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ESTUDIO DE CASO

## Diseño de controladores robustos para una columna de destilación

### Robust controllers design for a distillation column

Luini Leonardo Hurtado Cortés<sup>1</sup>, Jhon Alejandro Forero Casallas<sup>2</sup>, Luís Villarreal López<sup>3</sup>**Fecha de recepción:** 12 de septiembre de 2015**Fecha de aceptación:** 23 de agosto de 2016

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#### ABSTRACT

**Objective:** This paper presents the design of robust controllers for a distillation column using a graphical editor. The graphical tool simplifies the design process.

**Methods:** The graphical editor helps calculate weighting functions of the performance criteria and the uncertainty of the plant due to disturbances, and perform their interconnection. Subsequently, three types of controllers were designed and analyzed the robustness.

**Results:** We performed closed loop simulations on the non-linear model, modifying the values of the feed rate  $F$  and the feed composition  $zF$ , and assuming uncertainty in the parameters of the control valves and sensors.

**Conclusions:** Closed loop simulations showed that the process achieves its stability by meeting the performance criteria. The obtained results for bottom compositions showed errors under 0.8%. The *LMI* controller obtained satisfactory results for top and bottom compositions under any kind of disturbance.

**Keywords:** distillation process, nominal performance, nominal stability, parametric uncertainties, robust performance, structured singular value.

#### RESUMEN

**Objetivo:** Este artículo presenta el diseño de controladores robustos para una columna de destilación usando un editor gráfico. Con el desarrollo de la herramienta gráfica el proceso de diseño se simplifica.

**Métodos:** El editor gráfico ayuda a calcular las funciones de peso de los criterios de desempeño y de la incertidumbre de la planta debido a las perturbaciones y realiza su interconexión. Posteriormente, se diseñaron tres tipos de controladores y se analizó su robustez.

**Resultados:** Se realizaron simulaciones en lazo cerrado sobre el modelo no lineal, modificando los valores para la tasa de alimentación  $F$  y de la composición de alimentación  $zF$ , y se realizaron simulaciones suponiendo incertidumbre en los parámetros de las válvulas de control y los sensores.

**Conclusiones:** De las simulaciones en lazo cerrado, se muestra que el proceso logra su estabilidad cumpliendo con los criterios de desempeño. Los resultados obtenidos para la composición en el fondo, presentaron errores inferiores al 0.8%. El controlador *LMI* tuvo resultados satisfactorios para la composición en la cima y en el fondo bajo cualquier tipo de perturbación.

1 Mechanical Engineer, magister of Engineering, doctor of Engineering. Associate Professor Universidad Distrital Francisco José de Caldas. Bogotá, Colombia. Contact: [llhurtadoc@gmail.com](mailto:llhurtadoc@gmail.com)

2 Mechanical Engineer, magister of Engineering. Assistant Professor Universidad Distrital Francisco José de Caldas. Bogotá, Colombia. Contact: [jaforeroca@gmail.com](mailto:jaforeroca@gmail.com)

3 Industrial Engineer, magister of Engineering. Assistant Professor Fundación Universitaria Agraria de Colombia. Bogotá, Colombia. Contact: [luis595@yahoo.com](mailto:luis595@yahoo.com)

**Palabras clave:** proceso de destilación, desempeño nominal, estabilidad nominal, incertidumbres

paramétricas, desempeño robusto, valor singular estructurado.

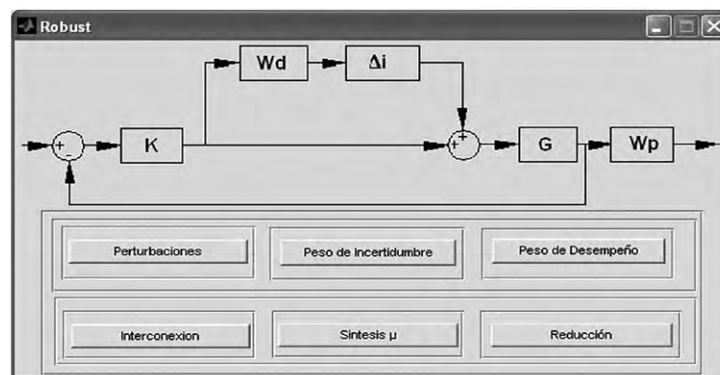
## INTRODUCTION

The unavoidable presence of uncertainties may jeopardize the achievement of the control objectives of a system. In the last thirty years there has been a growing interest for the design of uncertainty tolerant control systems. These uncertainties are usually present because of the approximation errors, poor information about the system parameters, nonlinearities and changes in the operation conditions. The unknown response and order of the systems outside the operation bandwidth and the imperfection of the process components also provides uncertainty to the manipulated inputs.

Robust Control Theory gives a set of techniques for the analysis and controller design for systems under disturbances using techniques as the *DK iteration* (Balas, Chiang, Packard, & Safonov, 2009), *Linear Matrix Inequalities (LMI)* (Gahinet, Nemirovski, Laub, & Chilali, 2004) and the *Riccati Algebraic Equation (REA)* (Zhou & Doyle, 1999), among others. The objective is to obtain a control law which maintains the system response and the error signal within preset limits despite the effect of disturbances.

However, during the design stage, these considerations may be challenging. Thus, it has been developed collaborative tools for controllers design. In (Lorenzo, López, & García, 2004), they developed a software tool for identification and PID,  $H_2$  and  $H_\infty$  controller design using Builder C++, this software can be used in real and simulated systems. On the other hand, (Vásquez, Morilla, & Sanmiguel, 2000) propose a graphical editor for PID controllers design for systems modeled using Matlab®/Simulink®, and (Puerto, Fernández, Jiménez, Ñeco, & García, 2002) describe a remote laboratory for controller design using the same software. Despite of these tools, they do not implement robust control strategies.

In this paper, it is introduced a new graphical tool under Robust Control Theory (Hurtado & Villarreal, 2008), developed from the “easy to use” approach (Hurtado & Forero, 2014), that allows the user to interpret partial results, reduce the calculations time, run multiple analysis and find an optimal controller. The developed interface is tested on a classical prototype problem in order to verify its functionality. The graphical editor was implemented using the GUI from Matlab® with commands from the *Robust Control Toolbox* (figure 1).



**Figure 1.** Start window for the developed graphical editor for robust controller design.

Source: own work.

## MATERIALS AND METHODS

### Description of the distillation process

Distillation is a separation process by which a mixture is heated to an intermediate temperature among the boiling point of its components in order to separate them. The most volatile component evaporates first and after the separation; it is cooled until its condensation point where the two components are in liquid state. In this paper, in order to test the functionality of the designed tool, a binary continuous column composed of inner plates is treated (figure 2).

The feed flow ( $F$ ), feed composition ( $zF$ ) and feed liquid fraction ( $q$ ) may be disturbed causing changes in the distillation process. The four remaining input variables, the reflux ( $L$ ), the distilled ( $D$ ) the evaporated ( $V$ ) and background ( $B$ ) are used to regulate the process (Luyben, 1992).

### Distillation process modeling

In this section we present the model of a distillation column proposed by (Skogestad, 1997) and revised in (Villarreal, 2005). Assuming some conditions,

the steam and liquid composition in the same stage can be related using the equation (1).

$$y_i = \frac{\alpha x_i}{1 + (\alpha - 1)x_i} \quad (1)$$

The liquid flux depends on the retained liquid in the previous step and the steam flux according to equation (2).

$$L_i = L0_i + \frac{M_i - M0_i}{\tau_L} + (V - V0)_{i-1} \times \lambda \quad (2)$$

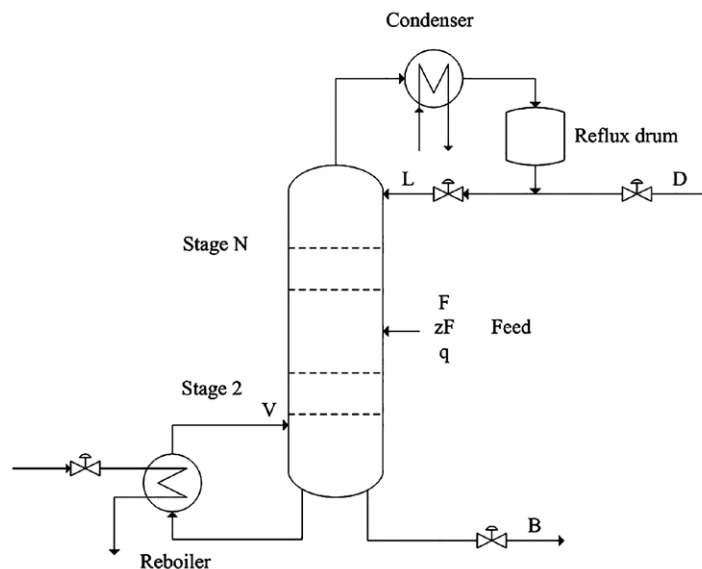
The total mass balance in the stage  $i$ , according (Himmelblau & Riggs, 2012) is:

$$\frac{dM_i}{dt} = L_{i+1} - L_i \quad (3)$$

And the mass balance for the lightweight component on each stage according to equation (4) as follows.

$$\frac{d(M_i x_i)}{dt} = L_{i+1} x_{i+1} + V_{i-1} y_{i-1} - L_i x_i - V_i y_i \quad (4)$$

This initial model has seven input variables ( $L$ ,  $V$ ,  $D$ ,  $B$ ,  $F$ ,  $zF$ ,  $q$ ) and four output variables distilled composition  $x_D$ , bottom product  $x_B$ , reboiler level  $M_B$



**Figure 2.** Continuous distillation column.

Source: Skogestad, 1997.

and condenser level  $M_D$ . The control system has to ensure the system stability and the product quality. However, the stability is compromised because of the condenser and the column, if one of these elements is empty or saturated, and then the system will collapse. Hence, two control systems has to be designed, one for the composition using the variables  $L$  and  $V$ , and other for the level using  $D$  and  $B$ . This control structure is known as the  $LV$  (Luyben, 1992).

With the implementation of the two uncoupled proportional controllers, the level control in the

condenser and in the column can be performed. On the other side, using a condenser in the column feed, the liquid feed fraction  $q$  can be 1. With these adjustments, it is obtained a stable nonlinear model with 6 inputs variables, two output variables and eighty-two states. After its linearization and reduction in order to decrease the low frequency error, it is obtained a simplified equivalent model with four input variables ( $L, V, D, B$ ), two output variables ( $x_D$  y  $x_B$ ) and six states (Villarreal, 2005), as shown in equation (5).

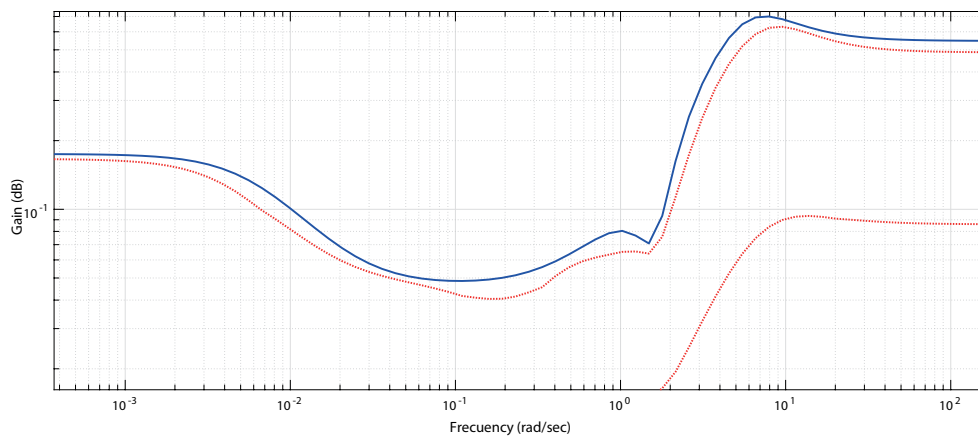
$$\mathbf{A} = \begin{bmatrix} -0.0051 & -0.0047 & 0.0077 & -0.0015 & 0.0025 & -0.0142 \\ 0.0009 & -0.2083 & 0.3764 & -0.3937 & 0.2024 & -0.4051 \\ -0.0055 & 0.0520 & -0.3930 & 0.6316 & -0.3874 & 1.0683 \\ -0.0025 & 0.3732 & -0.7112 & -0.2285 & 0.3625 & -0.7663 \\ 0.0024 & -0.0832 & 0.2836 & -0.1657 & -0.1574 & 2.2683 \\ 0.0081 & -0.5756 & 1.1078 & 1.3443 & -2.1471 & -6.4120 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0.0719 & 0.0706 & 0.7068 & -0.7151 \\ 0.1619 & -0.0192 & -0.5242 & -0.3863 \\ 0.2555 & 0.0038 & 0.7413 & 0.0089 \\ -0.0138 & 0.1717 & 0.3968 & 0.0652 \\ 0.0661 & 0.1518 & -0.1766 & 0.1755 \\ -0.2671 & 0.4570 & -1.3020 & -0.1427 \end{bmatrix} \quad (5)$$

$$\mathbf{C} = \begin{bmatrix} 0.6242 & -0.1060 & -0.3016 & -0.2349 & -0.0224 & 1.0348 \\ 0.7946 & 0.6628 & -0.7238 & 0.3690 & -0.2982 & 0.9617 \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} 0.0841 & -0.0469 & 0.0796 & 0.0654 \\ 0.0066 & -0.0296 & 0.0889 & -0.0060 \end{bmatrix}$$

### Uncertainty modeling of the plant

The valves and sensors considered in the distillation column are modeled with first order transfer functions (Creus, 2011). On the other side, the disturbance on each parameter causes a set of models that differ from

the nominal model; this can be observed with the “*Perturbaciones*” button. The highest difference represents the weight of the uncertainty of the plant. Figure 3 shows the result obtained with the graphical editor using the “*Peso de incertidumbre*” button.



**Figure 3.** Adjustment for uncertainty data of the plant for the distillation process.

Source: own work.

## Performance criteria modeling

The performance criteria are specifications given for the design of a control system. These criteria are considered for the disturbance and noise rejection and for determining the maximum steady state error (Dullerud & Paganini, 2000). They can be expressed through a weight function introduced into the plant. In this case, they are found by pressing the “Peso de desempeño” button in the graphical editor.

## Generalized plant obtaining

The connection between the nominal model, the plant disturbances and the performance criteria is known the generalized plant (Skogestad, 1997). The graphical editor can find this model with the “Interconexión” button. In this case, we present

the interconnection of nine systems forming a new system of 54<sup>th</sup> order.

## Synthesis and analysis of robust controllers

Through the “Análisis” button of the editor, three different kinds of controllers can be designed, *Optimal Control*  $H_\infty$ , *REA* and *LMI*. With each of them, a robustness test has to be conducted. First we estimate the maximum *singular structured value*  $\mu$  (Packard & Doyle, 1993). If  $\mu < 1$  the closed loop robustness is guaranteed. If this condition is not reached some adjustments have to be done changing the performance, the plant, valves and sensors uncertainty weights.

In this case, the table 1 shows the  $\mu$  values obtained for each controller for two different performance weights. In the second case, the three controllers reach closed loop robustness.

**Table 1.**  $\mu$  values for each controller with two different performance weights.

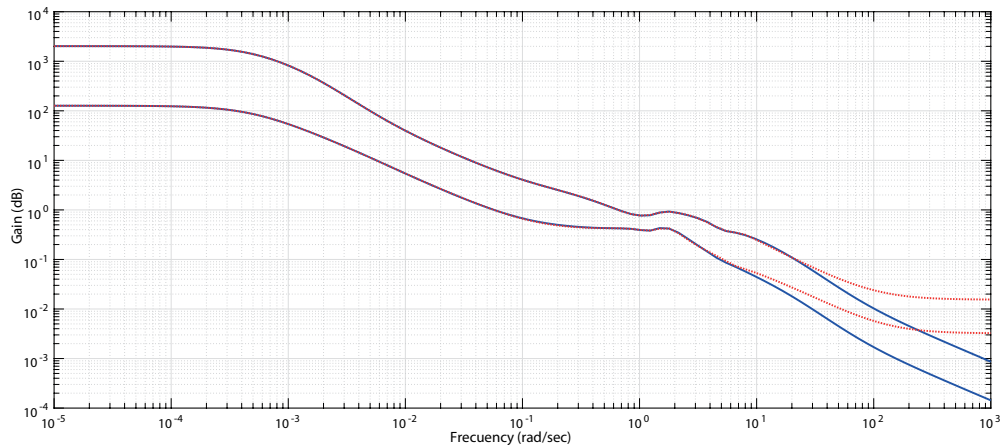
Performance weights	$\mu$ values		
	$H_\infty$	REA	LMI
$Wp(s) = \frac{0,02s + 0,1}{s + 0,005}$	1,04897	1,04951	0,982902
$Wp(s) = \frac{0,02s + 0,1}{s + 0,015}$	0,996424	0,995785	0,893883

Source: own work.

## Order reduction of the obtained controllers

The three designed controllers are of 54<sup>th</sup> order. To reduce the amount of calculations and required time, a reduction may be useful. The editor allows the selection of three different reduction methods

with the button “Reducción”: Residualization (used here), Truncation and Hankel norm approximation (Green & Limebeer, 1995). Figure 4 shows the comparison between the singular structured values of the reduced system with the non-reduced system.



**Figure 4.** Comparison between the reduced model (dotted line) and non-reduced model (solid line).

Source: own work.

## RESULTS AND DISCUSSION

To verify the robustness of the system with the obtained controllers, closed loop simulations on the non-linear model were performed, modifying the values of the feed rate  $F$  and the feed composition  $zF$ . From the obtained results in table 2, it may be inferred that the  $LMI$  controller meets the desired specifications, in the top and the bottom, while the  $H_\infty$  and  $REA$  controllers only satisfy the required specification for the product in the top.

Figure 5 shows the composition obtained with the  $LMI$  controller. Additionally, table 3 also shows that the  $LMI$  controller satisfies the requirements in the top and bottom compositions while  $H_\infty$  y  $REA$  only meets the product requirements in the top.

Finally, simulations were conducted supposing uncertainty in the valves parameters and sensors. These results are shown in tables 4 and 5 respectively.

**Table 2.** Controller response for  $F = 0.99$  y  $zF = 0.2$ .

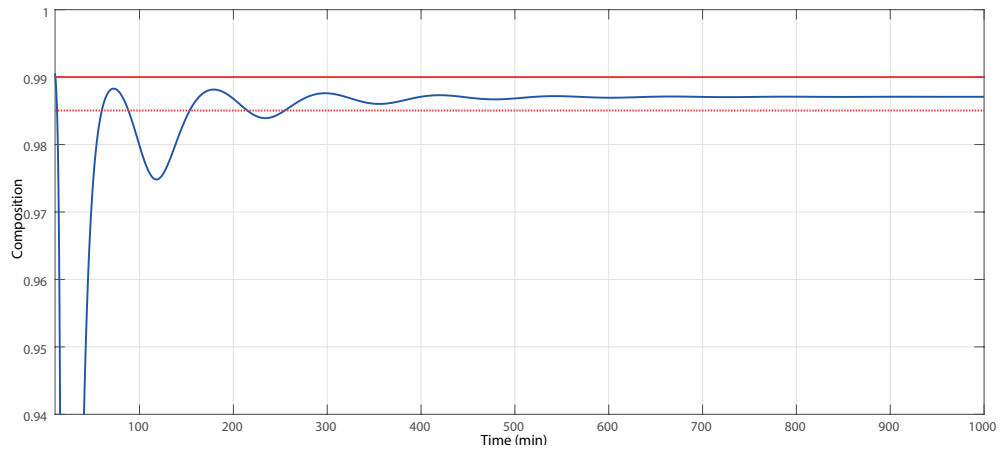
	$H_\infty$	REA	LMI
Top Composition Error	< 0,5%, t = 432 min	< 0,5%, t = 432 min	< 0,5%, t = 244 min
Bottom Composition Error	< 0,6%, t = 658 min	< 0,6%, t = 658 min	< 0,5%, t = 302 min

Source: own work.

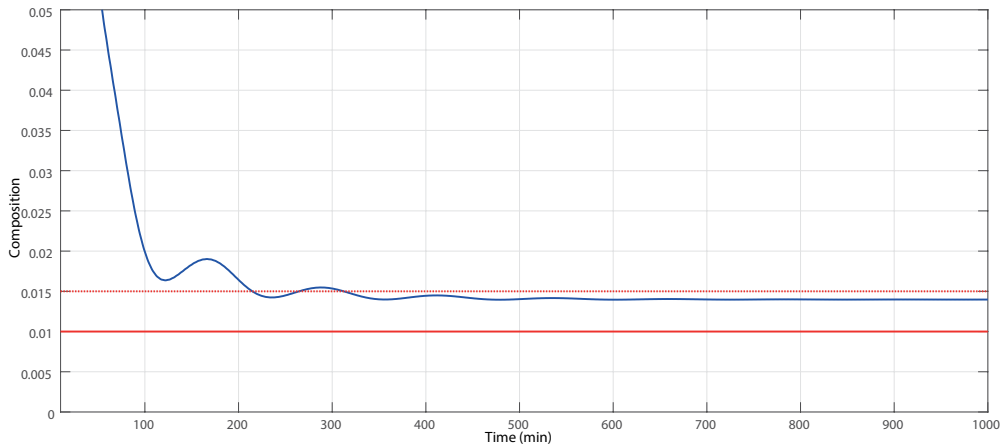
**Table 3.** Controller response for  $F = 1.01$  zF = 0.8.

	$H_\infty$	REA	LMI
Top Composition Error	< 0,5%, t = 312 min	< 0,5%, t = 310 min	< 0,5%, t = 194 min
Bottom Composition Error	< 0,8%, t = 532 min	< 0,8%, t = 523 min	< 0,5%, t = 520 min

Source: own work.



(a) top composition



(b) bottom composition

**Figure 5.** System response with  $F = 0.99$  y  $zF = 0.2$  (LMI controller).

**Source:** own work.

**Table 4.** Controller response for input uncertainty.

	$H_\infty$	REA	LMI
Top Composition Error	< 0,5%, t = 232 min	< 0,5%, t = 232 min	< 0,5%, t = 134 min
Bottom Composition Error	< 0,7%, t = 423 min	< 0,7%, t = 423 min	< 0,5%, t = 205 min

**Source:** own work.

**Table 5.** Controller response for output uncertainty.

	$H_\infty$	REA	LMI
Top Composition Error	< 0,5%, t = 318 min	< 0,5%, t = 317 min	< 0,5%, t = 152 min
Bottom Composition Error	< 0,8%, t = 468 min	< 0,8%, t = 468 min	< 0,5%, t = 348 min

**Source:** own work.

## CONCLUSIONS

In the last years, Robust Control Theory has shown its effectiveness in the study of systems under uncertainty in their parameters. However, the design process of these controllers may be a difficult task. With the development of the graphical tool the design process is simplified, because the user obviates the need to understand the syntax of the various commands and is limited only to enter the data requested for each dialog window.

There is always a difference between a nominal model of a process and a set of models under uncertainty due to disturbances. The introduced graphical tool established this difference as a weight function. Besides, another function guarantees the achievement of the performance requirements of the control systems, rejecting noise and process disturbances.

The general control configuration involves uncertainties, the controller and the generalized plant. With the graphical editor and the generalized plant, the connection of these functional blocks can be easily accomplished. Later, three different kinds of controllers were obtained allowing the robustness analysis using the singular structured value  $\mu$ . Usually, the higher order of a controller hinders its real implementation. This problem was addressed reducing the controller order using residualization.

Finally, from the results, it can be concluded that the  $H_\infty$  and REA controllers produce satisfactory results only for the top composition under any kind of disturbance (less than 0.5% of error). The obtained results for bottom compositions showed errors under 0.8%. The LMI controller obtained satisfactory results for top and bottom compositions (less than 0.5% of error), under any kind of disturbance. On the other hand, the complete set of designed controllers for the LV structure accomplishes the required composition in the top (less than 0.5% of error).

## ACKNOWLEDGEMENTS

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