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Control of Nonholonomic Mobile Robot Formations: Backstepping Kinematics into Dynamics [1]

Travis Dierks and S. Jagannathan

Abstract—***In this paper, we seek to expand framework developed to control a single nonholonomic mobile robot to include the control of formations of multiple nonholonomic mobile robots. A combined kinematic/torque control law is developed for leader-follower based formation control using backstepping in order to accommodate the dynamics of the robots and the formation in contrast with kinematic-based formation controllers. The asymptotic stability of the entire formation is guaranteed using Lyapunov theory, and numerical results are provided. The kinematic controller is developed around control strategies for single mobile robots and the idea of virtual leaders. The virtual leader is replaced with a physical mobile robot leader and the assumption of constant reference velocities is removed. An auxiliary velocity control is developed allowing the asymptotic stability of the followers to be proved without the use of Barbalat's Lemma which simplifies proving the entire formation is asymptotically stable. A novel approach is taken in the development of the dynamical controller such that the torque control inputs for the follower robots include the dynamics of the follower robot as well as the dynamics of its leader, and the case when all robot dynamics are known is considered.***

Index Terms —**Formation control, Lyapunov methods, kinematic/dynamic controller.**

I. INTRODUCTION

Over the past decade, the attention has shifted from the control of a single nonholonomic mobile robot [1-2] to the control of multiple mobile robots because of the advantages a team of robots offer such as increased efficiency and more systematic approaches to tasks like search and rescue operations, mapping unknown or hazardous environments, and security and bomb sniffing.

There are several methodologies [3-9] to robotic formation control which include behavior-based [3], generalized coordinates [4], virtual structures [5], and leader-follower [6-10] to name a few. Perhaps the most popular and intuitive approach is the leader-follower method. In this method, a follower robot stays at a specified separation and bearing from a designated leader robot.

In [6] and [9], local sensory information and a vision based approach to leader-following is undertaken respectively. In both the approaches, the sensory information was used to calculate velocity control inputs. A modified leader follower control is introduced in [7] where Cartesian coordinates are used rather than polar. In [8], it is acknowledged that the separation-bearing methodologies of leader-follower formation control closely resemble a tracking controller problem and a reactive tracking control strategy that converts a relative pose control problem into a tracking problem between a virtual robot and the leader is developed. A drawback of this controller is the need to define a virtual robot and the fact that dynamics are not considered. A characteristic that is common in many formation control papers [6-9] is the design of a kinematic controller thus requiring a perfect velocity tracking assumption and formation dynamics are ignored.

In this paper, we examine frame works developed for controlling single nonholonomic mobile robots and seek to expand them to be used in leader follower formation control. Specifically, we examine tracking controllers in the form of [1]. Like [8], we seek to convert a relative pose problem into a tracking control problem, but without the use a virtual robot for the follower. We also seek to bring in the dynamics of the robots themselves thus incorporating the formation dynamics in the controller design. In [10], the dynamics of the follower robot are considered, but the effect the leader's dynamics has on the follower (formation dynamics) is not incorporated. The leader's dynamics become apart of the follower robot's control torque input through the derivative of the follower's kinematic velocity control, which is a function of the leader's velocity. In other words, the dynamical extension introduced in this paper provides a rigorous method of taking into account the specific vehicle dynamics to convert a steering system command into control inputs via backstepping approach. Both feedback velocity control inputs and velocity following control laws are presented for asymptotic stability of the formation.

II. LEADER-FOLLOWER FORMATION CONTROL

The two popular techniques in leader-follower formation control include separation-separation and separation-bearing [9]. The goal of separation-bearing formation control is to find a velocity control input such that

$$\lim_{t\to\infty}(L_{ijd} - L_{ij}) = 0 \quad \text{and} \quad \lim_{t\to\infty}(\Psi_{ijd} - \Psi_{ij}) = 0 \qquad (1)$$

where L_{ij} and ψ_{ij} are the measured separation and bearing of the follower robot with L_{ijd} and ψ_{ijd} represent desired distance and angles respectively [6][9]. Only separation-bearing techniques are considered, but our approach can be extended to separation-separation control.

To avoid collisions, separation distances are measured from the back of the leader to the front of the follower, and

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the kinematic equations for the front of the j^{th} follower robot can be written as

$$\dot{q}_j = \begin{bmatrix} \dot{x}_j \\ \dot{y}_j \\ \dot{\theta}_j \end{bmatrix} = S_j(q_j)v_j = \begin{bmatrix} \cos\theta_j & -d_j\sin\theta_j \\ \sin\theta_j & d_j\cos\theta_j \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_j \\ \omega_j \end{bmatrix} \quad (2)$$

where d_j is the distance from the rear axle to the to front of the robot, x_j , y_j , and θ_j are actual Cartesian position and orientation of the physical robot, and v_j , and ω_j are linear and angular velocities respectively.

Many robotic systems can be characterized as a robotic system having an n-dimensional configuration space $\mathcal{C}$ with generalized coordinates $(q_1, \ldots q_n)$ and subject to m constraints described in detail in [1] and mathematically after applying the transformation described in [1] as

$$\bar{M}_j(q_j)\dot{v}_j + \bar{V}_{m_j}(q_j,\dot{q}_j)v_j + \bar{F}_j(v_j) + \bar{\tau}_{d_j} = \bar{B}_j(q_j)\tau_j. \quad (3)$$

where $\bar{M}_j \in \Re^{rxr}$ is a symmetric positive definite inertia matrix, $\bar{V}_{mj} \in \Re^{rxr}$ is the centripetal and coriolis matrix, $\bar{F}_j \in \Re^{rx1}$ is the friction vector, $\bar{\tau}_{dj}$ represents unknown bounded disturbances, and $\bar{\tau}_j = \bar{B}_j\tau \in \Re^{rx1}$ is the input vector. It is important to highlight the *skew symmetric property* common to robotic systems [1] as $\dot{\bar{M}}_j - 2\bar{V}_{m_j}(q_j,\dot{q}_j) = 0$.

A. Controller Design

The complete description of the behavior of a mobile robot is given by (2) and (3). Standard approaches to leader follower formation control deal only with (2) and assume that perfect velocity tracking holds. This paper seeks to remove that assumption by defining the nonlinear feedback control input

$$\tau_j = \bar{B}_j^{-1}(\bar{M}_j u_j + \bar{V}_{mj}v_j + \bar{F}_j(v_j) + \bar{\tau}_d) \quad (4)$$

where u_j is an auxiliary input. Applying this control law to (3) allows one to convert the dynamic control problem into the kinematic control problem [1] such that

$$\begin{aligned} \dot{q}_j &= S_j(q_j)v_j \\ \dot{v}_j &= u_j. \end{aligned} \quad (5)$$

Backstepping Design: Tracking controller frameworks have been derived for controlling single mobile robots, and there are many ways [1-2] to choose velocity control inputs $v_c(t)$ for steering system (2). To incorporate the dynamics of the mobile platform, it is desirable to convert $v_c(t)$ into a control torque, $\tau_j(t)$ for the physical robot. Contributions in single robot frameworks are now considered and expanded upon in the development a kinematic controller for the separation-bearing formation control technique. Our aim to design a conventional computed torque controller such that (2) and (3) exhibit the desired behavior for a given control $v_c(t)$ thus removing perfect velocity tracking assumptions.

Consider the tracking controller error system presented in [1] used to control a single robot as

$$\begin{bmatrix} e_{j1} \\ e_{j2} \\ e_{j3} \end{bmatrix} = \begin{bmatrix} \cos\theta_j & \sin\theta_j & 0 \\ -\sin\theta_j & \cos\theta_j & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x_j \\ y_r - y_j \\ \theta_r - \theta_j \end{bmatrix} \quad (6)$$

$$\dot{x}_r = v_r\sin\theta_r,\ \dot{y}_r = v_r\sin\theta_r,\ \dot{\theta}_r = \omega_r,\ \dot{q}_r = \begin{bmatrix}\dot{x}_r & \dot{y}_r & \dot{\theta}_r\end{bmatrix}^T \quad (7)$$

where x_j, y_j, and θ_j are actual position and orientation of the robot, and x_r, y_r, and θ_r are the positions and orientation of a virtual reference cart robot j seeks to follow [1].

In single robot control, a steering control input $v_c(t)$ is designed to solve three basic problems: path following, point stabilization, and trajectory following such that $\lim_{t\to\infty}(q_r-q_j)=0$ and $\lim_{t\to\infty}(v_c-v_j)=0$ [1]. If the mobile robot controller can successfully track a class of smooth control velocity inputs, then all three problems can be solved with the same controller structure [1].

The three basic tracking control problems can be extended to formation control as follows. The virtual reference cart is replaced with a physical mobile robot acting as the leader i, and x_r and y_r are defined as points at a distance L_{ijd} and a desired angle ψ_{ijd} from the lead robot. Now the three basic navigation problems can be introduced for leader-follower formation control as follows.

Tracking: Let there be a leader i for follower j such that

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \end{bmatrix} = \begin{bmatrix} \cos\theta_i & -d_i\sin\theta_i \\ \sin\theta_i & d_i\cos\theta_i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_i \\ \omega_i \end{bmatrix} \quad (8)$$

$$\begin{aligned} x_{jr} &= x_i - d_i\cos\theta_i + L_{ijd}\cos(\Psi_{ijd} + \theta_i) \\ y_{jr} &= y_i - d_i\sin\theta_i + L_{ijd}\sin(\Psi_{ijd} + \theta_i) \\ \theta_{jr} &= \theta_i \end{aligned} \quad (9)$$

$$v_{jr} = [|v_i| \quad |\omega_i|]^T \quad (10)$$

where v_{jr} is the time varying linear and angular speeds of the leader such that $v_{jr} > 0$ for all time. Then define the actual position and orientation of follower j as

$$\begin{aligned} x_j &= x_i - d_i\cos\theta_i + L_{ij}\cos(\Psi_{ij} + \theta_i) \\ y_j &= y_i - d_i\sin\theta_i + L_{ij}\sin(\Psi_{ij} + \theta_i) \\ \theta_j &= \theta_j \end{aligned} \quad (11)$$

where L_{ij} and Ψ_{ij} is the actual separation and bearing of follower j. In order to solve the formation tracking problem with one follower, find a smooth velocity input $v_{jc}=f(e_p, v_{jr}, K)$ such that $\lim_{t\to\infty}(q_{jr}-q_j)=0$, where e_p, v_{jr}, and K are the tracking position errors, reference velocity for follower j robot, and gain vector respectively. Then compute the torque $\tau_j(t)$ for the dynamic system of (3) so that $\lim_{t\to\infty}(v_{jc}-v_j)=0$. Achieving this for every leader i and follower $j=1,2,..N$ ensures that the entire formation tracks the formation trajectory.

Path Following: Given a path P_i for leader i as well as the entire formation to follow, define a path P_j relative to P_i as the points at a distance L_{ijd} and an angle ψ_{ijd} for the follower robot j to follow with a linear velocity $v_j(t)$. Find a smooth velocity control input $v_{jc}=f(e_{j\theta}, v_{jr}, b_{ji}, K)$, where $e_{j\theta}$ and $b_{ji}(t)$ are the orientation and distance errors between a reference point of the follower robot j and path P_j, respectively, such that $lim_{t\to\infty}(e_{j\theta})=0$ and $lim_{t\to\infty}(b_{ij})=0$. Then compute the torque $\tau_j(t)$ for the dynamic system given by (3) so that $lim_{t\to\infty}(v_{jc}-v_j)=0$. Achieving this for every leader i and follower $j=1,2,..N$ ensures that the entire formation follows a formation path P_i with a bounded error that is a function of L_{ijd} and ψ_{ijd}.

Point Stabilization: Given an arbitrary configuration of leader i denoted as q_{ir}, define a relative reference configuration for follower j as q_{jr}. Then find a smooth control velocity input $v_{jc}=f(e_p, v_{jr}, K,)$ such that $lim_{t\to\infty}(q_{jr}-q_j)=0$. Then compute the torque $\tau_j(t)$ for the dynamic system of (3) so that $lim_{t\to\infty}(v_{jc}-v_j)=0$. Achieving this for every leader i and follower $j=1,2,..N$ ensures the entire formation is stabilized about a reference point at the geometric center of the formation which is defined as the formation trajectory.

Leader-Follower Trajectory Tracking: Many solutions [6-9] to the leader-follower formation control problem of (1) and the kinematic model (2) have been suggested and smooth velocity control inputs for the follower have been derived. Unfortunately, dynamical models are rarely studied, and the effect of the dynamics of mobile robot leader i on follower j has not been well understood in the process of incorporating the dynamics of the formation. This paper will now address these issues.

The contribution in this paper lies in deriving an alternative control velocity, $v_{jc}(t)$, for separation-bearing leader follower formation control, and calculating the specific torque $\tau_j(t)$ to control (3) which accounts for the i^{th} leader's dynamics as well as the j^{th} follower's. It is common in the literature to assume perfect velocity tracking which does not hold in real applications. To remove this assumption, integrator backstepping is applied.

Using (9), (11) and simple trigonometric identities the error system (6) can be rewritten as

$$e_j = \begin{bmatrix} e_{j1} \\ e_{j2} \\ e_{j3} \end{bmatrix} = \begin{bmatrix} L_{ijd}\cos(\Psi_{ijd}+e_{j3}) - L_{ij}\cos(\Psi_{ij}+e_{j3}) \\ L_{ijd}\sin(\Psi_{ijd}+e_{j3}) - L_{ij}\sin(\Psi_{ij}+e_{j3}) \\ \theta_i - \theta_j \end{bmatrix} \quad (12)$$

The transformed error system now acts as a formation tracking controller which not only seeks to remain at a fixed desired distance L_{ijd} with a desired angle ψ_{ijd} relative to the lead robot i, but also achieves the same orientation as the lead robot which is desirable when $\omega_i = 0$.

In order to calculate the error dynamics given in (12), it is necessary to calculate the derivatives of L_{ij} and ψ_{ij}, and it is assumed that L_{ijd} and ψ_{ijd} are constant. Consider the two robot formation depicted in Figure 1. The x and y components of L_{ijd} can be defined as

$$\begin{aligned} L_{ijx} &= x_{i_{rear}} - x_{j_{front}} = x_i - d_i\cos\theta_i - x_j \\ L_{ijy} &= y_{i_{rear}} - y_{j_{rear}} = y_i - d_i\sin\theta_i - y_j \end{aligned} \quad (13)$$

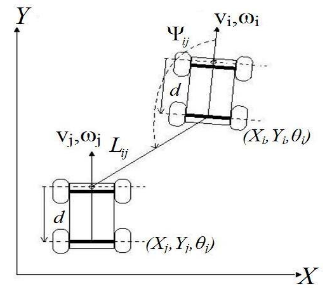

Figure 1: Leader-follower formation control

and the derivative of the x and y components of L_{ij} can be found to be

$$\begin{aligned} \dot{L}_{ijx} &= v_i\cos\theta_i - v_j\cos\theta_j + d_j\omega_j\sin\theta_j \\ \dot{L}_{ijy} &= v_i\sin\theta_i - v_j\sin\theta_j - d_j\omega_j\cos\theta_j \end{aligned} \quad (14)$$

Noting that $L_{ij}^2 = L_{ijx}^2 + L_{ijy}^2$ and $\Psi_{ij} = \arctan\left(\frac{L_{ijy}}{L_{ijx}}\right) - \theta_i + \pi$, it can be shown that derivatives of the separation and bearing are consistent with [6] and [9] even when using the kinematics described in (2) such that

$$\begin{aligned} \dot{L}_{ij} &= v_j\cos\gamma_j - v_i\cos\Psi_{ij} + d_j w_j\sin\gamma_j \\ \dot{\Psi}_{ij} &= \frac{1}{L_{ij}}(v_i\sin\Psi_{ij} - v_j\sin\gamma_j + d_j w_j\cos\gamma_j - L_{ij}w_i) \end{aligned} \quad (15)$$

where $\gamma_j = \Psi_{ij} + e_{j3}$.

Now, using the derivative of (12), equation (15) and applying simple trigonometric identities, the error dynamics can be expressed as

$$\begin{bmatrix} \dot{e}_{j1} \\ \dot{e}_{j2} \\ \dot{e}_{j3} \end{bmatrix} = \begin{bmatrix} -v_j + v_i\cos e_{j3} + \omega_j e_{j2} - \omega_i L_{ijd}\sin(\Psi_{ijd}+e_{j3}) \\ -\omega_j e_{j1} + v_i\sin e_{j3} - d_j\omega_j + \omega_i L_{ijd}\cos(\Psi_{ijd}+e_{j3}) \\ \omega_i - \omega_j \end{bmatrix}. \quad (16)$$

Examining (16) and the error dynamics of a tracking controller for a single robot in [1], one can see that dynamics of a single follower with a leader is similar to [1], except additional terms are introduced as a result of (2) and (15).

To stabilize the kinematic system, we propose the following velocity control inputs for follower robot j to achieve the desired position and orientation with respect to leader i as

$$v_{jc} = \begin{bmatrix} v_{jc} \\ \omega_{jc} \end{bmatrix} = \begin{bmatrix} v_i\cos e_{j3} + k_1 e_{j1} \\ \omega_i + (v_i + k_v)k_2 e_{j2} + (v_i + k_v)k_3\sin e_{j3} \end{bmatrix} + \begin{bmatrix} \gamma_{vjc} \\ \gamma_{wjc} \end{bmatrix} \quad (17)$$

where

$$\gamma_{vjc} = -\omega_i L_{ijd}\sin(\Psi_{12d} + e_{j3}) \quad (18)$$

$$\gamma_{\omega jc} = -\frac{|e_{j2}|\left(\omega_i(d_j + L_{ijd}) + (v_i + k_v)k_3 d_j + 1\right)}{1/k_2 + |e_{j2}|d_j} \quad (19)$$

Comparing this velocity control with the tracking controller designed for a single robot in [1], one can see that the two are similar except for the novel auxiliary terms which ensure stability for the formation of two robots using kinematics alone. Additionally, the design parameter k_v was added to ensure asymptotic stability holds even when $v_i=0$.

Before we proceed, the following assumptions are needed.
Assumption 1. Complete knowledge of the j^{th} follower and i^{th} leader dynamics are known.
Assumption 2. Each follower has full knowledge of its leader's dynamics.
Assumption 3. Follower j is equipped with sensors capable of measuring the separation distance L_{ij} and bearing Ψ_{ij} and that both leader and follower are equipped with instruments to measure their linear and angular velocities as well as there orientations θ_i and θ_j.
Assumption 4. Wireless communication is available between the j^{th} follower and i^{th} leader with communication delays being zero.
Assumption 5. The i^{th} leader communicates its linear and angular velocities v_i, w_i as well as its orientation θ_i and control torque $\tau_i(t)$ to its j^{th} follower.
Assumption 6. For the nonholonomic system of (2) and (3) with n generalized coordinates q, m independent constraints, and r actuators, the number of actuators is equal to the number of degrees of freedom ($r = n - m$).
Assumption 7. The reference linear and angular velocities measured from the leader i are bounded and $v_{jr}(t) \geq 0$ for all t.
Assumption 8. $K = [k_1 \quad k_2 \quad k_3]^T$ is a vector of positive constants.
Assumption 9. Let perfect velocity tracking hold such that $\dot{v}_j = \dot{v}_{jc}$ (this assumption is relaxed later).

Theorem 1: Given the nonholonomic system of (2) and (3) with n generalized coordinates q, m independent constraints, and r actuators, along with the leader follower criterion of (1), let *Assumption 1-9* hold. Let a smooth velocity control input $v_{jc}(t)$ for the j^{th} follower be given by (17), (18), and (19). Then the origin $e_j=0$ consisting of the position and orientation error for the follower is asymptotically stable.

Proof: Consider the following Lyapunov function candidate

$$V_j = \frac{1}{2}(e_{j1}^2 + e_{j2}^2) + \frac{1-\cos e_{j3}}{k_2} \tag{20}$$

Clearly, $V_j > 0$ and $V_j = 0$ only when $e_j=0$.
Differentiating (20) and substitution of (16), (17), and (18) yields

$$\begin{aligned}\dot{V}_j = &-k_1 e_{j1}^2 - d_j(v_i + k_v)k_2 e_{j2}^2 - (v_i + k_v)k_3 \sin^2 e_{j3} \\ &- e_{j2} d_j\left(\omega_i + (v_i + k_v)k_3 \sin e_{j3} + \omega_i L_{ijd} \cos(\Psi_{ijd} + e_{j3}\right) \\ &- e_{j2}\sin e_{j3} - \gamma_{\omega jc}\left(\frac{\sin e_{j3}}{k_2} + e_{j2} d_j\right)\end{aligned} \tag{21}$$

which can be rewritten as

$$\begin{aligned}\dot{V}_j \leq &-k_1 e_{j1}^2 - d_j(v_i + k_v)k_2 e_{j2}^2 - \frac{k_3}{k_2}(v_i + k_v)\sin^2 e_{j3} \\ &+ |e_{j2}|d_j\left(\omega_i(1 + \frac{L_{ijd}}{d_j}) + (v_i + k_v)k_3 + \frac{1}{d_j}\right) + \gamma_{\omega jc}\left(\frac{1}{k_2} + |e_{j2}|d_j\right)\end{aligned} \tag{22}$$

Clearly, the first three terms in (22) are strictly less than zero for $e_j \neq 0$. Now consider the last two terms of (22) in the inequality

$$|e_{j2}|d_j\left(\omega_i(1 + \frac{L_{ijd}}{d_j}) + (v_i + k_v)k_3 + \frac{1}{d_j}\right) + \gamma_{\omega jc}\left(\frac{1}{k_2} + |e_{j2}|d_j\right) \leq 0 \tag{23}$$

Substitution of (19) into (23) reveals

$$\dot{V}_j \leq -k_1 e_{j1}^2 - d_j(v_i + k_v)k_2 e_{j2}^2 - \frac{k_3}{k_2}(v_i + k_v)\sin^2 e_{j3} \tag{24}$$

Clearly $\dot{V}_j < 0$ for all $v_i \geq 0$, and the velocity control (17), (18) and (19) provides asymptotic stability for the error system (12) and (16) and $e_j \to 0$ as $t \to \infty$.

Remark: The asymptotic stability of the error system (12) and (16) is proved without the use of Barbalat's Lemma which is required in [1]. Proving the stability of the formation is greatly simplified without the need for Barbalat's Lemma for every follower robot.

Now assume that the perfect velocity tracking assumption does not hold making *Assumption 9* invalid. Define the velocity tracking error as

$$e_{jc} = v_{jc} - v_j \tag{25}$$

Differentiating (25) and adding and subtracting $\overline{M}_j(q_j)\dot{v}_{jc}$ and $\overline{V}_{mj}(q_j)v_{jc}$ to (3) allows the mobile robot dynamics to be written in terms of the velocity tracking error and its derivative as

$$\overline{M}_j(q_j)\dot{e}_{jc} = -\overline{V}_{mj}(q_j, \dot{q}_j)\dot{e}_{jc} - \overline{\tau}_j + f_j(x) + \overline{\tau}_{dj} \tag{26}$$

where $$f_j(x_j) = \overline{M}_j(q_j)\dot{v}_{jc} + \overline{V}_{mj}(q_j, \dot{q}_j)v_{jc} + \overline{F}_j(v_j) \tag{27}$$

with $x_j = [\dot{v}_i, \dot{\omega}_i, v_i, \omega_i, q_j, v_j, w_j, e_j, \dot{e}_j]$. The function $f_j(x_j)$ in (27) will be used to bring in the dynamics of leader i through $\dot{v}_{jc}$ by observing that

$$\dot{v}_{jc} = f_{vcj}(\dot{v}_i, \dot{\omega}_i, v_i, \omega_i, e_j, \dot{e}_j). \tag{28}$$

The leader i's dynamics can be written in the form of (3)

$$\dot{v}_i = \overline{M}_i^{-1}(q_i)(\overline{B}_i(q_i)\tau_i - \overline{V}_{m_i}(q_i, \dot{q}_i)v_i - \overline{F}_i(v_i) - \overline{\tau}_{d_i}) \tag{29}$$

Substituting (29) into (28) results in the dynamics of the i^{th} leader robot to become apart of $\dot{v}_{jc}$ as

$$\dot{v}_{jc} = f_{vcj}(v_i, \omega_i, \theta_i, \tau_i, e_j, \dot{e}_j) \tag{30}$$

Under *Assumptions 1-5*, follower j is able to construct $\dot{v}_{jc}$. Defining the auxiliary control input u_j from (5) to be [1]

$$u_j = \dot{v}_{jc} + K_4 e_{jc}, \tag{31}$$

the control torque for the j^{th} follower robot can be written in the form

$$\tau_j = \overline{B}_j^{-1}(\overline{M}_j K_4 e_{jc} + f_j(x_j)) \tag{32}$$

where K_4 is a positive definite matrix defined by

$$K_4 = k_4 I \quad (33)$$

Substituting (32) into the dynamics of follower robot j (3) produces the closed loop error dynamics shown below.

$$\overline{M}_j \dot{e}_{jc} = -(\overline{M}_j K_4 + \overline{V}_{mj}) e_{jc} + \overline{\tau}_d \quad (34)$$

Remark: In [1], the reference velocity is considered to be constant, therefore the dynamics of the reference cart are never considered. That assumption is not be made here since the reference cart has been replaced by a physical robot i. Thus, the dynamics of leader robot i must be considered and become an important term in follower j's torque command.

Theorem 2: Let *Assumptions 1-8* hold, and let k_4 in (33) be a sufficiently large positive constant. Let a smooth velocity control input $v_{jc}(t)$ for the j^{th} follower be defined by (17), (18) and (19). Let the torque control for the j^{th} follower robot (32) be applied to the mobile robot system (3). Then the origin e_j=0 and $e_{jc}=0$ which are the position, orientation and velocity tracking errors for follower j are asymptotically stable.

Proof: Consider the following Lyapunov candidate:

$$V'_j = V_j + \frac{1}{2} e_{jc}^T \overline{M}_j e_{jc} \quad (35)$$

where V_j is defined as (20). Differentiating yields

$$\dot{V}'_j = \dot{V}_j + e_{jc}^T \overline{M}_j \dot{e}_{jc} + \frac{1}{2} e_{jc}^T \dot{\overline{M}}_j e_{jc} \quad (36)$$

In *Theorem 1,* it was proved that $\dot{V}_j < 0$. Assuming an ideal case such that the disturbance $\overline{\tau}_d = 0$, substituting (34) into (36), and applying the *skew symmetric property* yields

$$\dot{V}'_j = \dot{V}_j - e_{jc}^T (\overline{M}_j K_4) e_{jc} \quad (37)$$

Examining (37), it is clear that $\dot{V}'_j < 0$ and the position tracking error system $e_j=0$ and velocity tracking error system e_{jc}=0 are asymptotically stable.

Leader Control Structure: In every formation, we assume there is leader i such that the following assumptions hold:
Assumption 10. The formation leader follows no physical robots, but follows the virtual leader described in [1].
Assumption 11. The formation leader is capable of measuring its absolute position via instrumentation like GPS so that tracking the virtual robot is possible.

The kinematics and dynamics of the formation leader i are defined similarly to (2) and (3) respectively. From [1], the leader tracks a virtual reference robot with the kinematic constraints of (7), and the control velocity $v_{ic}(t)$ can be defined as

$$v_{ic} = \begin{bmatrix} v_{ir} \cos e_{i3} + k_{i1} e_{i1} \\ \omega_{ir} + k_{i2} v_{ir} e_{i2} + k_{i3} v_{ir} \sin e_{i3} \end{bmatrix} \quad (38)$$

Using similar steps and justification to form (26) and (27), the leader's error system can be formed similarly to follower *j's* and the leader's torque $\overline{\tau}_i$ is defined as [1]

$$\tau_i = \overline{B}_i^{-1} (\overline{M}_i (K_{i4} e_{ic} + \dot{v}_{ic}) + \overline{V}_{mi} v_i + \overline{F}_i(x_i)) \quad (39)$$

where e_{ic} and K_{i4} are defined similarly to (25) and (33). The following additional mild assumptions are needed before proceeding.

Assumption 12. The reference linear velocity v_{ir} is greater than zero and bounded and the reference angular velocity ω_{ir} is bounded for all t.
Assumption 13. $K=[k_{i1}\ k_{i2}\ k_{i3}]^T$ is a vector of positive constants.

Theorem 3: Given the kinematic system of (8) and dynamic system in the form of (3) for leader i with n generalized coordinates q_i, m independent constraints, and r actuators, let *Assumptions 1-6* and *Assumptions 10-13* hold. Let k_{i4} be a sufficiently large positive constant. Let there be a smooth velocity control input $v_{ic}(t)$ for the leader i given by (38), and let the torque control for the lead robot i (39) be applied to the mobile robot system in the form of (3). Then the origin e_i=0 and $e_{ic}=0$ which are the position, orientation and velocity tracking errors for leader i are asymptotically stable. This theorem is proved in [1].

*Remark: T*he asymptotic stability of a formation consisting of 1 leader and N followers can be proved as well as the asymptotic stability of the formation for the case when follower j becomes a leader to follower $j+1$. Proofs of these claims are not presented here due to length constraints, but they follow as a result of *Theorems 2* and *3*.

III. SIMULATION RESULTS

A wedge formation of five identical nonholonomic mobile robots is considered where leader's trajectory is the desired formation trajectory, and simulations are carried out in MATLAB under two scenarios. First, only the kinematic steering system (2) under perfect velocity tracking such that $v_j = v_{jc}$ and $\dot{v}_j = \dot{v}_{jc}$ is considered for the leader and its followers in the absence of all dynamics. Then, the full dynamics as well as the kinematics of all the robots are considered. Under both scenarios, the leader's reference linear velocity is 5 m/s while the reference linear velocity is allowed to vary. Results for the leader's tracking ability are presented in [1] and are therefore not shown here.

A simple wedge formation is considered such that follower j should track its leader at separation of L_{12d}=2 meters and a bearing of $\Psi_{12d} = \pm 120^o$ depending on the follower's location, and the formation leader is located at the apex of the wedge. The following gains are utilized for the controllers:

Leader	K_{i4}=*daig{40}*	K_{i1}=10	K_{i2}=5	K_{i3}=4	
Follower j	K_4=*diag{40}*	k_1=10	k_2=50	k_3=.5	k_v=1

The following robotic parameters are considered for the leader and its followers: m=5 kg, I = 3 kg^2, R=.175 m, r = 0.08 m, and d=0.45 m. Friction is added to both the leader's and its followers' dynamics and modeled as

$$F = \begin{bmatrix} .5sign(v) + .25v \\ .75sign(\omega) + .3\omega \end{bmatrix}$$

Figure 2 shows the resulting trajectories for both scenarios. In both cases, the robots start in the bottom left corner of Figure 2 and travel toward the top right corner of the figure. A steering command in the form of angular acceleration is given to the formation at *x=2* symbolizing an obstacle avoidance maneuver. From Figure 2, it is apparent that the wedge formation can be achieved under both scenarios.

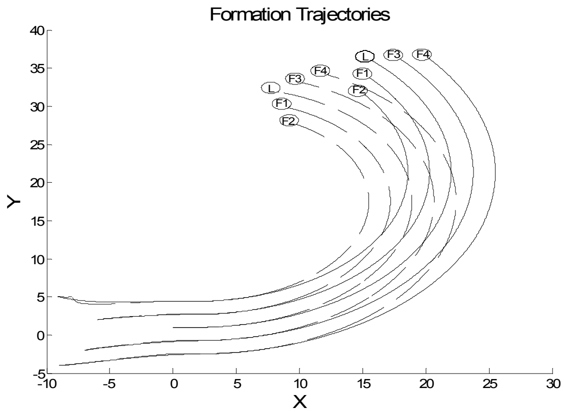

Figure 2: Trajectory when dynamics are included: solid, Trajectory when only kinematics are considered: dashed

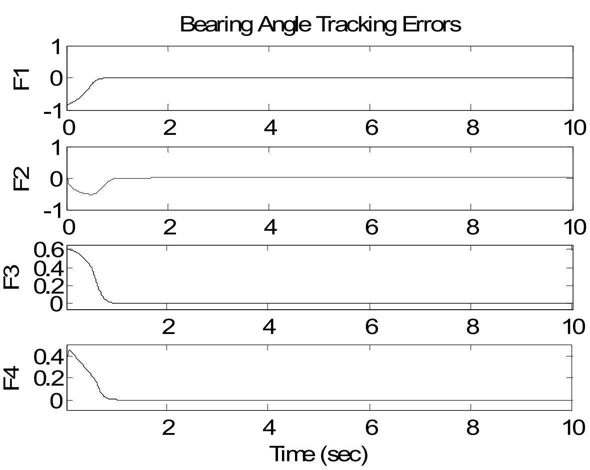

Figure 3: Bearing Errors for Scenario 2

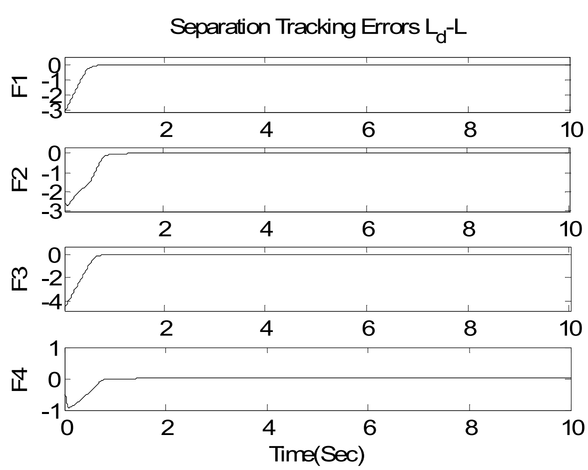

Figure 4: Separation Errors for Scenario 2

However, the formation trajectories are not the same. When the steering command is issued, the dynamics of the robots become an apparent influence on the formation trajectory. This is an important result that displays the importance of incorporating the dynamics of the robots into the control law. In an obstacle ridden environment, it is important that the formation follows a specific trajectory to ensure safe passage. Ignoring the dynamics of the robots, one cannot guarantee the trajectory the formation follows is the desired trajectory.

Figures 3 and 4 display the bearing and separation errors for the dynamical scenario. It is evident that both the bearing errors and separation errors converge to zero very quickly and remain there so that the wedge formation is maintained.

IV. CONCLUSION

An asymptotically stable tracking controller for leader-follower based formation control was presented that considers the dynamics of the leader and the follower using backstepping. The feedback control scheme is valid as long as the complete dynamics of the followers and their leader are known. Numerical results were presented and the stability of the system was verified. Simulation results verify the theoretical conjecture and expose the flaws in ignoring the dynamics of the mobile robots.

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