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Channel Equalization and Symbol Detection for Single Carrier Broadband MIMO Systems with Multiple Carrier Frequency Offsets

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Channel Equalization and Symbol Detection for Single Carrier Broadband MIMO Systems with Multiple Carrier Frequency Offsets

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Abstract—**We consider the frequency-domain channel equalization and symbol detection of multiple input multiple output (MIMO) single-carrier broadband wireless system in the presence of severe frequency-selective channel fading and multiple unknown carrier frequency offsets (CFOs). We show that the constellation of the equalized data is rotating due to multiple CFOs, therefore, the equalized data can not be reliably detected without removing the rotating phases caused by the multiple unknown CFOs. Instead of estimating the CFOs, we propose a method to estimate the rotating phases caused by multiple CFOs, remove the rotating phases from the equalized data, and perform symbol detection. Numerical example indicates that the proposed method provide very good results for a 4×2 wireless system with 8PSK modulation and 75-tap Rayleigh fading channels.**

I. INTRODUCTION

In the last few years, single carrier (SC) frequency domain equalization (FDE) has attracted considerably attention for both single input single output (SISO) and multiple input multiple output (MIMO) broadband wireless systems. It has been shown to be effective to combat extended intersymbol interference (ISI) caused by severe frequency selective fading channels, and has been proposed in a few standards including IEEE 802.16e [1]. Compared to time-domain equalization, SC-FDE has higher computational efficiency and better convergence properties to achieve the same or better performance for a wireless system over severe frequency selective fading channels [2]. Moreover, compared to orthogonal frequency division multiplexing (OFDM), SC-FDE has similar performance and signal processing complexity, and less sensitivity to carrier frequency offset (CFO) [3]. In the literature, most, if not all, the existing SC-FDE algorithms assumed that CFO is not an issue for reliable symbol detection. However, in this paper, we will show that CFO can be a troublesome for SC-FDE if the DFT (discrete Fourier transform) block size is large and/or the constellation size of signal modulation is high. We will further show that the constellation of the equalized data symbols is rotating due to the CFO, and phase compensation has to be done before symbol detection. Furthermore, we present a new algorithm to estimate the rotating phases caused by multiple unknown CFOs in MIMO wireless systems.

II. SYSTEM MODEL AND PRELIMINARIES

Consider a broadband MIMO wireless system with N_t transmit antennas and N_r receive antennas. The Baseband equivalent signal received at the p-th antenna can be expressed in discrete-time domain as follows

$$r_p(k) = \sum_{q=1}^{N_t} \sum_{l=1}^{L} \alpha_{p,q}(l,k) s_q(k+1-l) e^{j(\omega_{p,q}kT+\theta_{p,q;0})} + v_p(k) \quad (1)$$

where $s_q(k)$ is the k-th symbol from the q-th transmit antenna, $\alpha_{p,q}(l,k)$ is the impulse response of the fading channel linking the q-th transmit antenna and p-th receive antenna with l being the tap index and k being the time index, $\omega_{p,q}$ and $\theta_{p,q;0}$ are respectively the angular carrier frequency offset and timing error phase between the q-th transmit antenna and the p-th receive antenna, $v_p(k)$ is the additive white Gaussian noise with average power σ^2, T is the symbol interval, L is the fading channel length in terms of T. It is noted that the fading channel coefficients $\alpha_{p,q}(l,k)$ include the effects of the transmit pulse-shape filter, physical multipath fading channel response, and the receive matched filter.

To facilitate frequency-domain channel equalization for the system described in (1), the transmitted data sequence $\{s_q(k)\}$ are partitioned into blocks of size N. Each block is then padded with N_{zp} zeros, forming a transmission block given by

$$\mathbf{s}_q^{bk} = \begin{bmatrix} s_q(1) & s_q(2) & \cdots & s_q(N) & 0 & \cdots & 0 \end{bmatrix}^t \quad (2)$$

The corresponding received signal $\mathbf{r}_p^{bk}$ of the block is denoted by

$$\mathbf{r}_p^{bk} = \begin{bmatrix} r_p(1) & r_p(2) & \cdots & r_p(N) & \cdots & r_p(N+N_{zp}) \end{bmatrix}^t \quad (3)$$

where the superscript $[\cdot]^t$ is the transpose. The length of zero padding N_{zp} is chosen to be at least $L-1$ so that the inter-block interference is avoided in the received signal.

Adopting the overlap-add method [4] for N-point DFT, we define two signal vectors

$$\mathbf{s}_q = \begin{bmatrix} s_q(1) & s_q(2) & \cdots & s_q(N) \end{bmatrix}^t \quad (4)$$

$$\mathbf{r}_p = \begin{bmatrix} r_p(1) & \cdots & r_p(N_{zp}) & r_p(N_{zp}+1) & \cdots & r_p(N) \end{bmatrix}^t$$
$$+ \begin{bmatrix} r_p(N+1) & \cdots & r_p(N+N_{zp}) & 0 & \cdots & 0 \end{bmatrix}^t \quad (5)$$

then the time-domain signal vectors $\mathbf{r}_p$ and $\mathbf{s}_q$ are related as

$$\begin{bmatrix} \mathbf{r}_1 \\ \vdots \\ \mathbf{r}_{N_r} \end{bmatrix} = \begin{bmatrix} \mathbf{D}_{1,1}\mathbf{A}_{1,1} & \cdots & \mathbf{D}_{1,N_t}\mathbf{A}_{1,N_t} \\ \vdots & \ddots & \vdots \\ \mathbf{D}_{N_r,1}\mathbf{A}_{N_r,1} & \cdots & \mathbf{D}_{N_r,N_t}\mathbf{A}_{N_r,N_t} \end{bmatrix} \begin{bmatrix} \mathbf{s}_1 \\ \vdots \\ \mathbf{s}_{N_t} \end{bmatrix} + \begin{bmatrix} \mathbf{v}_1 \\ \vdots \\ \mathbf{v}_{N_r} \end{bmatrix} \quad (6)$$

where $\mathbf{A}_{p,q}$ is given by (7) on the top of next page and

$$\mathbf{A}_{p,q} = \begin{bmatrix} \alpha_{p,q}(1,1) & 0 & \cdots & 0 & \alpha_{p,q}(L,N+1) & \cdots & \alpha_{p,q}(2,N+1) \\ \alpha_{p,q}(2,2) & \alpha_{p,q}(1,2) & 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \alpha_{p,q}(L,N+L-1) \\ \alpha_{p,q}(L,L) & \ddots & \ddots & \alpha_{p,q}(1,L) & 0 & \ddots & 0 \\ 0 & \alpha_{p,q}(L,L+1) & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \alpha_{p,q}(L,N) & \cdots & \cdots & \alpha_{p,q}(1,N) \end{bmatrix} \tag{7}$$

$$\mathbf{D}_{p,q} = \mathrm{diag}\left\{ e^{j(\omega_{p,q}T+\theta_{p,q;0})} \quad \cdots \quad e^{j(\omega_{p,q}NT+\theta_{p,q;0})} \right\} \tag{8}$$

$$\mathbf{v}_p = \begin{bmatrix} v_p(1) & \cdots & v_p(N_{zp}) & v_p(N_{zp}+1) & \cdots & v_p(N) \end{bmatrix}^t$$
$$+ \begin{bmatrix} v_p(N+1) & \cdots & v_p(N+N_{zp}) & 0 & \cdots & 0 \end{bmatrix}^t. \tag{9}$$

Let $\mathbf{F}_N$ be the normalized DFT matrix of size $N \times N$, i.e., its (m,n)-th element is given by $\frac{1}{\sqrt{N}} \exp\left(\frac{-j2\pi(m-1)(n-1)}{N} \right)$, one concludes that $\mathbf{F}_N^h \mathbf{F}_N = \mathbf{I}_N$ with $\mathbf{I}_N$ being the $N \times N$ identity matrix. Left multiplying (6) by $\mathrm{diag}\left\{ \mathbf{F}_N \cdots \mathbf{F}_N \right\}$, one obtains the frequency-domain representation as follows

$$\begin{bmatrix} \mathbf{Y}_1 \\ \vdots \\ \mathbf{Y}_{N_r} \end{bmatrix} = \begin{bmatrix} \mathbf{\Phi}_{1,1}\mathbf{H}_{1,1} & \cdots & \mathbf{\Phi}_{1,N_t}\mathbf{H}_{1,N_t} \\ \vdots & \ddots & \vdots \\ \mathbf{\Phi}_{N_r,1}\mathbf{H}_{N_r,1} & \cdots & \mathbf{\Phi}_{N_r,N_t}\mathbf{H}_{N_r,N_t} \end{bmatrix} \begin{bmatrix} \mathbf{X}_1 \\ \vdots \\ \mathbf{X}_{N_t} \end{bmatrix} + \begin{bmatrix} \mathbf{V}_1 \\ \vdots \\ \mathbf{V}_{N_r} \end{bmatrix} \tag{10}$$

where $\mathbf{Y}_p \triangleq \mathbf{F}_N \mathbf{r}_p$, $\mathbf{X}_q \triangleq \mathbf{F}_N \mathbf{s}_q$, $\mathbf{V}_p \triangleq \mathbf{F}_N \mathbf{v}_p$, $\mathbf{H}_{p,q} = \mathbf{F}_N \mathbf{A}_{p,q} \mathbf{F}_N^h$, $\mathbf{\Phi}_{p,q} = \mathbf{F}_N \mathbf{D}_{p,q} \mathbf{F}_N^h$, and $\mathbf{F}^h$ is the conjugate transpose of $\mathbf{F}$.

Remark 1: For a given L, if N is chosen to have the block time duration $(N+L-1)T$ less than the channel coherence time, then $\mathbf{A}_{p,q}$ is a circulant matrix, and $\mathbf{H}_{p,q}$ is a diagonal matrix with n-th diagonal element being $h_{p,q;n} = \sum_{l=1}^{L} \alpha_{p,q}(l,1) \exp\left(\frac{-j2\pi(l-1)(n-1)}{N} \right)$. i.e., $\mathbf{H}_{p,q} = \mathrm{diag}\{ h_{p,q;1} \quad h_{p,q;2} \quad \cdots \quad h_{p,1;N} \}$.

Remark 2: Although $\mathbf{\Phi}_{p,q}$ is generally a non-diagonal matrix, its diagonal elements, $\{\mathbf{\Phi}_{p,q}(n,n)\}_{n=1}^{N}$ are identical and equal to

$$\mathbf{\Phi}_{p,q}(n,n) = \frac{1}{N} \sum_{k=1}^{N} e^{j(\omega_{p,q}kT+\theta_{p,q;0})}, \quad n = 1,2,\cdots,N. \tag{11}$$

These two properties of the single-carrier frequency-domain representation enables effective channel estimation and equalization.

III. FD CHANNEL ESTIMATION

There are two commonly used methods [5], [6] for MIMO channel estimation assisted with training symbols. One is to estimate the fading channel by sequentially transmitting one small block of N_{ts} training symbols plus N_{zp} padded zeros ($N_{zp} < N_{ts}$) from each transmit antenna, while all other transmit antennas remain silent, which means that transmitting N_t small blocks with time duration of $N_t(N_{ts} + N_{zp})T$ are required for the MIMO channel estimation. This method can be referred to as time-domain and frequency-domain orthogonal training method. The other is time-domain overlap and frequency-domain orthogonal training method. This method employs an arbitrary length-N_{ts} Chu sequence as a base sequence denoted by $\mathbf{s}_{ts}$. Then one can construct N_t phase-shifted sequences whose k-th symbol is multiplied by $e^{j2\pi k(q-1)/N_{ts}}$ with $q = 1,2,\cdots,N_t$. Then the first transmit antenna repeatedly transmits the base sequence for N_t times, at the same time, the q-th transmit antenna repeatedly transmits the q-th phase-shifted sequence for N_t times. Although these N_t sequences are transmitted simultaneously at the N_t transmit antennas, they are orthogonal in the $N_t N_{ts}$ frequency tones as shown in [5], and the channel can be estimated with a bit higher bandwidth efficiency compared to the former training method if there is no carrier frequency offset or timing phase error.

In this paper, we adopt the first training method. That means, during the training mode, the training signals across transmit antennas are orthogonal in both time domain and frequency domain. Moreover, we choose the number of training symbols N_{ts} such that the training block time duration $(N_{ts} + N_{zp})T$ is smaller than a third of the quantity $2\pi/\max(\omega_{p,q})$ for all p and q. Then, the non-diagonal elements of $\mathbf{\Phi}_{p,q}$, i.e., $\mathbf{\Phi}_{p,q}(n,l)$ with $n \neq l$, are negligible comparing to the diagonal elements $\mathbf{\Phi}_{p,q}(n,n)$. Therefore, the frequency-domain representation (10) can be simplified as

$$Y_p(n) = \mathbf{\Phi}_{p,q}(n,n)H_{p,q}(n,n)X_q(n) + V_p(n)$$
$$= \lambda_{p,q}h_{p,q;n}X_q(n) + V_p(n), \quad n = 1,2,\cdots,N_{ts} \tag{12}$$

where $\lambda_{p,q} = \frac{1}{N_{ts}} \sum_{k=1}^{N_{ts}} e^{j(\omega_{p,q}kT+\theta_{p,q;0})}$ is a complex-valued unknown parameter with amplitude close to unit.

The channel transfer function $\lambda_{p,q}h_{p,q;n}$ is estimated by the least squares criterion as

$$\lambda_{p,q}\tilde{h}_{p,q;n} = \frac{Y_p(n)}{X_q(n)}, \quad n = 1,2,\cdots,N_{ts}. \tag{13}$$

The estimate $\lambda_{p,q}\tilde{h}_{p,q;n}$ can be further improved by a frequency-domain filter to reduce noise and converted into an N-point transfer function $\lambda_{p,q}\hat{\mathbf{H}}_{p,q}$ with $\hat{\mathbf{H}}_{p,q} = \mathrm{diag}\left\{ \hat{h}_{p,q;1} \quad \hat{h}_{p,q;2} \quad \cdots \quad \hat{h}_{p,q;N} \right\}$. Details are omitted for brevity.

We employ the frame structure designed in [8], where one frame contains one small pilot block and a number of

N-symbol data blocks. Then the estimated N-point transfer function matrices of the pilot block at the current frame and the pilot block at the next pilot block can be utilized to estimate the transfer function matrices of the data blocks via interpolation method proposed in [8]. The estimated transfer function matrices at the data blocks are used for frequency-domain channel equalization which is discussed in the next section.

IV. FD CHANNEL EQUALIZATION

In the data transmission mode, we choose the data block time duration $(N + N_{zp})T$ smaller than the channel coherence time, therefore the frequency-domain channel matrices $\{\mathbf{H}_{p,q}\}$ are diagonal matrices.

Based on the estimated channel transfer functions, we define $\mathcal{H}$ as follows

$$\mathcal{H} = \begin{bmatrix} \lambda_{1,1}\hat{\mathbf{H}}_{1,1} & \cdots & \lambda_{1,N_t}\hat{\mathbf{H}}_{1,N_t} \\ \vdots & \ddots & \vdots \\ \lambda_{N_r,1}\hat{\mathbf{H}}_{N_r,1} & \cdots & \lambda_{N_r,N_t}\hat{\mathbf{H}}_{N_r,N_t} \end{bmatrix}. \quad (14)$$

Applying the minimum mean square error (MMSE) criterion, we obtain the FD equalized block data as

$$\begin{bmatrix} \hat{\mathbf{X}}_1 \\ \vdots \\ \hat{\mathbf{X}}_{N_t} \end{bmatrix} = \mathcal{H}^h \left(\mathcal{H}\mathcal{H}^h + \sigma^2 \mathbf{I}_{N_r N} \right)^{-1} \begin{bmatrix} \mathbf{Y}_1 \\ \vdots \\ \mathbf{Y}_{N_r} \end{bmatrix}. \quad (15)$$

It is important to note that (15) is not computationally efficient due to the inversion of an $N_r N \times N_r N$ matrix. However, taking into consideration of the diagonal property of $\mathbf{H}_{p,q}$ and $\hat{\mathbf{H}}_{p,q}$ and keeping in mind that $\mathbf{\Phi}_{p,q}$ are diagonal dominant, we can obtain a much more computationally efficient MMSE FD equalization as follows

$$\begin{bmatrix} \hat{X}_1(n) \\ \vdots \\ \hat{X}_{N_t}(n) \end{bmatrix} = \mathbb{H}_n^h \left(\mathbb{H}_n\mathbb{H}_n^h + \sigma^2 \mathbf{I}_{N_r} \right)^{-1} \begin{bmatrix} Y_1(n) \\ \vdots \\ Y_{N_r}(n) \end{bmatrix} \quad (16)$$
$$n = 1, 2, \cdots, N$$

where

$$\mathbb{H}_n = \begin{bmatrix} \lambda_{1,1}\hat{h}_{1,1;n} & \cdots & \lambda_{1,N_t}\hat{h}_{1,N_t;n} \\ \vdots & \ddots & \vdots \\ \lambda_{N_r,1}\hat{h}_{N_r,1;n} & \cdots & \lambda_{N_r,N_t}\hat{h}_{N_r,N_t;n} \end{bmatrix}. \quad (17)$$

As can be seen from (16), the improved MMSE FD equalization involves N-times inversions of $N_r \times N_r$ matrices, this is certainly much more computationally efficient and numerically robust compared to (15) which involves the inversion of an $N_r N \times N_r N$ matrix.

Applying inverse DFT to the equalized data vector $\hat{\mathbf{X}}_q$, we have the equalized time-domain data vector $\hat{\mathbf{s}}_q$. Similarly to [9], we can prove that the k-th symbol of $\hat{\mathbf{s}}_q$ can be expressed by

$$\hat{s}_q(k) = \left[\sum_{p=1}^{N_r} \gamma_{p,q} e^{j\left(\omega_{p,q}kT + \theta_{p,q;0} \right)} \right] s_q(k) + \hat{v}_q(k)$$
$$= |\beta_q(k)| e^{j\angle\beta_q(k)} s_q(k) + \hat{v}_q(k) \quad (18)$$

where $\beta_q(k) = \sum_{p=1}^{N_r} \gamma_{p,q} e^{j\left(\omega_{p,q}kT + \theta_{p,q;0} \right)}$.

From (18), we can conclude that the complex-valued symbol-wise scaling factor $\beta_q(k)$ is actually a diversity combining factor determined by the N_r-receive channel transfer functions, carrier frequency offsets, time-error phases, and Doppler. In other words, the equalized data symbol $\hat{s}_q(k)$ is an amplitude-scaled and phase-rotated version of the transmitted data symbol $s_q(k)$. The rotating phase $\angle\beta_q(k)$ is a collection of all the contributions from the carrier frequency offsets $\omega_{p,q}$, and timing-error phases $\theta_{p,q;0}$ of all the N_r fading channels. For each individual fading channel, the rotating phase $\angle\beta_{p,q}(k) = \omega_{p,q}kT + \theta_{p,q;0} + \angle\gamma_{p,q}$, which represents the p-th channel's CFO-driven shifting phase, timing-error phase, and the channel transfer function effect. This is a physical interpretation for the single carrier frequency-domain equalized data.

If $s(k)$ is phase shift keying (PSK) modulated data, then the time-varying rotating phase $\angle\beta_{p,q}(k)$ must be compensated at the receiver after FDE and before demodulation and detection. This will be discussed in detail in the next section.

V. PHASE-COHERENT DETECTION

In this section, we present an effective and robust algorithm for estimating the phases $\angle\beta_q(k)$, which are crucial for successful data detection of PSK modulated symbols. The challenge of the phase estimations is that we face to multiple fading channels, in which each individual channel has different carrier frequency offset, timing-error phase and Doppler, and the rotating phases $\angle\beta_q(k)$ represents a nonlinearly composed effect of these random (or time-varying) factors of all the fading channels. Therefore, directly estimating these carrier frequency offsets, timing-error phases and Doppler will be very costly if any possible.

What we know from the nature of fading channels is that the carrier frequency offsets are either constants or change gradually from time to time, *i.e.*, it does not change arbitrary in a short period of time. Therefore, the rotating phase $\angle\beta_q(k)$ is also changing gradually from time to time. We treat $\angle\beta_q(k)$ to be a constant for a small number of N_s consecutive received symbols, and to another constant in the next N_s consecutive received symbols, and so on so forth.

We partition the equalized N-symbol block data $\hat{\mathbf{s}}_q$ into N_g groups, each with N_s data symbols, except for the last group might have less than N_s symbols if N/N_g is not an integer.

Let $\psi_q(g)$ be the estimated average rotating phase for the g-th group of $\{\angle\beta_q((g-1)N_s+1), \angle\beta_q((g-1)N_s+2), \cdots, \angle\beta_q((g-1)N_s+N_s)\}$, with $g = 1, 2, \cdots, N_g$, and let $\psi_q(0)$ denote the initial rotating phase, $\Delta\psi_q(g)$ the phase difference $\psi_q(g) - \psi_q(g-1)$. Hence

$$\psi_q(g) = \psi_q(g-1) + \Delta\psi_q(g), \quad g = 1, 2, \cdots, N_g. \quad (19)$$

For MPSK modulation with symbols taken from an M_m-ary constellation $\mathcal{A}_M \triangleq \left\{ \exp\left[\frac{j(m-1)2\pi}{M_m} \right], m = 1, 2, \cdots, M_m \right\}$, we

define a phase quantization function $\mathbb{Q}[\phi]$ as follows

$$\mathbb{Q}[\phi] = \frac{2(m-1)\pi}{M_m}, \qquad \begin{array}{c} \frac{2m\pi-3\pi}{M_m} < \phi \leq \frac{2m\pi-\pi}{M_m} \\ m = 1, 2, \cdots, M_m. \end{array} \qquad (20)$$

We are now in a position to present our algorithm as follows:

Algorithm: Group-wise Rotating Phase Estimation
Step 1. Designate the first N_p symbols $\{s_q(k)\}_{k=1}^{N_p}$ of each N-symbol block data $\mathbf{s}_q$ at the q-th transmit antenna as pilot symbols for phase reference to determine the initial rotating phase $\psi_q(0)$ given by

$$\psi_q(0) = \frac{1}{N_p} \sum_{k=1}^{N_p} \angle \hat{s}_q(k) - \angle s_q(k). \qquad (21)$$

Compensate the phase of the first group data by $e^{-j\psi_q(0)}$, yielding

$$\tilde{s}_q(1,k) = \hat{s}_q(k)e^{-j\psi_q(0)}, \qquad k = 1, 2, \cdots, N_s. \qquad (22)$$

Calculate the individual phase deviation from its nominal phase of each symbol in the first group

$$\varphi_q(1,k) = \angle \tilde{s}_q(1,k) - \mathbb{Q}\left[\angle \tilde{s}_q(1,k)\right], \quad k = 1, 2, \cdots, N_s. \quad (23)$$

Calculate the average phase deviation and compute the rotating phase for the first group as follows

$$\Delta\psi_q(1) = \frac{1}{N_s} \sum_{k=1}^{N_s} \varphi_q(1,k) \qquad (24)$$

$$\psi_q(1) = \psi_q(0) + \Delta\psi_q(1). \qquad (25)$$

Set $g = 2$ for next step.

Step 2. Compensate the phase of the g-th group data by $e^{-j\psi_q(g-1)}$, yielding

$$\tilde{s}_q(g,k) = \hat{s}_q((g-1)N_s+k)e^{-j\psi_q(g-1)}, \quad k = 1, 2, \cdots, N_s. \quad (26)$$

Calculate the individual phase deviation from its nominal phase of each symbol in the g-th group

$$\varphi_q(g,k) = \angle \tilde{s}_q(g,k) - \mathbb{Q}\left[\angle \tilde{s}_q(g,k)\right], \quad k = 1, 2, \cdots, N_s. \quad (27)$$

Calculate the average phase deviation and estimate the rotating phase for the g-th group as below

$$\Delta\psi_q(g) = \frac{1}{N_s} \sum_{k=1}^{N_s} \varphi_q(g,k) \qquad (28)$$

$$\psi_q(g) = \psi_q(g-1) + \Delta\psi_q(g). \qquad (29)$$

Step 3. Update $g = g+1$, repeat Step 2 until $g = N_g$. □

After estimating the N_g group phases of equalized block data from the q-th transmit antenna, we can compensate the phase rotation of the equalized data $\hat{s}_q$ in group basis:

$$\check{s}_q(g,k) = \hat{s}_q((g-1)N_s+k)e^{-j\psi_q(p)}, \quad \begin{array}{c} k = 1, 2, \cdots, N_s \\ g = 1, 2, \cdots, N_g \end{array} . \quad (30)$$

Finally, the binary information data of the block can be obtained via standard MPSK demodulation procedure on the phase-compensated signal $\check{s}_q(g,k)$ of the block. Different block data with different transmit antenna index q can be processed in a similar manner, details are omitted here for brevity.

We make two remarks before leaving this section.

Remark 1: The choice of N_s symbols in a group needs to satisfy the condition: $2\pi |f_{d,q}| N_s T < \frac{\pi}{M_m}$, to ensure that the maximum rotating phase does not exceed a decision region of MPSK, where $|f_{d,q}|$ is the absolute value of the maximum composite CFO, in Hz, linking to the q-th transmit antenna.

Remark 2: The group-wise estimation of the rotating phase is insensitive to noise perturbations due to its averaging process (28), which is an implicit low-pass filtering process.

VI. NUMERICAL RESULTS

In this section, we present a numerical example to show that our algorithm provides very good results for fading channels having long delay spread and multiple unknown carrier frequency offsets. We consider a MIMO wireless system with 2 transmit antennas and 4 receive antennas. We employ a 75-tap frequency-selective Rayleigh fading channel, where the average power of the first 25 taps ramps up linearly and the last 50 taps ramps down linearly, and the fading channel is normalized to have total average power as one. We employ the improved Clarke's Rayleigh fading model [7] with maximum Doppler being 50 Hz. We choose FFT size $N = 4096$, symbol interval $T_s = 0.25\mu s$ and 8PSK modulation. The multiple CFOs are listed in Table 1.

Table 1: Multiple CFOs for the 4×2 wireless system

$CFO_{1,1}$	-200 Hz	$CFO_{1,2}$	170 Hz
$CFO_{2,1}$	-190 Hz	$CFO_{2,2}$	180 Hz
$CFO_{3,1}$	-180 Hz	$CFO_{3,2}$	190 Hz
$CFO_{4,1}$	-170 Hz	$CFO_{4,2}$	200 Hz

The constellations of the equalized data is depicted in Fig. 1, which shows that the data can not be reliably demodulated right after the equalization. Fig. 2 shows the constellation of the phase-corrected data, which clearly indicates that the data can now be reliably demodulated using conventional procedure. Fig. 3 depicts the uncoded bit error rates for the following cases: 1) the channel coefficients are estimated by the interpolation-based algorithm [8] with CFO values given in Table 1; 2) the channel coefficients are perfectly known with CFO values given in Table 1 as well. For comparison purpose, these two channel conditions with no CFO are demonstrated in the figure. As indicated by these figures, the proposed method is very effective and crucial for mitigating the multiple unknown carrier frequency offsets and extended intersymbol interference caused by severe frequency selective channel fading, especially when the DFT block size is large and/or the CFOs are significant.
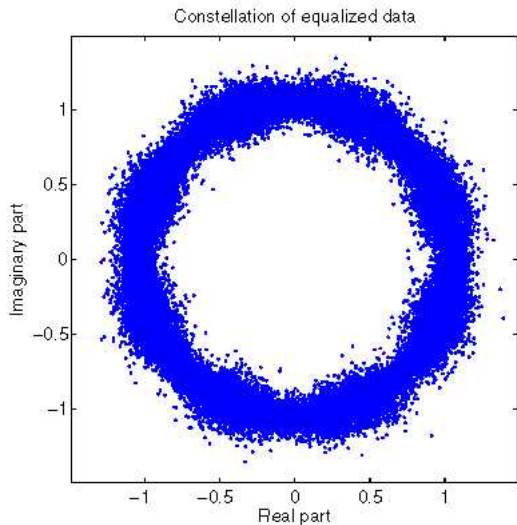
Constellation of equalized data

Fig. 1. Scatter plot of equalized 8PSK signals, SNR= 20 dB. Observation: the 8PSK signal can not be reliably demodulated.
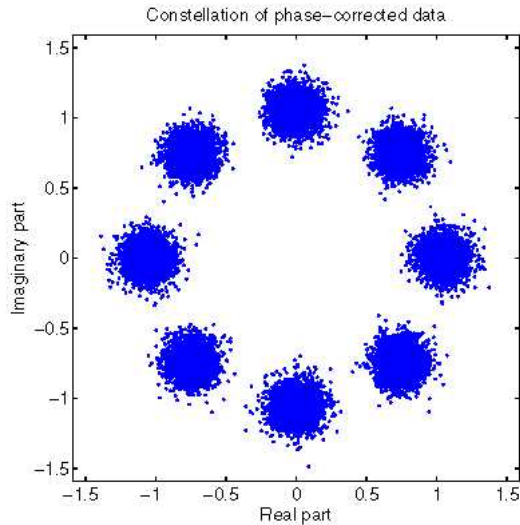
Constellation of phase−corrected data

Fig. 2. Scatter plot of equalized and phase-corrected 8PSK signals, SNR= 20 dB. Observation, the phase-corrected signal can be reliably demodulated using conventional demodulation procedure.

VII. CONCLUSION

In this paper, we demonstrated that multiple carrier frequency offsets (CFOs) in MIMO systems can be a troublesome for single carrier frequency domain equalization if the discrete Fourier transform (DFT) block size is large and/or the constellation size of signal modulation is high. The multiple CFOs will cause the constellation of the equalized data rotating, therefore the equalized data can not be reliably detected if the effect of multiple CFOs is not mitigated. Instead of directly estimating the CFOs, which is very costly, we proposed a new method to estimate the rotating phases caused by the multiple CFOs, and employed the estimated phases to correct the phase rotation of the equalized data, then performed the symbol detection. Numerical examples showed that the proposed method
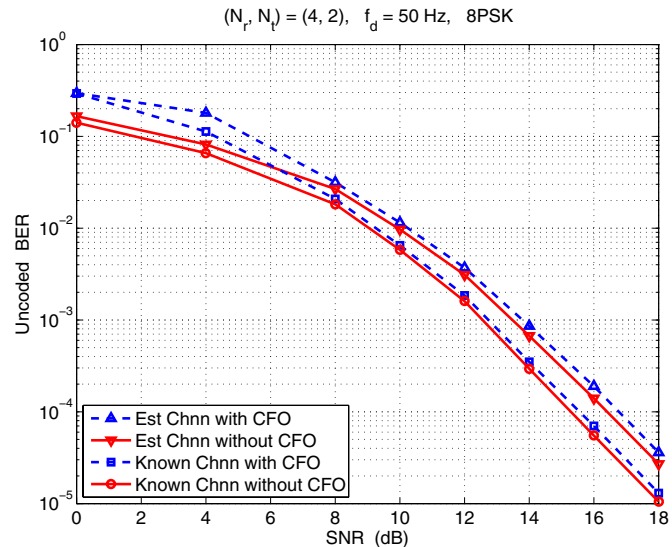
$(N_r, N_t) = (4, 2),\quad f_d = 50$ Hz,   8PSK

Fig. 3. Uncoded bit error rate of equalized and phase-corrected 8PSK modulated signals.

has provided very good results for a 4×2 wireless system with 8PSK modulation over 75-tap Rayleigh fading channels, at the maximum Doppler frequency of 50 Hz. The rotating phase estimation algorithm is not only computationally efficient, but also numerically robust in a wide range of signal-to-noise ratio.

REFERENCES

[1] IEEE Std 802.16e-2005, "IEEE Standard for local and metroplolitan area networks, part 16: air interface for fixed and mobile broadband wireless access systems," Feb. 2006.
[2] M. V. Clark, "Adaptive frequency-domain equalization and diversity combining for broadband wireless communications," *IEEE JSAC*, vol.16, pp.1385-1395, Oct. 1998.
[3] D. Falconer, S. L. Ariyavisitakul, A. Benyamin-Seeyar, and B. Eidson, "Frequency domain equalization for single-carrier broadband wireless systems," *IEEE Commun. Mag.*, vol.40, no.4, pp.58-66, Apr. 2002.
[4] B. Muquet, Z. Wang, G. Giannakis, M. de Courville, and P. Duhamel, "Cyclic prefix or zero padding for wireless multicarrier transmissions?" *IEEE TCOM*, vol.50, pp.2136-2148, Dec. 2002.
[5] J. Siew, J. Coon, R. J. Piechocki, A. Dowler, A. Nix, M. Beach, S. Armour, and J. McGeehan, "A channel estimation algorithm for MIMO-SCFDE," *IEEE ComLet*, pp.555-557, Sept. 2004.
[6] J. Coon, S. Armour, M. Beach, and J. McGeehan, "Adaptive frequency-domain equalization for single-carrier multiple-input multiple-output wireless transmissions," *IEEE T-SP*, vol.53, pp.3247-3256, Aug. 2005.
[7] C. Xiao, Y. R. Zheng, and N. C. Beaulieu, "Novel sum-of-sinusoids simulation models for Rayleigh and Rician fading channels," *IEEE Trans. Wireless Commun.*, vol.5, pp.3667-3679, Dec. 2006.
[8] Y. R. Zheng and J. Zhang, "Improved frequency-domain channel estimation for fast time-varying MIMO-SCFDE channels," in *Proc. IEEE ICC08*, Beijing, China, May 19-23, 2008.
[9] Y. R. Zheng, C. Xiao, T. C. Yang, and W. B. Yang, "Frequency-domain channel estimation and equalization for single carier underwater acoustic communications," in *Proc. IEEE/MTS OCEANS'07*, Vancouver, BC, Canada, Oct. 2007.