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FDTD Method Capable of Attaching Rectangular Domains

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Abstract
A finite-difference time-domain (FDTD) method capable of attaching rectangular computational domains is proposed herein. The aim of this approach is to reduce the size of computational domain according to the geometry of the modeled structure so as to reduce the simulation time with keeping precision of the result in an acceptable range. A multigrid algorithm is applied on the attaching interface between two rectangular domains so that different resolution can be achieved in different domains.

Keywords
FDTD, domain attaching, multigrid

INTRODUCTION
The finite-difference time-domain (FDTD) method is a prevailing numerical tool in electromagnetics for its simplicity and capacity of modeling problems with complicated geometries. A three-dimensional FDTD modeling is usually implemented in one properly meshed rectangular region containing all the objects to be modeled [1][2][3]. The limitation that the field update is conducted in one rectangular region sometimes leads to a very large, finely meshed domain, which results in long computational time or even exceeds the availability of the computing resources. However, such a large domain may not always be necessary, for example, a PCB or an enclosure connected with a long cable of relatively small cross-section. In this case, instead of only one large domain, two attached rectangular domains fitting to the dimensions of the big and small objects respectively can be used to improve the calculation efficiency. Since the computational time of FDTD modeling is approximately proportional to the number of cells, reduction of the domain size can significantly save the calculation time.

In order to achieve a better resolution in the domain containing the object with small geometries and avoid dramatic increase of the number of cells, the smaller domain can be meshed more finely both in space and time. As the meshing schemes in the two attached domains are different, a multigrid algorithm needs to be applied on the coarse-fine grid boundary at the attaching surface.

The key issue in a multigrid algorithm is the coupling of field values between fine and coarse grids on the boundary. A number of papers proposed different multigrid methods[4], which can be generally classified as two-step computation, subgridding only in space, and subgridding in both time and space. Since the method of subgridding both in time and space has the highest calculation efficiency, it attracts the most interest of the recent research. Linear and higher order interpolation is widely used to estimate the electric fields or magnetic fields in the fine mesh on the fine-coarse boundary[5][6]. An algorithm proposed in [7] used a weighted current method to update the fine-region tangential electric fields on the boundary, which is based on the idea of current conservation.

Domain Attaching Method (DAM)
The main purpose of domain attaching is to reduce the size of the computational domain, and consequently save the calculation time. First of all, for a valid attaching, all the objects must be included in the attached domains. Then, the attaching interface should be contained in a boundary surface of the bigger rectangular domain. In other words, the smaller domain can be viewed as an extrusion from the larger domain. Figure 1 gives an example of two attached domains, and it shows the cross-section of a three-dimensional model.

The attached domains overlap each other at the attaching interface, and the boundaries of different domains also have overlapping cells, the black-colored cells in Figure 1. The key problem in implementation of DAM is the boundary conditions at the attaching interface. Both the 1st order and 2nd order MUR absorbing boundary conditions were tested. The 2nd order MUR algorithm needs the present and previous field values of both the current cell and its neighboring cells. For an overlapped boundary cell, however, the fields of adjacent cells are calculated in different domains, which may cause instability.

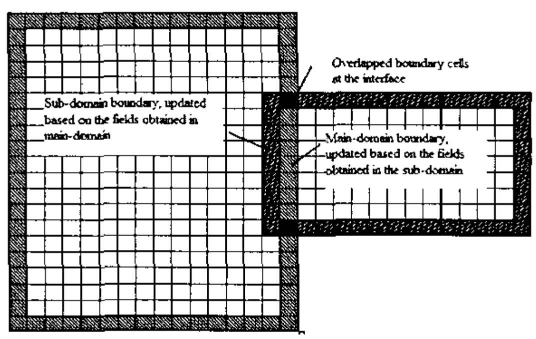

Figure 1. Attached domain-

The update scheme is given as below. For convenience purpose, only the case with two attached domains is considered, but it is easy to extend the method to multiple attached domains. Let's name the bigger rectangular domain the main-domain, and the smaller one the sub-domain. The field update is conducted in two steps:

- Fields in the main-domain are updated using the standard Yee algorithm, and the boundary conditions are reinforced. The boundary of sub-domain at the attaching interface is updated using the fields obtained in main-domain.
- Fields and the boundary conditions are updated in the sub-domain. Based on the results obtained in sub-domain, the boundary of main-domain at the attaching interface is updated.

DAM is suitable for modeling the structures consisted of a geometrically large object attached with relatively small but long objects. A test case was selected and calculated using the developed DAM. The geometries of the model are given in Figure 2. The model consists of a lossy dielectric block and a non-lossy slice. In this case, the number of cells is reduced by about ¼ compared to that of regular one-domain method, which means that the computational time can be also reduced by ¼ accordingly. The computational domain is illuminated with an incident plane wave from -z direction. The excitation source is a Gaussian pulse.

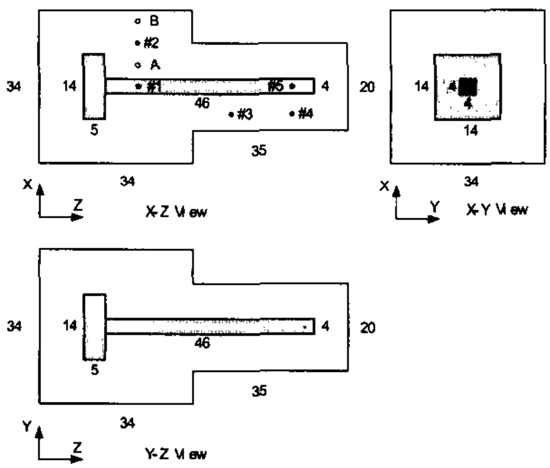

Figure 2. Test case (unit: cell number)

Several probes are set to record the E-field. Their positions are marked with # and shown in Figure 2. The E_x at probe 5 obtained from one-domain method and domain attaching method are compared in Figure 3. As probe 5 is at the far end of the sub-domain, the effect of domain reduction should be more serious at this position. From the figure, the two curves match sufficiently well, which indicates that the DAM can be a feasible method.

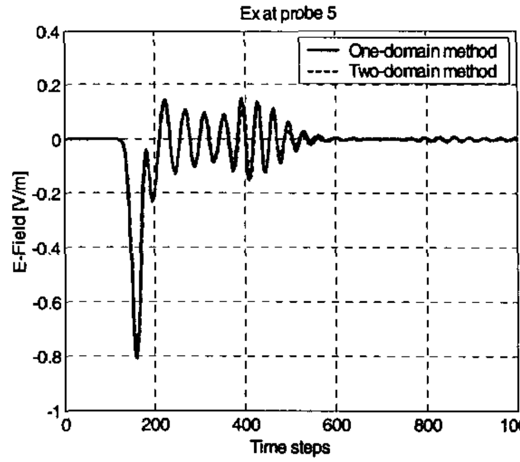

Figure 3. Comparison of E-field results in time domain

For DAM, the attaching interface is the only path for the field coupling between the main-domain and sub-domain. As a result, some couplings between the attached domains may be lost, which will compromise the accuracy of the result. To test the effect of source position on the result accuracy, an E-field hard source was placed at two different positions marked with A and B, as shown in Figure 2. These two source positions were selected so that position A is close to the center of the attaching interface, and position B is at the corner. Compared to the source at position A, the source at position B cannot illuminate all of the structures directly. Figure 4 and Figure 5 give the E-fields at probe 1 in the main-domain and probe 4 in the sub-domain when the source is at point A, respectively. Results show that the E-fields at probe 1 obtained by the two methods are nearly identical. At the same time, the two curves in Figure 5 differ from each other in the first few peaks, and then converge. The difference is probably caused by the loss of coupling and the imperfect absorbing boundary conditions at the attaching interface.

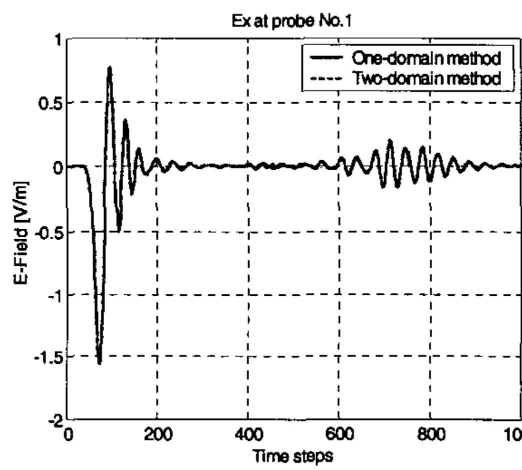

Figure 4. E-fields at probe 1 with the source at position A

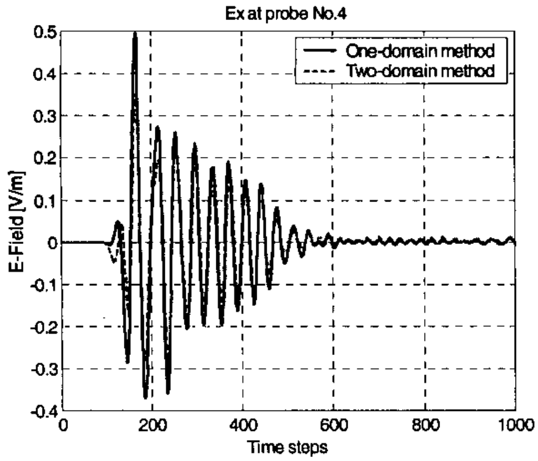

Figure 5. E-fields at probe 4 with the source at position A

The E-field results when the source is placed at position B are compared in Figure 6 and Figure 7. Figure 7 shows that the fields at probe 4 obtained using DAM has less amplitude at the first several peaks, which should be mainly due to the loss of direct coupling from the source. This is can also be verified by the E-field at probe 1 shown in Figure 6, in which the two curves fit each other well at the beginning, and then they differ a little from time step 200 to 600 before converging again. This difference is caused by the reflection of the fields from the sub-domain whose accuracy is compromised by the loss of direct coupling from the source.

The above results show that the source position can significantly affect the accuracy of the results, especially for the rectangular domain that does not contain the source. Therefore, the attaching surface should be carefully chosen so that the source can illuminate the whole computational domain directly. This limitation indicates there is a trade-off between accuracy and the domain size for DAM.

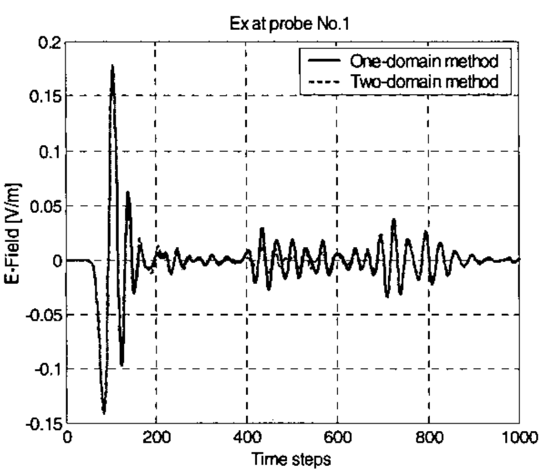

Figure 6. E-fields at probe 1 with the source at position B

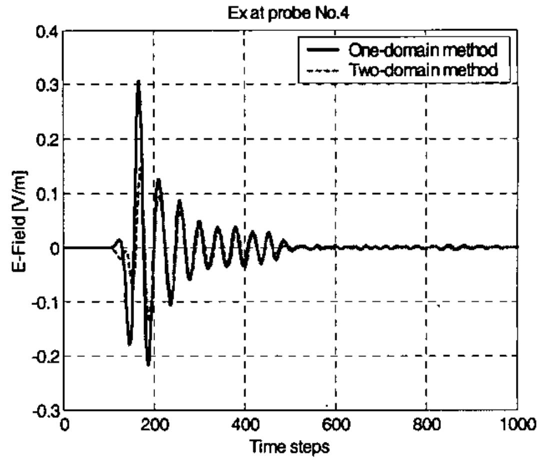

Figure 7. E-fields at probe 4 with the source at position B

Multigrid on the attaching interface

Since the objects in the sub-main are smaller in size compared to those objects in the main-domain, fine mesh can be generated in the sub-domain to achieve more precise model and better results. This requires a multigrid algorithm applied at the attaching interface. The key issue for a multigrid algorithm is the field coupling on the coarse-fine mesh boundary. Since the Yee cells in coarse mesh and fine mesh differ in size, the field obtained in coarse mesh cannot directly provide the complete boundary information for the fine mesh. Numerical methods for the estimation of the boundary fields are needed to initiate the field update in the fine mesh region. The estimation, however, can cause the reflection on the coarse-fine mesh boundary, which may result in instability.

Compared to the general subgridding algorithm, the algorithm for DAM is applied to the entire attaching interface, which makes it have two difficulties: material transverse capability and subgridding on boundary cells. The algorithm used in this paper is based on the idea of current conservation[7], and the subgridding is implemented both in time and space.

Consider a two-dimensional subgridded cell with a fine-coarse ratio of three as shown in Figure 8, and assume that the cell is source free. The Ampere's law in integral form is shown in Equation 1.

$$\oint H \cdot dl = \frac{\partial}{\partial t}\int \varepsilon E \cdot dS + \int \sigma E \cdot dS \qquad [1]$$

Then, the update equation for E-field can be written as,

$$E^{n+1} = \left(\frac{1 - \frac{\sigma \Delta t}{2\varepsilon}}{1 + \frac{\sigma \Delta t}{2\varepsilon}} \right) E^{n} + \left(\frac{\frac{\Delta t}{\varepsilon}}{1 + \frac{\sigma \Delta t}{2\varepsilon}} \right) \frac{I}{\Delta x \Delta y} \qquad [2]$$

where $I = \oint H \cdot dl$ is the current through the coarse cell, Δt is the update time interval, and Δx , Δy define the cell size. Equation 2 indicates that if the current through a cell is known, the field update can be continued. At the coarse-fine mesh boundary, the E-fields and H-fields of a coarse cell can be updated normally using the standard Yee scheme, and the current through the entire coarse cell can be calculated using loop integral of H-fields in coarse mesh. The current through the fine cells, however, cannot be calculated directly. Therefore, the essential problem for the subgridding algorithm used in this paper is how to distribute the current in a coarse cell to the fine cells it contains both in time and space.

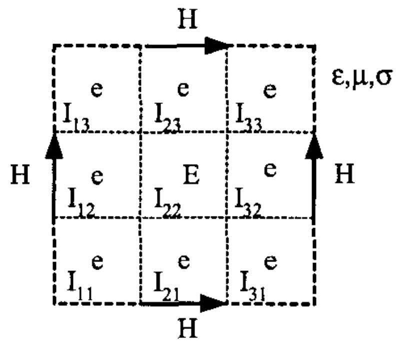

Figure 8. Two-dimensional subgridded cells of 3:1 ratio

E: the co-located E-filed; H: the co-located H-field; e: the E-field of the subgridded cells; I_{ij}: the current flowing through the subgridded cells

For example, suppose that the present time step is n + 1. For a coarse cell on the subgridding boundary, based the E-field at time step n and the H-field at time step n + ½, the E-field at time step n + 1 can be calculated by using standard update equations (See Figure 9). The current through the coarse cell at time step n + ½ can also be obtained from the loop integration of H-field. For a fine cell at the boundary, however, the E-field needs to be updated three times from time step n to n + 1. To complete the update, the current through the fine cell at time step n + 1/6, n + 1/2, and n + 5/6 needs to be estimated using the known currents and field information in the neighboring coarse cells.

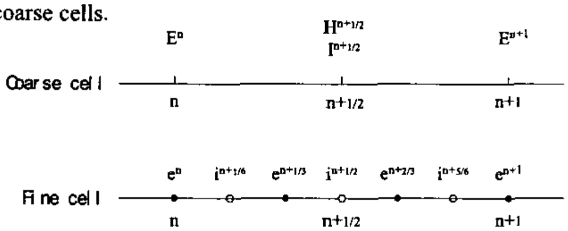

Figure 9. Subgridding scheme

In the estimation of the current, at least two constraints must be satisfied,

- Current conservation, i.e. $I = \sum_{i,j} I_{ij}$.
- The co-located E-fields in coarse and fine mesh should remain the same. Co-located fields exist when the fine to coarse mesh ratio is an odd number.

A variety of schemes can be used to distribute the current to the fine cells. For a homogeneous cell, the easiest way is to assume that the current in a coarse cell distributes uniformly in space, and remains unchanged within one time step. This is also the scheme used here.

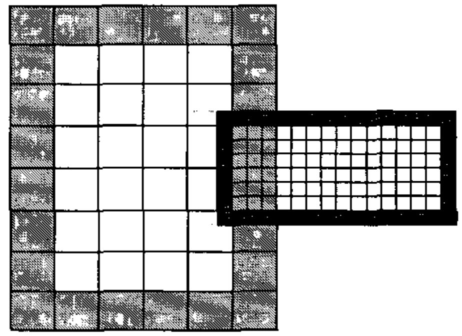

Figure 10. An example of meshing scheme for subgridded DAM

Figure 10 shows an example of the meshing scheme for the subgridded DAM. To improve the stability of the subgridding algorithm, the transition from coarse to fine mesh is smoothened by making the coarse mesh and fine mesh have two overlapping layers. In the first layer, the E-fields of coarse cells remain unchanged, and the E-fields of coarse cells in the second layer are replaced by the average of the values obtained from coarse and fine meshes.

The multigrid algorithm was also tested using the case given in Figure 2. E-field result at probe 5 obtained by using DAM with the sub-domain subgridded, is compared with the result from uniformly fine mesh in Figure 11.

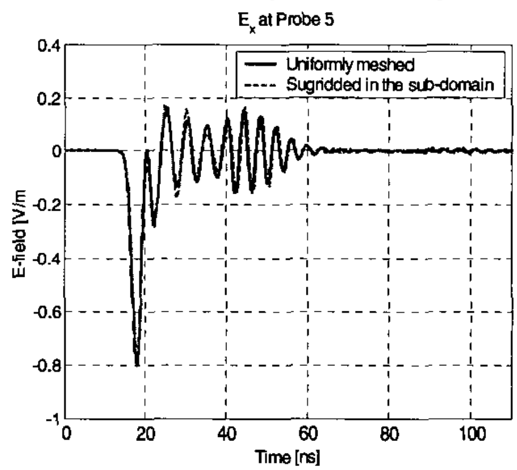

Figure 11. E-field results of the test case using DAM with the subgridded sub-domain

It shows that the subgridding algorithm is stable, and the two curves match each other well. Since the algorithm does multigrid both in time and space, the multigridded DAM can reduce computational time greatly. For the test case, the number of finely meshed cells in the multigridded DAM is about ¼ of that in the uniformly finely meshed one-domain method; therefore the computational time is approximately reduced by ¾.

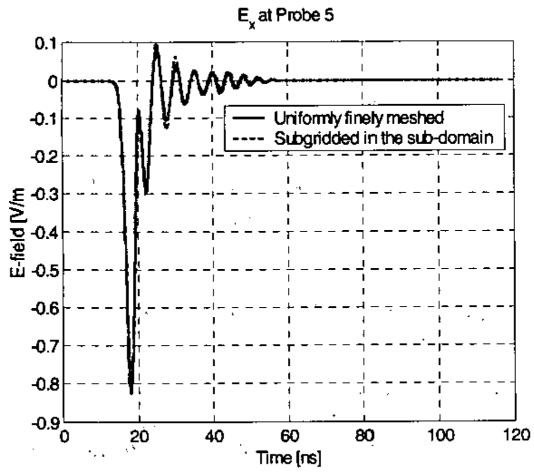

Figure 12. E-fields of the modified test case with the glossy material transverse across the attaching interface

To test the material transverse capability of the subgridding algorithm, the material across the attaching interface was changed from lossless to lossy with conductivity of 0.01S. The results of this test, shown in Figure 12, are also sufficiently good.

However, the algorithm currently has problems in dealing with high-lossy materials or PEC transverse at the attaching surface. The reason is that the assumption of uniformly distribution of the current in one cell will no longer hold in the region close to such material boundaries, for example free space to PEC boundary. This requires a new current distribution scheme with the material properties of adjacent cells taken into account. Moreover, more and complex test cases need to be run to justify the algorithm. All these are the ongoing work.

Conclusion

An FDTD method with the capacity of attaching domains was proposed in this paper. The developed DAM is suitable for modeling structures consisting of a geometrically big object attached with a long object of relative small cross-section. Results show that DAM is a feasible method that can significantly reduce the computational time while keeping the accuracy in an acceptable range. To improve the accuracy of the results, the size of the sub-domain and position of attaching must be properly chosen so that the source can illuminate the whole structure directly.

A multigrid algorithm with material transverse capability is implemented on the attaching interface. This algorithm is based on the idea of current conservation. The results of a test case show that it is a stable algorithm with material transverse capacity.

More cases with complex structures need to be tested to verify the DAM. The scheme of distribution of the current through a coarse cell to the subgridded fine cells needs to be improved so that more accurate results can be achieved.

The proposed attaching domain method also provides a way for FDTD parallel computation for its inherent parallel capacity. A rectangular domain can be divided into several attached small domains. The FDTD update can be simultaneously conducted in different domains, and the exchange of data is only needed between two adjacent domains to update the fields at the attaching interface.

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