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Reconstruction of the parameters of Debye and Lorentzian dispersive media using a genetic algorithm

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Abstract

A method for reconstruction of the parameters of the Debye or Lorentzian dispersive media is proposed. In this method, S-parameters of a simple parallel-plate fixture filled with the dispersive medium are measured and modeled using the transmission line equations, provided a single TEM mode propagating in this parallel-plate waveguide. The genetic algorithm is used for searching the parameters of the dispersive medium by means of minimizing the discrepancy between the measured and modeled Sparameters. The results are verified using the full-wave FDTD modeling technique.

Keywords

Debye and Lorentzian dispersion laws, genetic algorithm, S-parameters, telegrapher's equations, FDTD modeling

INTRODUCTION

Complex electromagnetic structures containing different kinds of dispersive media, including novel composite dielectric, magnetic, and magneto-dielectric materials have been analyzed using modern numerical methods. To simulate wideband performance of a dispersive material, the corresponding frequency dependency of constitutive parameters must be expressed in terms of an analytical dispersion law, while the parameters of this law can be found from measurements. The Debye and Lorentzian dispersion laws are in conventional use for the numerical simulation of the materials.

However, the extraction of the parameters characterizing the Debye and Lorentzian media from a set of measured or reference data requires the solution of systems of non-linear equations, as described in [1]. This might be cumbersome. Besides, exact data for the real and/or imaginary parts of permittivity and/or permeability of materials are not available in many cases. It is important to develop a simple, accurate, and reliable method for the Debye or Lorentzian dispersive media parameter reconstruction.

In the method proposed herein, the scattering matrix parameters of a simple parallel-plate fixture filled with the dispersive medium are measured and modeled using the transmission line (telegrapher's) equations [2], provided a single TEM mode propagating in this parallel-plate waveguide. A powerful, robust, and efficient in global searching and optimization genetic algorithm (GA) [3-5] is used for searching the parameters of the dispersive medium by means of minimizing the discrepancy between the measured and modeled S-parameters. This is a straightforward method for a single-component Debye or Lorentzian material. The results are verified using the full-wave FDTD modeling technique.

MODELING OF THE TRANSMISSION LINE CONTAINING A DISPERSIVE MEDIUM

For a parallel-plate transmission line shown in Figure 1, the telegrapher's equations [2] in frequency domain are

$$\frac{dV(z)}{dz} = -(R + j\omega L)I(z), \tag{1}$$

$$\frac{dI(z)}{dz} = -(G + j\omega C)V(z). \tag{2}$$

The wave equations obtained from (1) and (2) are

$$\frac{d^2V(z)}{dz^2} - \gamma^2 V(z) = 0; \tag{3}$$

$$\frac{d^2I(z)}{dz^2} - \gamma^2 I(z) = 0, \tag{4}$$

where

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}. \tag{5}$$

For the parallel-plate transmission line shown in Figure 1, the per-unit-length (p.u.l.) parameters are

$$L = \frac{\mu d}{w}, \tag{6}$$

$$C = \frac{\varepsilon'(\omega) w}{d}, \tag{7}$$

$$R = \frac{2R_S}{w}, \tag{8}$$

$$G = \frac{\omega\varepsilon'(\omega)w}{d}, \quad (9)$$

where d is the distance between the two metal plates; w is the width of the plate; R_s is the total surface resistance of the plates; $\varepsilon'(\omega)$ and $\varepsilon?(\omega)$ are the real part and imaginary part of the permittivity of the medium between the two plates; and ω is the circular frequency. The medium is supposed to be without magnetic loss, and its permeability $\mu = \mu_0 \cdot \mu_r$ is real.

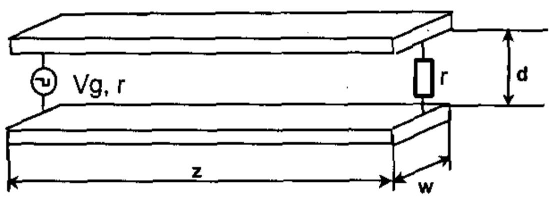

Figure 1. Parallel-plate transmission line model.

A sum of Lorentzian and Debye terms can approximate the frequency-domain permittivity function of almost any multi-component composite dielectric material,

$$\varepsilon(\omega) = \varepsilon_0\varepsilon_\infty + \varepsilon_0 \sum_{k=1}^{M} \frac{(\varepsilon_{sk} - \varepsilon_\infty)\omega_{0k}^2}{\omega_{0k}^2 - \omega^2 + j\omega(2\delta_k)} + \varepsilon_0 \sum_{i=1}^{N} \frac{\varepsilon_{si} - \varepsilon_\infty}{1 + j\omega\tau_i} - \frac{j\sigma_e}{\omega\varepsilon_0}, \quad (10)$$

where ε_{Sk} is the static dielectric constant for the k-th Lorentzian or Debye resonance line, ε_∞ is an optical region permittivity, ω_{0k} is the resonance frequency of the k-th Lorentzian peak, and $(2\delta_k)$ is the width of the k-th Lorentzian resonance line. The loss constant for the i-th Debye component is τ_i. The last term in (10) is responsible for the low-frequency conductivity loss.

For a single-component linear isotropic homogeneous Debye dielectric medium, the frequency-dependent permittivity can be described as,

$$\varepsilon(\omega) = \varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{1 + j\omega\tau_p}, \quad (11)$$

where τ_p is the pole relaxation time.

For a single-component dielectric Lorentzian material, the frequency-dependent permittivity is

$$\varepsilon(\omega) = \varepsilon_\infty + \frac{(\varepsilon_s - \varepsilon_\infty)\omega_0^2}{\omega_0^2 - \omega^2 + 2j\omega\delta}. \quad (12)$$

If the d.c. conductivity loss is substantial, then the term $\left(-\frac{j\sigma_e}{\omega\varepsilon_0}\right)$ is added to (11) and (12).

The real ε' and imaginary $\varepsilon?$ parts of the permittivity of a Debye or Lorentzian material can be obtained from the equations (11) and (12), respectively. Substituting R, L, G, C, ε', and $\varepsilon?$ into (5), the following equation is obtained,

$$\gamma = \sqrt{\omega\left(\frac{2R_s}{d} + j\omega\mu\right)(\varepsilon'' + j\varepsilon')}, \quad (13)$$

Assume only the TEM wave is excited in the geometry shown in Figure 1 by the voltage source having the amplitude V_g at the circular frequency ? . The S-parameters for the geometry can be calculated in the frequency domain as [2]

$$|S_{11}| = 20\,log_{10}\left|\frac{Z_{in} - r}{Z_{in} + r}\right|, \quad (14)$$

$$|S_{21}| = 20\,log_{10}\left|\frac{2V_0^+(1 + \Gamma_l)}{V_g}\right|, \quad (15)$$

where

$$V_0^+ = \frac{V_g Z_{in}}{(Z_{in} + r)(e^{-\gamma z} + \Gamma_l e^{\gamma z})}, \quad (16)$$

$$Z_{in} = Z_0 \frac{r + Z_0\,tanh(-\gamma\, z)}{Z_0 + r\,tanh(-\gamma\, z)}, \quad (17)$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}, \quad (18)$$

$$\Gamma_l = \frac{r - Z_0}{r + Z_0}. \quad (19)$$

Here r is the source resistance, which is the same as the resistance of the load. For the frequency range of interest, the S-parameters can be obtained from the equations (14) and (15) at different frequencies in the range of interest.

LIMITATIONS OF THE TRANSMISSION LINE METHOD

The transmission line method is not a full-wave technique, and it deals only with the single-mode (TEM) propagation, while the fringing fields due to finite size of parallel plates and higher order modes are neglected. This is true for every parallel-plate structure only over a limited frequency range, depending on its dimensions. Thus, to study frequency behavior of different dielectrics in different frequency ranges based on the proposed method, it may be necessary to design parallel-plate structures of different sizes.

The assumption that the fringing fields in the parallel-plate geometry, shown in Figure 1, are negligible, is true only when the ratio of the transmission line width to its thickness is $w/d>>1$. To satisfy this condition, the ratio w/d is usually taken at least as 10 or 20. It is reasonable to assume that the acceptable ratio is $w/d \geq 15$.

The transmission line model deals with only TEM mode. However, in the parallel-plate waveguide the higher order modes can be excited. Consideration should be limited to the single-mode case. Thus, the critical wavelength for the 1-st order modes (TE_1 and TM_1) of the structure are defined as [6]

$$\lambda_{c1} = 2w. \tag{20}$$

Then, the frequency where these 1-st order modes start propagating is

$$f_1 = \frac{c}{2w\sqrt{\varepsilon_r}}. \tag{21}$$

For example, to consider the material having the dielectric constant ε_r of about 2 in the frequency range up to 10 GHz, the parallel-plate waveguide with width w =10 mm should be constructed.

Summarizing the above discussion, the highest frequency of the single-mode propagation f_1 [Hz] or the thickness of the dispersive medium layer d [mm] can be estimated as

$$d \cdot f_1 \leq \frac{100 c_0}{3\sqrt{\varepsilon_r}} = \frac{10^{10}}{\sqrt{\varepsilon_r}}, \tag{22}$$

where c_0 [m/s] is the free space light velocity, and ε_r is the relative permittivity of the medium, or for the dispersive medium, the maximum real part permittivity in the frequency range of interest.

GENETIC ALGORITHM APPLICATION

Genetic Algorithms (GAs) belong to the class of stochastic search techniques for global search and optimization [3-5]. Most of the stochastic search methods deal with a single solution, while the GAs operate on a population solution. The GAs are associated with the directions and chances in the searching process, and combine "useful" information inherited from the individual parameters ("parents") to produce a new generation [3,4].

To implement the GAs, the objective function, representation, and operators should be defined very well, then the variation is very small, and the result is stable. In our case, the objective function is the measured scattering matrix parameter - $|S_{11}|$ or $|S_{21}|$. *The representation* is an array where the parameters of the dispersive medium are presented. The real value representation is used in the arrays, since it works much quicker than the binary representation, and its format is much closer to the problem solution in the case. The operators are based on the representation to complete initialization, mutation, and recombination for those individuals in the problem domain, as well as assigning fitness to individuals and producing new generations. If the discrepancy between the objective function and the search results is in the desired range, then the GA stops search. The expected parameters of the dispersive medium are extracted. Otherwise, the search will be continued. The program flow chart of the GA is shown in Figure 2.

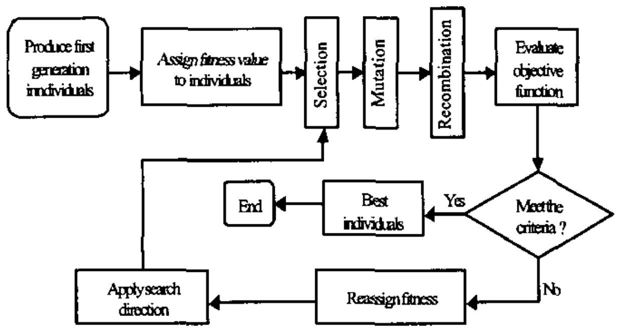

Figure 2. GA program flow chart.

The first generation population is selected randomly, and the initial fitness indices assigned to individuals are all equal to avoid bias introduced in the search domain. The population number of each parameter of the dispersive medium should be chosen appropriately to maintain the GA converging to the optimum while it works effectively. The choice of the population number is complicated, and it depends on the desired convergent speed, an expected accuracy of the solution, and the selection method employed. For example, to reconstruct the parameters of one of the Debye dispersive media, the population number is 80 for both static and optical region permittivity values, and 200 for both relaxation time and effective conductivity. A set of "good" optimized parameters that satisfy the optimization criterion is achieved at the 5th generation in the case with the selection method of truncation [3,4].

Fitness indices and *search directions* are two important issues in the GA. The fitness index implies the chance for staying in the search pool to produce next generation. It is in the format of *1/(abs(delta)+1)*, where the $delta = \Delta(f)$ is the discrepancy between the objective function and the calculated Sparameter for a certain set of individuals in frequency domain. The criterion of the S_{ij} parameter restoration at the current frequency f is

$$\Delta(f) = \left\| S_{ij}^{m}(f) \right| - \left| S_{ij}^{c}(f) \right\| < \eta, \tag{23}$$

where $\left|S_{ij}^{m}(f)\right|$ is the amplitude of the measured S-parameter, $\left|S_{ij}^{c}(f)\right|$ is the amplitude of the calculated S-parameter using the transmission line equations, and η [dB] is the demanded accuracy.

The fitness index shows which set contains "good" data, and which set contains "bad" data for further excluding from the search pool. The search direction is some physical rules embedded in the GA. These rules are intended to avoid producing meaningless simulation results and to make the GA converge quickly. For example, there is a causality condition $\varepsilon_S > \varepsilon_\infty$ for both Debye and Lorentzian media. There are two ways to maintain the GA search along this way. One is to keep only the individual of ε_S larger than the individual of ε_∞ having a chance to be in mating set. The other one is to combine the search direction to fitness. If the individual of ε_S is larger than the individual of ε_∞ in a mating set, the higher fitness index is assigned to the individuals in the mating set. Otherwise, a lower fitness is assigned to those individuals. Both methods work well in the GA.

Selection is a key procedure in the GA. The known selection methods are the "roulette wheel", "tournament", "ranking", and "truncation" selection [3,4]. The truncation method is used herein, since it makes the GA converge quickly, and is suitable for a wide-range parameter search. But it is easy to lose diversity in the problem domain. In order to avoid this problem, a 25% extension of the search range is applied at the both ends of each approximation of the parameter range.

The mutation operator introduces a certain amount of randomness to the search in the problem domain, and the recombination produces new individuals in combining the information contained in the parents. After these two steps, a new searched value is produced. This value will be compared with the objective function. The difference between the objective function and the searched value will be included in the fitness function, and the fitness will be reassigned to those individuals in the mating set. This procedure repeats again and again until reaching the search steps defined.

RECONSTRUCTION OF THE DEBYE DIELECTRIC PARAMETERS

Below there is an example of the Debye dielectric parameters reconstruction using the proposed method for an FR-4 dielectric, which is one of the glass-filled epoxy resin materials, and is widely used in printed circuit board designs. It is known that its relative permittivity varies substantially with frequency and temperature, and is different for different samples of the material. Three simple test boards for the parameters of the dielectric extraction were designed. These are double-sided copper-clad test boards with an FR-4 substrate. Their dimensions are represented in Table 1.

Table 1. Dimensions of the test boards

# Board	Length, mm	Width, mm	Thickness of dielectric, mm
1	82	14	0.39
2	83	13.5	1.05
3	73	14	1.45

The top and side views of a test board are schematically shown in Figure 3.

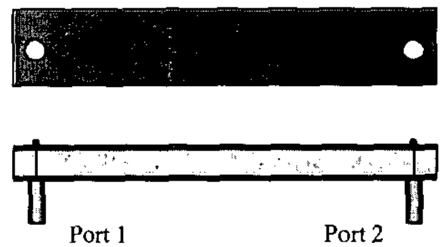

Figure 3. Test board with two SMA connectors.

Figure 4 shows the parameter fitness plot for static dielectric constant. The analogous parameter fitness plots were also obtained for the optical range permittivity, relaxation time, and the d.c. conductivity of the same material.

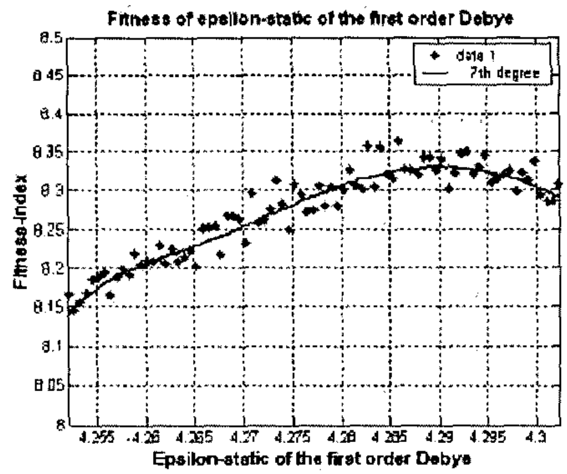

Figure 4. Fitness of the static dielectric constant of the Debye dispersion law.

Figures 5 and 6 represent the reconstructed real and imaginary parts of the permittivity of the FR-4 dielectric versus frequency in the test board #1. The d.c. conductivity loss essentially influences the form of the imaginary part of permittivity at lower frequencies.

The results of the extracted Debye parameters for the FR-4 dielectric in different test boards are summarized in Table 2.

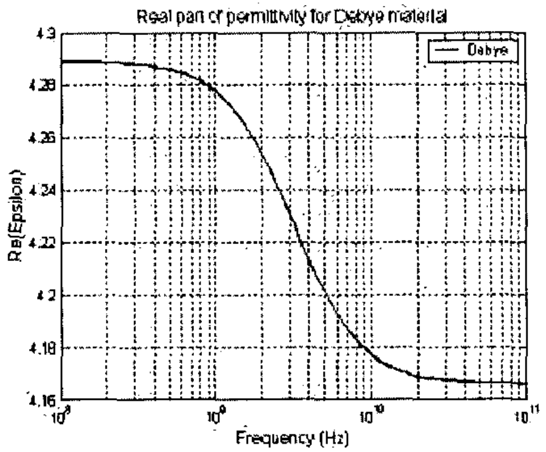

Figure 5. Reconstructed real part of permittivity of the Debye dielectric (FR-4) of the test board #1.

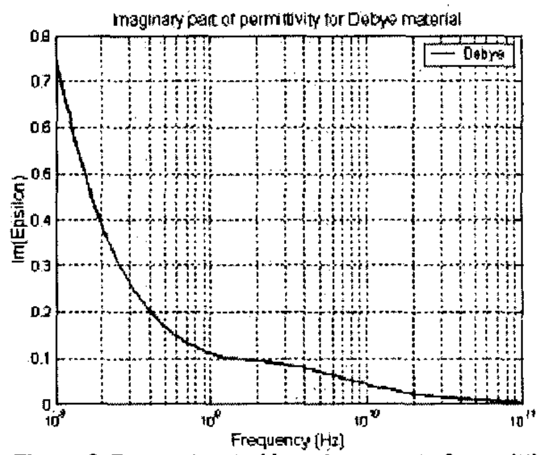

Figure 6. Reconstructed imaginary part of permittivity of the Debye dielectric (FR-4) of the test board #1.

Table 2. Extracted Debye parameters for the test boards

# Board	ε_s	ε_∞	τ , s	σ , S/m
1	4.289	4.166	$5.05 \cdot 10^{-11}$	0.00421
2	4.178	4.07	$1.15 \cdot 10^{-12}$	$7.15 \cdot 10^{-3}$
3	3.784	3.728	$1.85 \cdot 10^{-12}$	$5.519 \cdot 10^{-3}$

The extracted FR-4 Debye parameters are checked by comparing the measured and the FDTD modeling results. In the FDTD full-wave modeling, the copper plates of the boards were modeled as perfect electric conductors (PEC). The parameters of the dielectric substrate were extracted using the proposed GA method and were used in the FDTD modeling of the scattering matrix parameters of the test boards. The FDTD codes (called EZ-FDTD) were developed at the UMR, and they implement the recursive convolution procedure for taking into account the Debye dispersive law. The S-parameters were also measured using an HP 8753D network analyzer over the frequency range from 100 MHz to 5 GHz.

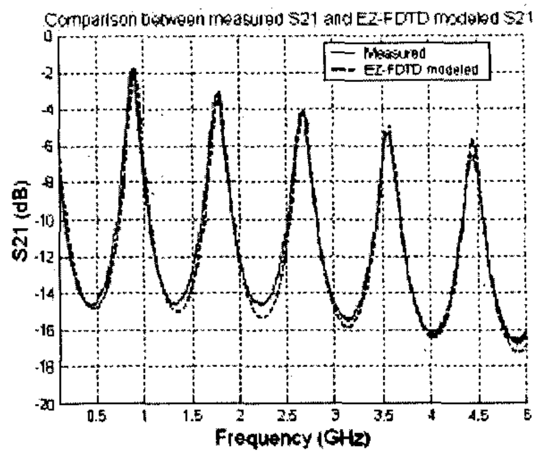

Figure 7. Measured and modeled $|S_{21}|$ for the board #1.

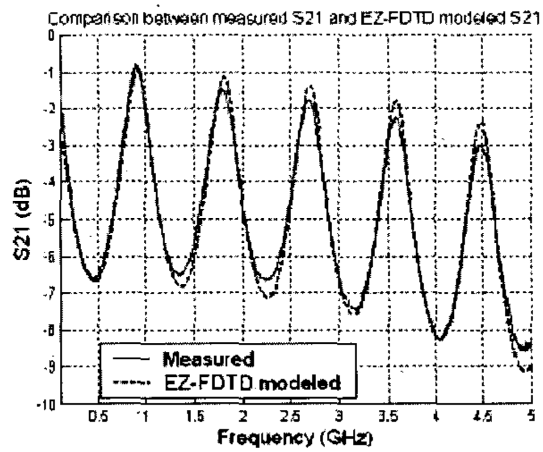

Figure 8. Measured and modeled $|S_{21}|$ for the board #2.

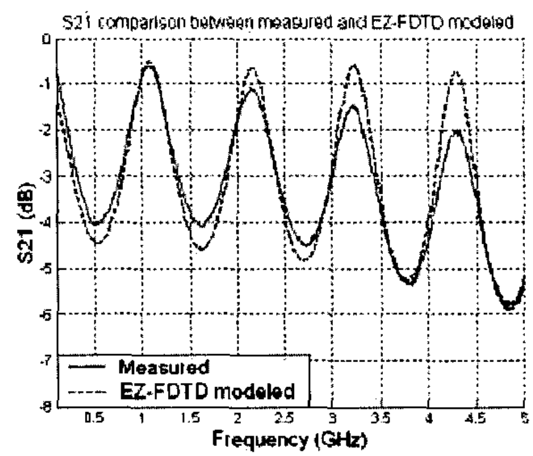

Figure 9. Measured and modeled $|S_{21}|$ for the board #3.

Figures 7, 8, and 9 represent the measured and modeled $|S_{21}|$ parameters for test boards #1, 2, and 3 (see Tables 1, 2). The results agree well. The agreement for test board #1 is within 0.5 dB in amplitude, and there is less than 0.05 GHz shift in resonance frequencies. For test board #2 having the dielectric layer 1.05-mm thick, the agreement is within the limits of 1 dB and 0.1 GHz. Figure 9 for test board #3 with the dielectric 1.45-mm thick shows that the discrepancy between the measured and FDTD modeled curves increases when the dielectric layer becomes thicker. This is a consequence of the assumption that the fringing fields in the transmission line model are neglected. In all the three considered cases, the highest frequency is about 5 GHz. The corresponding half-wavelength is 30 mm in free space, or about 15 mm in FR-4 medium with ε_r of about 4. Thus, the maximum thickness of the FR-4 layer should be less than approximately 1 mm. If the layer is 1.45 mm thick, the result is not as good as for thickness of 0.39 mm and 1.05 mm.

LORENTZIAN DIELECTRIC MEDIUM

An example for the Lorentzian dielectric parameters reconstruction using the proposed method is represented below. A 0.6 mm-thick sheet of the composite dielectric is placed between two copper plates 60 mm x 10 mm. This is test board #4. The dielectric material is composed of the polymer matrix of Teflon type filled with a mixture of long aluminum and short carbon fibers. The material is anisotropic; however, its parameters were studied only in the direction normal to its plane. The extracted parameters for the Lorentzian curves are represented in Table 3. This is a wideband Lorentzian dielectric, since the ratio of its resonance line width to the resonance frequency is greater than unity, $\delta / \omega_0 > 1$ [7].

Table 3. Extracted Lorentzian parameters

# Board	ε_s	ε_∞	$\Delta f_r = \delta / \pi$, GHz	f_0, GHz	σ, S/m
4	4.289	4.166	$5.05 \cdot 10^{-11}$	1.08	0.0056

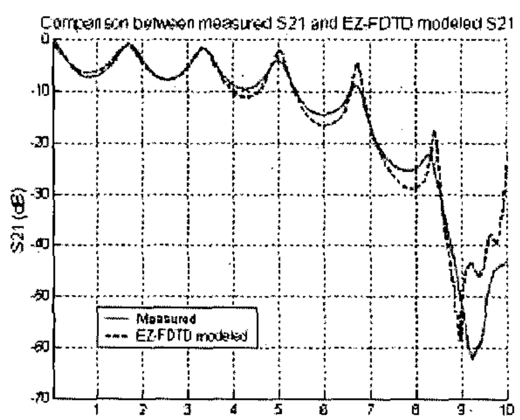

Figure 10. Measured and modeled $|S_{21}|$ of the board #4.

Figure 10 represents the measured and FDTD modeled $|S_{21}|$ parameter for test board #4. There is a good agreement at frequencies below 9 GHz. At higher frequencies the discrepancy is due to the increasing effect of higher-order modes neglected in the transmission line model. Besides, at higher microwave frequencies the influence of the ports on the S-parameter measurement might become noticeable.

CONCLUSIONS

The method for reconstruction of the parameters of the Debye and Lorentzian dispersive media was developed herein. It is based on transmission line theory and genetic algorithm application. The good agreement between the measured and the FDTD modeled scattering matrix parameters in the simple parallel-plate test fixture with a dielectric layer between two metal plates has been achieved when the dispersive media parameters were extracted using the proposed method. The parallel-plate test fixture is the simplest structure for both measurements and modeling, and it is suitable for reconstructing the parameters of dispersive media under condition for the highest frequency of single-mode (TEM) propagation in the parallel-plate waveguide.

The frequency range of the presented method can be expanded using test fixtures on the base of the microwave cavities or waveguides, such as metal waveguide. Some frequency-domain numerical techniques (e.g., method of moments) can be applied to calculate the propagating modes. Also, the method represented here, can be used for extracting the parameters of magnetic and magneto-dielectric dispersive media.

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