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## Determination of the Thickness and Dielectric Constant of a Dielectric Slab Backed by Free-Space or a Conductor through Inversion of the Reflection Coefficient of a Rectangular Waveguide Probe

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**Abstract** – Evaluation of thickness and material properties of coatings and dielectric slabs is an important practical issue. Microwave nondestructive testing techniques, using open-ended rectangular waveguide and coaxial probes have shown great potential for this purpose. However, to evaluate one parameter requires that the other be known a priori. This paper discusses the use of a relatively efficient method for evaluating both parameters simultaneously from measurements of the reflection coefficient of a test material. Results of two cases as well as a brief discussion of the limitations of the technique are provided in this paper.

**Keywords** – microwaves, nondestructive testing, composites

### I. INTRODUCTION

Microwave nondestructive testing and evaluation (NDT&E) techniques, utilizing open-ended rectangular waveguide (OERW) probes, have been shown in the past to be effective means for quality control and in-service inspection of composite materials. In particular, microwave techniques are capable of detecting delaminations, disbonds, and impact damage in both thick composite panels and complex composite sandwich structures, in addition to many other relevant properties associated with such composites (i.e., resin cure monitoring) [1].

For quality control or on-line monitoring of composite laminates backed by free-space or a conducting plate, geometrical information (i.e., laminate thickness) as well as property information (i.e., porosity) about the state of laminate is required. An example of such an environment is the on-line inspection of thermal barrier coatings used in aircraft engines or other critical components. However, when using microwave NDT techniques sufficient knowledge of the dielectric properties of the composite must be known *a priori* if its thickness is to be accurately evaluated, and conversely its thickness must be known *a priori* if its dielectric properties are to be determined. From the knowledge of the dielectric properties, one may then be able to determine desired properties such as porosity, state of cure, etc. An electromagnetic model, evaluating the reflection properties of such composites using an open-ended rectangular waveguide, has already been developed and extensively implemented for the two applications mentioned above [2].

Microwave techniques are primarily based on the measurement of complex dielectric properties of a material. The complex dielectric constant of a given material (referenced to the permittivity of free-space) consists of the permittivity (i.e. real part,  $\epsilon_r'$ ), which represents the material's ability to store microwave energy, and the loss factor (i.e. imaginary part,  $\epsilon_r''$ ), which represents the material's ability to absorb microwave energy. The dielectric constant is heavily influenced by the chemical and physical makeup of a material. Additionally, the dielectric constant of complex structures consisting of a mixture of several materials can be macroscopically modeled using well established mixing models [3]. Based on the correlation of these properties, microwave techniques which measure the dielectric constant of a material can be used to indirectly classify or investigate changes in material properties. Furthermore, the change in the dielectric constant of the material may also reflect changes in the physical structure and indicate the presence of delamination, disbond, or porosity.

A closed form solution or simple means of determining both parameters of interest currently does not exist. Hence, an inversion or root-solving technique for simultaneously determining the thickness and dielectric properties of a dielectric slab, either backed by a conductor or free space, will result in an important tool for comprehensive inspection of these types of structures. A simple iterative technique has been used in conjunction with the above forward electromagnetic model to simultaneously determine the thickness and dielectric properties of dielectric slabs.

### II. FORWARD MODEL

The determination of the reflection coefficient measured at the aperture of an open-ended rectangular waveguide probe radiating into a layered structure has been investigated extensively in the past [2]. This model can be applied for a single layer of a generally lossy dielectric slab, either backed by a conductor or an infinite half-space, as shown in Fig. 1.

This technique is robust and has been verified with various types of structures [4-8]. However, the modeling of this technique is highly complex, as it requires an electromagnetic

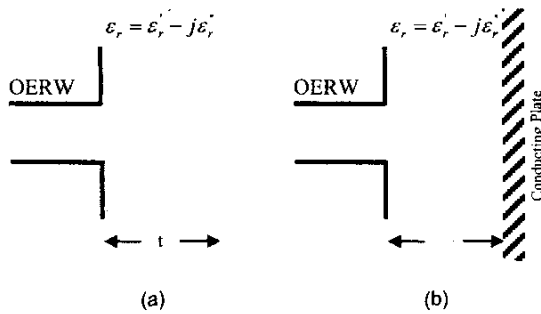


Fig. 1. A generally lossy dielectric slab backed by a) free-space and b) a conducting plate, interrogated by an open-ended rectangular waveguide.

analysis in the near-field region of the aperture of the waveguide [2]. Thus, direct inversion of the model is very complex and may not be possible. Furthermore, any inversion method used should rapidly converge, as the evaluation of a single set of data is both time and computationally intensive.

### III. INVERSE APPROACHES

#### A. Brute Force

The forward problem involves determining the reflection coefficient at the aperture of an open-ended waveguide radiating into a layered structure if the constitutive parameters of the medium are known (i.e. thickness and dielectric properties). If the thickness and dielectric properties of the material can be expected to lie within a finite range, then a brute force method may be employed to solve the inverse problem.

This method of solving the inverse problem requires that the set of data for thickness, permittivity, and loss factor be varied independently over the specified solution range. There are three independent variables to be calculated; therefore the solution domain has three degrees of freedom. If each variable is formed into an array, all possible combinations of these three arrays form a three dimensional matrix. Therefore, the reflection coefficient can be calculated for each combination of the three parameters and also forming a three dimensional matrix. However, as mentioned in the background of the forward model, the calculation of the reflection coefficient from the forward problem is complex. Therefore, calculating the reflection coefficient over a large set of data can be computationally expensive in terms of processing time. For example, a data set for a 50-element cube matrix (125,000 data points) took approximately 18 hours on an Athlon 1.2 GHz computer with 256 MB RAM.

Once the initial calculation of the reflection coefficient for the various combinations of thickness, dielectric constant, and loss factor is complete, the inverse problem is rather straightforward. In general, the reflection coefficient is a complex number. The calculated and measured reflection coefficients,  $\Gamma_{\text{calculated}}$  and  $\Gamma_{\text{measured}}$ , can be compared (i.e.,

$\Gamma_{\text{difference}}$ ). As the calculated reflection coefficient approaches the measured reflection coefficient, the calculated constitutive parameters approach the actual values from the measurement. Therefore, as the difference in the two reflection coefficients approaches zero, a solution to the inverse problem is obtained. It is also true that the magnitude of the difference in the reflection coefficients will also approach zero. This fact is the foundation for solving the inverse problem using the brute force method.

Practically, the magnitude of the difference in the measured and calculated reflection coefficients may never actually be zero. However, it is true that the trend will be that  $|\Gamma_{\text{difference}}|$  will be a minimum at the optimum solution to the inverse problem over the specified solution range. The solution to the inverse problem is complex due to the fact that three parameters can vary independently. In terms of the value for  $|\Gamma_{\text{difference}}|$ , there may be several local minima over the selected solution range. However, there will be only one global minimum over the solution range and this point corresponds to the values for the thickness, permittivity, and loss factor for the material in question.

The brute force method is limited to calculation of material properties within the initially selected solution range. There will always be one minimum in the calculation of  $|\Gamma_{\text{difference}}|$ , and this method will extract the values of thickness, permittivity, and loss factor that correspond to this minimum value. However, if the actual properties of the material lie outside of the specified solution range, then an erroneous result will be obtained. A minimum of  $|\Gamma_{\text{difference}}|$  will be calculated, but this value will correspond to the material properties which best fit the reflection coefficient within the solution range initially specified. This problem can be overcome by over processing the initial data set to encompass a larger solution set.

It is clear that the brute force method may be very front-end intensive in terms of computational time. However, once the initial calculations are complete, the actual processing of data can be done in real-time with measurements. In a manufacturing, production or on-line inspection setting, this method of solving the inverse problem may be the most time efficient.

#### B. The Downhill Simplex Method

The simplex method is a multi-parameter minimization algorithm of a given function, and was first introduced in 1965 by Nelder and Mead [9]. This technique is not as efficient in terms of the number of iterations required when compared to other minimization techniques. However, unlike other minimization functions, such as the conjugate gradient method or Powell's method, this technique requires only direct evaluation of the function to be minimized, and does not require knowledge of the slope or gradient of the function [10]. This is specifically important for any inversion method used in this investigation, as the direct evaluation of the

function is sufficiently complex. Determination of the gradient of the function as well would add further complex calculations. Hence, while fewer iterations may be required for these techniques, each iteration would be more complex, and the time required to converge to a solution may be ultimately longer using such techniques.

A simplex is a geometric figure consisting of  $N+1$  vertices, where  $N$  is the number of parameters to be determined. The three parameters to be determined in this investigation are permittivity, loss factor, and thickness. The function of each of these parameters is evaluated, and the highest vertex (i.e., the vertex furthest from the actual solution) is relocated in an attempt to converge the simplex around a minimum. At the beginning of an iteration, the highest and lowest points are determined. The high point is first reflected through the opposite face of the simplex. If the reflection produces a new low point, an expansion of the point is also performed to attempt to further minimize the point. If, however, the solution is not improved, the original high point is contracted towards the opposite face. Finally, if the high point cannot be improved, the entire simplex is contracted around the lowest point. The simplex will contract around the low point with a steep slope, which would not be otherwise possible using the other operations.

The simplex algorithm is continued until a minimum is obtained. Unfortunately, for multidimensional minimization, the global minimum of the function cannot be bracketed in the complex topology, and the method may converge around a local minimum. In this case, new initial guesses should be provided, and the method should be reevaluated until the absolute minimum is obtained. For a given run in the algorithm, then, the criterion for exiting should be based on the simplex converging around a point (i.e., very little variation in the vertices), rather than whether the simplex is within the tolerance of the actual answer.

#### IV. SIMPLEX METHOD RESULTS

A thick rubber sample and a thin rubber sample were examined to investigate the potential for using the Simplex Method for determining the thickness and dielectric properties of a dielectric slab. The reflection coefficient of the samples were measured at S-band using an open-ended rectangular waveguide over the entire frequency band ranging from 2.6 to 3.95 GHz, employing a vector network analyzer. From these measurements, the measured reflection coefficients at two different frequencies were used to calculate the thickness and dielectric constant of the samples using the Simplex Method. The reflection coefficient over the entire frequency band was recalculated [2] using the output of the Simplex Method and compared with the measured results for accuracy.

#### C. Conductor-Backed Thick Rubber Slab

The first sample used was a thick rubber slab with a conductor backing. The two frequencies used for this case were 2.90 GHz and 3.41 GHz. Table I summarizes the results from using the Simplex Method for this case. The error represents the deviation of the recalculated reflection coefficients from the measured reflection coefficients.

Table I. Results from the thick rubber slab backed by a conducting plate.

Frequency	$ \Gamma $	$\theta$	
2.90 GHz	0.85	-163.37°	
3.41 GHz	0.77	179.62°	
# of Iterations	45	Time (min.)	19
$\epsilon_r'$	6.31	$\epsilon_r''$	-0.73
Thickness	1.88 cm	% Error	3.1%

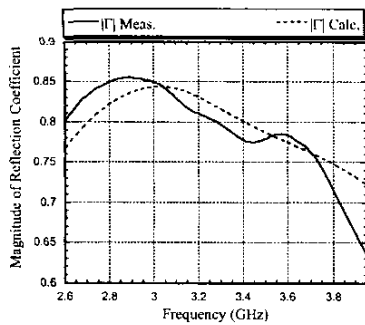
The thickness of the sample was measured (using a micrometer) to be 1.9 cm, which agrees well with the calculated value. The dielectric properties of the sample correlate with a rubber compound with approximately 10-15% carbon black [11]. The reflection coefficient was recalculated over the entire frequency band using the calculated material parameters [2]. These results were compared with the measured values. Figure 2 shows the magnitude and phase of measured and calculated reflection coefficients. As can be seen, the results correlate very well in phase, and marginally well in magnitude. Figure 3 shows the percent difference in magnitude and the difference in phase. The magnitudes are generally within 5% of one another, except at the higher end of the band, where it reaches 15% error. The difference in phase is less than 3.5° in all cases, and is less than 2° for the majority of the frequency band.

#### D. Conductor-Backed Thin Rubber Slab

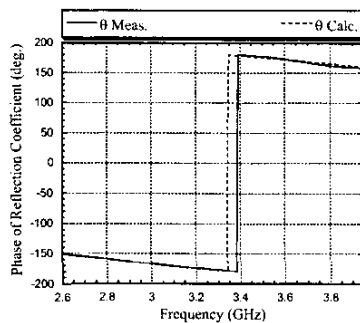
A thin rubber slab was also measured in the conductor backed case. The reflection coefficient was measured over the entire S-band. Similarly, the frequencies chosen were 2.90 GHz and 3.41 GHz. Table II summarizes the results from using the Simplex Method for this case. The error represents the deviation of the recalculated reflection coefficients from the measured reflection coefficients.

Table II. Results from the thin rubber slab backed by a conducting plate.

Frequency	$ \Gamma $	$\theta$	
2.90 GHz	0.95	169.38°	
3.41 GHz	0.89	165.18°	
# of Iterations	40	Time (min.)	18
$\epsilon_r'$	1.13	$\epsilon_r''$	-1.77
Thickness	0.201 cm	% Error	2.6%



(a)



(b)

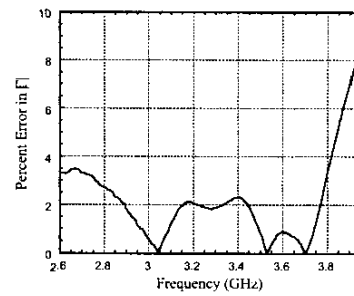
Fig. 2. a) Magnitude and b) phase of measured and calculated reflection coefficient for the thick rubber sample.

The thickness of the sample was measured (using a micrometer) to be 0.20 cm, which corresponds well with the calculated value from the Simplex Method.

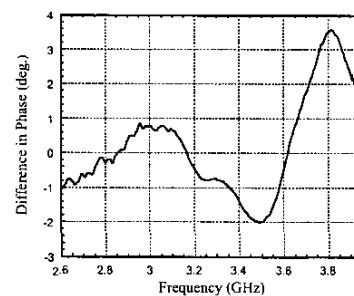
The calculated dielectric properties do not match the expected range of values. The slab thickness in this case is very thin comparison to the relative size of a wavelength for measurements in the S-band. Therefore, this technique may not provide accurate results an electrically thin sample. Measurements at higher frequencies (e.g. X-band) may provide more accurate and reliable results because the wavelength would be better matched to the sample thickness. Additionally, for such cases more than two frequencies may be required to accurately evaluate the thickness and the dielectric properties of a slab. These issues are currently being investigated.

## V. CONCLUSIONS

It has been shown that the Simplex Routine is able to accurately determine both the thickness and dielectric properties from the reflection coefficient measured at the aperture of an open-ended rectangular waveguide. The method is able to converge to a relatively good solution with a reasonable number of iterations. The properties of the thick rubber sample backed by a conductor were determined with good accuracy. The results were less accurate for slabs which were electrically thin. Further investigation into using



(a)



(b)

Fig. 3. Difference in a) magnitude (as a percentage), and b) phase (in degrees) between the measured and calculated reflection coefficients.

this technique should involve finding optimal frequencies, or the difference in frequencies, for use.

The Simplex routine is limited by factors such as frequency, dielectric properties and thickness. It is important to choose a frequency which closely matches the expected dielectric constant and thickness of the sample under test. In general, a lower frequency should be chosen to measure electrically thick materials and higher frequencies for electrically thin materials (with dielectric properties considered also, i.e. electrical thickness). The data provided in Table 2 may have been improved by modifying the Simplex routine to utilize additional frequency points to limit the effect of local minima when converging to the actual values.

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