveness for nonlinear constraint optimization problems with penalty functions.

The most important characteristics of DE is that it uses the differences of randomly sampled pairs of object vectors to guide the mutation operation instead of using the probability distribution function as other evolutionary algorithms (EAs). Consequently, the object vectors' differences will pass the objective functions topographical information toward the optimization process, and therefore provide more efficient global optimization. As a robust and powerful adaptive tool for solving search and optimization problems they have been proposed for various power system applications such as generation expansion [12], capacitor placement [13], etc.

The proposed DE tool for reactive power and voltage control has been developed using MATLAB Version 7.1 R14 and tested on the Nigerian transmission grid modeled on the power world simulator in detail. This provides a platform to preset a multitude of scenarios under operational realism.

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Three of the multitudes of case studies are then considered. The simulation results depict the effectiveness of DE based reactive power control tool in removing the voltage problem while simultaneously reducing the active power losses. In comparison with established computational intelligence (CI) techniques applied to the same problem [5, 6], DE promises to procure a superior computational effectiveness; thus, on line application by direct implementation into the EMS system is seen as a realistic perspective.

II. PROBLEM FORMULATION

The optimal reactive power dispatch is to optimize the steady state performance of a power system in terms of one or more objective functions while fulfilling both the equality and inequality constraints. The problem is formulated as follows:

$$\text{Min } P_{\text{loss}}(X, U) = \sum_{j=1}^{nl} P_j \quad (1)$$

Where: P_j is the real power losses in line j , nl is the number of transmission lines.

$$\begin{aligned} X^T &= [V_{L1}, V_{L2}, \dots, V_{L_{nd}}, Q_{g1}, Q_{g2}, \dots, Q_{g_{ng}}] \\ U^T &= [V_{g1}, V_{g2}, \dots, V_{g_{ng}}, Q_{C1}, Q_{C2}, \dots, Q_{C_{nc}}, T_1, T_2, \dots, T_{nt}] \end{aligned} \quad (2)$$

X is the vector of dependent variables consisting of load bus voltages V_L , and generator reactive power outputs Q_g . U is the vector of control variables comprising generator voltages V_g , shunt VAR compensation Q_C and transformer tap settings T . ng , nt , nc and nd are the number of generators, transformers, switchable VAR sources, and loads respectively.

The minimization is subject to the following operating constraints:

Power flow constraints: This represents typical load flow equations given by [13]:

$$\begin{aligned} P_{gi} - P_{di} - f_P(X, U) &= 0 \\ Q_{gi} - Q_{di} - f_Q(X, U) &= 0 \end{aligned} \quad (3)$$

Where: P_g and Q_g are the generator real and reactive power; P_d and Q_d are the load real and reactive power respectively. This is solved using the Newton Raphson load flow and DE is used to optimize the process.

Generation constraints: Generator terminal voltage set-points V_g and reactive power outputs Q_g are restricted by their limits as follows:

$$\begin{aligned} V_{gi}^{\min} \leq V_{gi} \leq V_{gi}^{\max} \\ Q_{gi}^{\min} \leq Q_{gi} \leq Q_{gi}^{\max} \end{aligned} \quad i=1, 2, \dots, ng \quad (4)$$

Transformer constraints: Transformer tap settings T are bounded as follows:

$$T_i^{\min} \leq T_i \leq T_i^{\max} \quad i=1, 2, \dots, nt \quad (5)$$

Switchable VAR source constraints: Switchable VAR compensation Q_C are restricted by their limits as follows:

$$Q_{Ci}^{\min} \leq Q_{Ci} \leq Q_{Ci}^{\max} \quad i=1, 2, \dots, nc \quad (6)$$

Security constraints: This is the constraints of the voltage at the load buses V_L as follows:

$$V_{Li}^{\min} \leq V_{Li} \leq V_{Li}^{\max} \quad i=1, 2, \dots, nd \quad (7)$$

III. OVERVIEW OF DIFFERENTIAL EVOLUTION

Differential evolution is a stochastic direct search optimization method. DE uses floating point numbers to encode the parameter variables in contrast with conventional GA that uses binary coding. It was introduced by Storn and Price in 1995 as heuristic optimization method which can be used to minimize nonlinear and non-differentiable continuous space functions with real-valued parameters. It has been extended to handle mixed integer discrete continuous optimization problem [10]. Design principles in DE's are [9]:

- Simple structure, ease of use and robustness.
- Operating on floating point format with high precision.
- Effective for integer, discrete and mixed parameter optimization.
- Handling non-differentiable, noisy and/or time dependent objective functions.
- Effective for nonlinear constraint optimization problems with penalty functions, etc.

Like the other evolutionary algorithm (EA) family, DE also relies on initial random population generation, which is then improved using selection, mutation, and crossover repeated through generations until the convergence criterion is met.

An initial population composed of vectors U_i^0 , $i=1, 2, \dots, np$, is randomly generated within the parameter space. The adaptive scheme used by the DE ensures that the mutation increments are automatically scaled to the correct magnitude. For reproduction, DE uses a tournament selection where the offspring vectors compete against one of their parents. The parallel version of DE maintains two arrays, each of which holds a population of np , D -dimensional, real value vectors. The primary array holds the current population vector, while the secondary array accumulates vectors that are selected for the next generation. In each generation, np competitions are held to determine the composition of the next generation. Every pair of randomly chosen vectors U_1 and U_2 defines a vector differential: $(U_1 - U_2)$. Their weighted differential is used to perturb another randomly chosen vector U_3 according to (8) given by:

$$U_3' = U_3 + F * (U_1 - U_2) \quad (8)$$

Where: F is the scaling factor for mutation and its value is typically $(0 \leq F \leq 1.2)$. It controls the speed and robustness of the search; a lower value increases the rate of convergence but also the risk of being stuck at the local optimum. The crossover is a complementary process for DE. It aims at reinforcing the prior successes by generating the offspring vectors out of the object vectors. In every generation, each

primary array vector U_i , is targeted for crossover with a vector like U_3' to produce a trial vector U_i according to (9).

$$U_i = \begin{cases} U_3' & \text{if } \text{rand} < CR \\ U_i & \text{otherwise} \end{cases} \quad (9)$$

Where: CR ($0 \leq CR \leq 1.0$) is a crossover constant. The newly created vector will be evaluated by the objective function and the corresponding value is compared with the target vector. The best fit vector is kept for the next generation as given by (10). The best parameter vector is evaluated for every generation in order to track the progress made throughout the minimization process; thus making the DE elitist method.

$$U_i(t+1) = \begin{cases} U_i(t) & \text{if } \text{fit}(U_i(t)) \leq \text{fit}(\text{offspring}(t)) \\ \text{offspring}(t) & \text{otherwise} \end{cases} \quad (10)$$

IV. DE BASED REACTIVE POWER DISPATCH

Differential evolution based reactive power dispatch is developed as follows:

A. Initial Population and Parameters Selection

Floating point numbers are used to encode the parameter variables. Population size (np) is one of the most important operators of the DE. In this study, a fixed population size was employed throughout the search process. Selection of the population size involves conflicting objectives. The population size can be in the range as low as $2 \times D$ to as high as $100 \times D$, depending on the problem and the available computing facilities, where D is the total number of control devices. An initial population composed of $U_i = [V_i, T_i, nc_i]$; $i=1,2,\dots,np$, is randomly generated within the parameter space using:

$$u_i = u_i^{\min} + \text{rand}_1 * (u_i^{\max} - u_i^{\min}) \quad (11)$$

Where: u_i^{\min} and u_i^{\max} are respectively the minimum and the maximum values of the parameter variables and rand_1 is a uniform random number generator in $[0, 1]$.

B. Treatment of Control Variables

Within the DE algorithm, mixed integer nonlinear programming formulation was used. The distinction between the continuous and discrete control variables is made as follows:

- Generating units' voltage set-points as continuous variables assumed to operate within the range ($0.9 \leq V_i \leq 1.1$).
- OLTC transformers considered to have 20 tap positions with a discrete step of 0.01 within the range ($0.9 \leq T_i \leq 1.1$).
- Number of reactors/condensers assumed to vary between 0 and the step size (nc_i) at each bus. Each step value is also specified. For the network used in this study the values of reactors are 30 MVar, 50 MVar and 75 MVar with step size of ranging between 1 and 4 located at 8 different buses.

C. Handling of Constraints

The reproduction operation of DE can extend the search outside the range of the parameters. A simple strategy is adopted in this study to ensure that the parameter values lie within the allowable range after reproduction. Any parameter that violates the limits is replaced with random values using:

$$u_i = \begin{cases} u_i^{\min} + \text{rand}(u_i^{\max} - u_i^{\min}) & \text{if } u_i < u_i^{\min} \text{ or } u_i > u_i^{\max} \\ u_i & \text{otherwise} \end{cases} \quad (12)$$

A penalty function approach proposed in [10] is adopted in this study to handle the voltage limits violations. The objective function is formulated as follows:

$$f_{obj} = (P_{loss} + a) * \prod_{i=1}^{nd} c_i^{b_i} \quad (13)$$

Where:

$$c_i = \begin{cases} 1 + s_i * V_{Ld} & \text{if } V_{Li} > V_{Li}^{\max} \text{ or } V_{Li} < V_{Li}^{\min} \\ 1 & \text{otherwise} \end{cases}$$

$$V_{Ld} = \begin{cases} V_{Li} - V_{Li}^{\max} & \text{if } V_{Li} > V_{Li}^{\max} \\ V_{Li}^{\min} - V_{Li} & \text{if } V_{Li} < V_{Li}^{\min} \end{cases}$$

$s_i \geq 1$ and $b_i \geq 1$. The constant a is used to ensure that only a non-negative value is assigned to the objective function. Constant s is used for appropriate scaling of the constraint function value. The exponent b modifies the shape of the optimization surface.

D. Realization of DE Based Reactive Power Dispatch

The flowchart of the developed tool is shown in Fig. 1 and its computational procedure is described as follows:

Step I: At the initialization stage, the relevant DE parameters as shown in Table I are defined. Also relevant power system data required for the computational process are actualized from the data files.

TABLE I: OPTIMUM PARAMETER SETTINGS FOR DE BASED TOOL

Control Parameters	Differential Evolution
Maximum generation, gen^{\max}	200
Number of control devices, D	22
Population size, np	$3D$
Scaling factor for mutation, F	0.8
Crossover constant, CR	0.6
Objective function scaling constant, a	7.0
Constraint function scaling constant, s	1
Optimization surface shape modifiers, b	1

Step II: Run the base case Newton Raphson load flow on the power world simulator [15] to determine the initial load bus voltage and active power losses respectively.

Step III: Each control device is treated as described in subsection B above. The randomly generated initial population comprises the control device variables within the parameter space using (10). The objective function for each vector of the population is computed using (12). The vector with the

minimum objective function value (the best fit) so far is determined.

Step IV: Update of the generation count.

Step V: Mutation, crossover, selection and evaluation of the objective function as described in Section III are performed. If parameter violation occurs, (10) is applied appropriately to generate randomly the parameter value. The elitist strategy is also applied: keeping track of the fittest vector.

Step VI: If the generation count is less than the preset maximum number of generations, go to step IV. Otherwise the parameters of the fittest vector are returned as the desired optimum settings. With the optimal settings of the control devices, run the final load flow to obtain the final voltage profiles and the corresponding system active power losses.

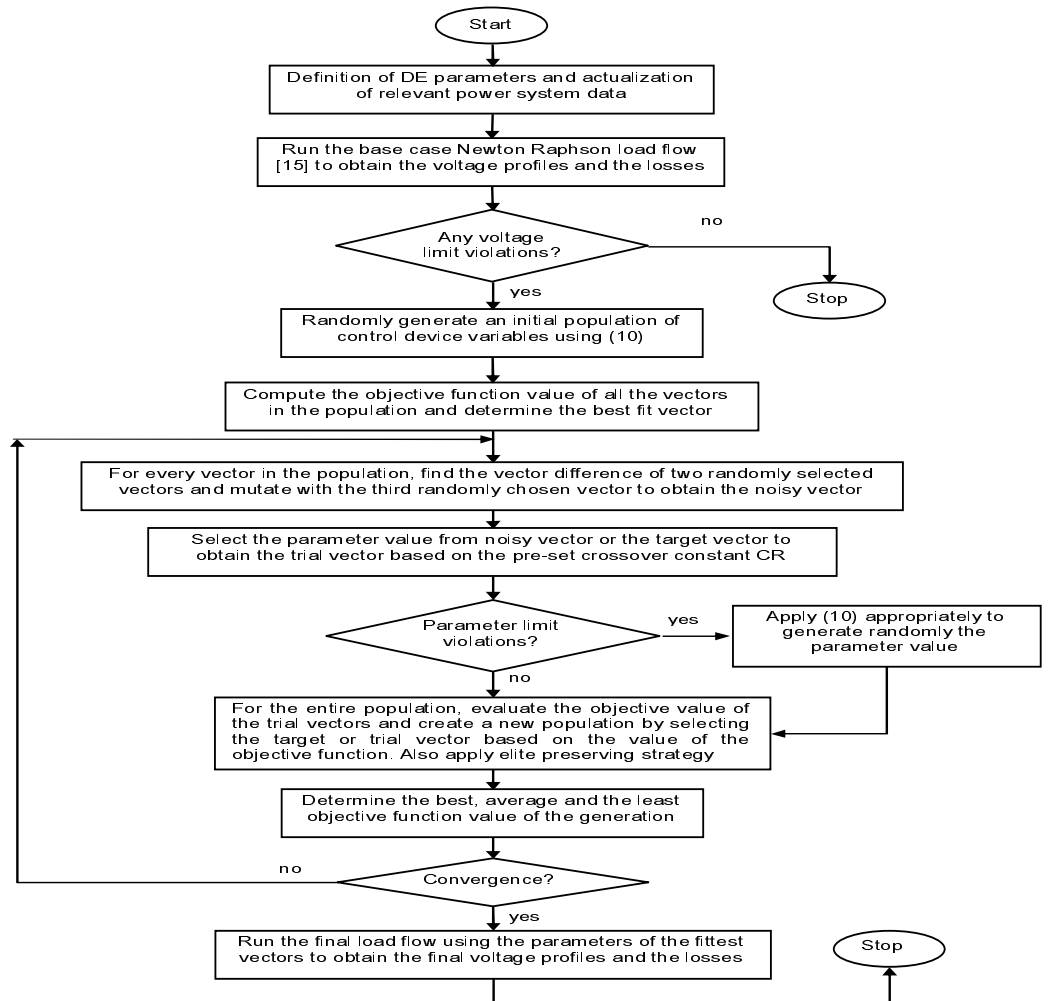


Fig. 1: Flowchart of DE based reactive power dispatch

V. SIMULATION RESULTS AND DISCUSSION

The feasibility and the effectiveness of the developed tool is demonstrated on the Nigerian 330 kV, 31-bus transmission grid. The above described DE procedure was implemented using MATLAB V 7.1 R14 for Windows whereby the power flow calculation needed for fitness check of solutions (individuals) is calculated via embedded calls of the power world simulator on which the Nigerian grid was replicated in operational detail. This provided a platform to preset a multitude of scenarios under operational realism.

The power system comprises: 7 generating units (4 thermal units and 3 hydro), 7 machine transformers equipped with tap changers, and compensation reactors of different discrete values located at 8 buses. The single line diagram of the network is depicted in Fig. 2 and the network data can be obtained in [16]. Three of the multitudes of case studies are then considered and the results presented.

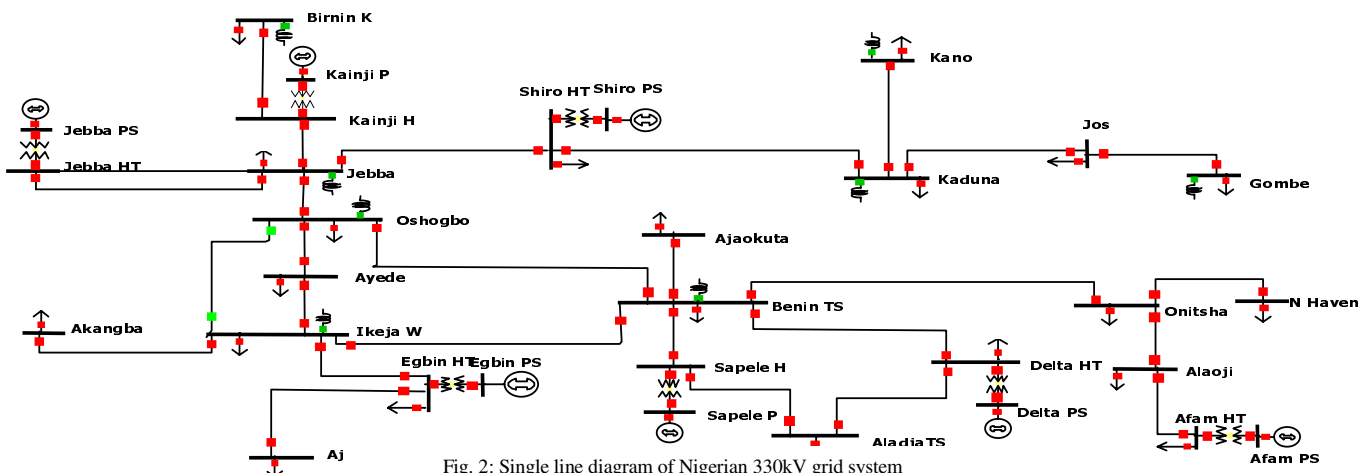


Fig. 2: Single line diagram of Nigerian 330kV grid system

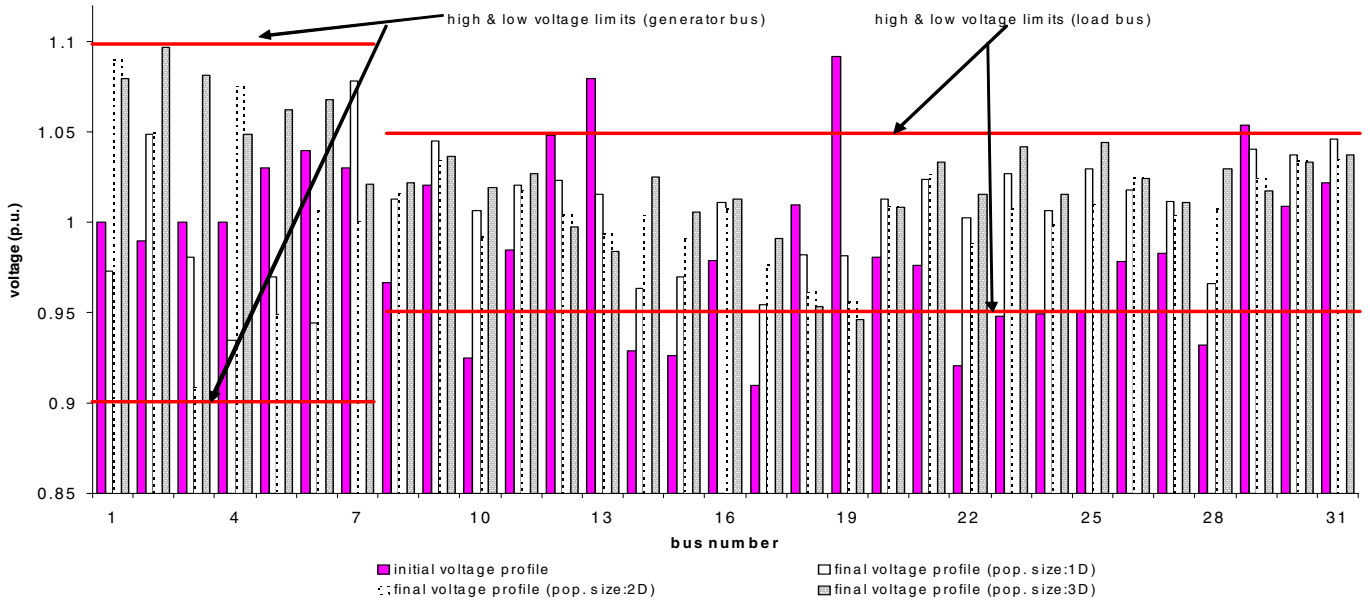


Fig. 3: Voltage profile correction for different population sizes

A. Case Study 1: Wrong Tap Settings of Transformers and Inductors

With all the 33 transmission lines operated, a scenario was preset on the power world simulator by heuristic based wrong tap settings of the machine transformer taps. Two of the four 75 MVar reactors at bus 8 (Benin TS), and bus 10 (Ikeja W) were wrongly switched on. There were also load reductions at some load points. These actions led to voltage limit violations in 10 nodes.

The developed DE tool was applied to solve this problem. The effect of different DE parameters such as population size, np and scaling factor for mutation F were investigated using this case study for 200 generations. Simulation results of the voltage profiles for three different population sizes: 1D, 2D and 3D, are shown in Fig. 3.

Fig. 4 also shows the convergence characteristics with the different population sizes.

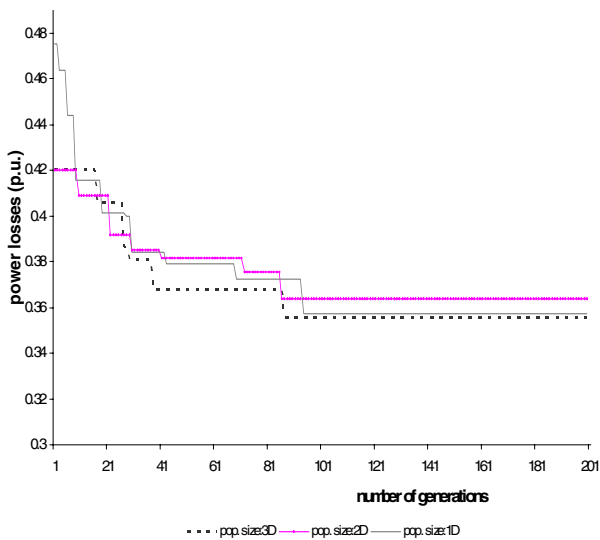


Fig. 4: Comparison of convergence characteristics (Case study 1)

Other useful results of the effect of population size are summarized in the Table II. The effect of scaling factor was

equally investigated on this scenario (results not shown here). Based on the results obtained, the optimum parameter settings were determined and depicted in Table I. It can be seen that the approach was able to keep the voltage at all buses within limits for all the three population sizes. Maximum power loss reduction of 11.16% (from 40.07 MW to 35.59 MW) was achieved using the approach for the population size 3D.

TABLE II: EFFECT OF POPULATION SIZES ON DE PERFORMANCE

Population Size	D	2D	3D
Initial power losses (MW)	40.07	40.07	40.07
Final power losses (MW)	35.74	36.40	35.59
Power loss reduction (%)	10.81	9.16	11.18
Total no. of function evaluations	4,400	8,800	13,200
Gen. at minimum loss reduction	95	88	87
No. of function evaluation at minimum loss reduction	2090	3872	5742

B. Case Study 2: Disconnection of a Transmission Line

Here, the system was initially operating as in case study 1. Interrupting the transmission line between Ikeja W and Benin TS (Fig. 2) resulted in voltage limits violations at 12 buses.

The DE based reactive power dispatch tool was used to solve this problem. The algorithm is able to solve the voltage problem connected with 14.03% power loss reduction (from 42.05 MW to 36.15 MW). This value was obtained in 148 generations. The voltage profile correction is as shown in Fig. 5.

C. Case Study 3: Load Modifications and Line Removal

A number of loads at some buses were changed in order to modify the load situation in the network and the voltage controllers of the seven generating units randomly set at values ranging from 1.0 to 1.04 p.u.. The transformer taps were all set at the nominal values of 1.0. Two transmission lines: Oshogbo – Benin TS (11-8) and Oshogbo - Ikeja W (11-10) were opened. As a result of this action, both under and over voltage problems were induced in the power system.

The result of the application of DE is depicted in Fig. 6. It can be seen clearly that the approach succeeded in keeping the voltage at all buses within the limits. Power loss reduction of 4.20% (from 46.45 MW to 44.58MW) was achieved with this approach in 168 generations.

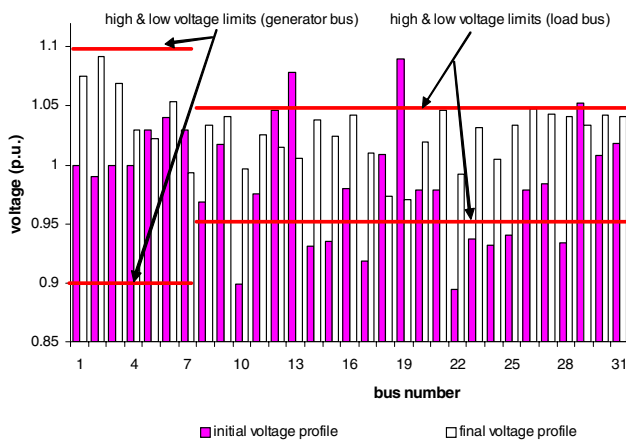


Fig. 5: Voltage profile correction using DE for case study 2

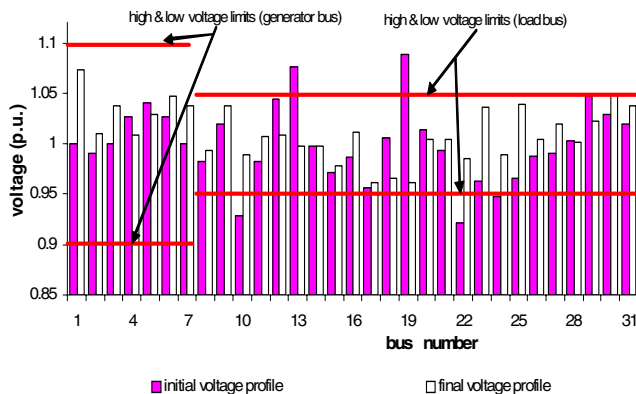


Fig. 6: Voltage profile correction using DE for case study 3

D. System Performance

The case studies have shown that the goals of voltage profile correction and power loss reduction were achieved within moderate numbers of generations. Rather, the pilot implementation presented here was made in the MATLAB environment which does not lead to practicable computation times; but it can be assessed that an EMS implementation would take few minutes for grids of reasonable size. Anyway, in comparison with other existing approaches based on computational intelligence techniques this seems to be superior; work of direct comparison is presently conducted.

VI. SUMMARY AND CONCLUSIONS

This paper has presented the application of the differential evolution technique for solving the reactive power / voltage control problem, featuring the advantages of established computational intelligence techniques. Differential evolution seems to provide a better computational performance, thus raising the aptitude for on line application in the control centers. Three case studies were conducted on the Nigerian power system replicated on the power world simulator which was linked with DE for power flow calculation (fitness check of solutions). The results revealed that the DE based reactive

power dispatch is an efficient tool in keeping the abnormal bus voltages within the prescribed limits at the same time returning lower system transmission power losses. From the practical point of view, it is pertinent to curtail the number of control devices employed to alleviate bus voltage problems. It is also feasible to integrate a pre-selection mechanism into the DE to select the control devices *a priori*. This will be an added advantage to the computational time of the DE since the population size depends on the number of control variables. This will be pursued in the future research thrust.

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