01 Jan 2004

A Preliminary Study of Maximum System-Level Crosstalk at High Frequencies for Coupled Transmission Lines

H. Weng

Daryl G. Beetner
Missouri University of Science and Technology, daryl@mst.edu

Todd H. Hubing
Missouri University of Science and Technology

R. Wiese

et. al. For a complete list of authors, see https://scholarsmine.mst.edu/ele_comeng_facwork/1394

Recommended Citation

H. Weng et al., "A Preliminary Study of Maximum System-Level Crosstalk at High Frequencies for Coupled Transmission Lines," *Proceedings of the International Symposium on Electromagnetic Compatibility (2004, Silicon Valley, CA)*, Institute of Electrical and Electronics Engineers (IEEE), Jan 2004.
The definitive version is available at https://doi.org/10.1109/ISEMC.2004.1349828

A Preliminary Study of Maximum System-level Crosstalk at High Frequencies for Coupled Transmission Lines

Xiaopeng Dong, Haixiao Weng, Daryl G. Beetner and Todd Hubing
Electromagnetic Compatibility Laboratory
University of Missouri-Rolla
Rolla, MO 65409
Email: xdrg4@umr.edu

Richard Wiese and Joseph McCallum
Electromagnetic Compatibility Electrical Center
General Motors
Milford, MI 48380

***Abstract*— Simple formulas were derived to quickly estimate maximum crosstalk between wires in a harness at high frequencies, where the length of circuits is comparable with or greater than the wavelength of the signals of interest. Formulas were derived from multi-conductor transmission line theory. When the source and load resistances are either both large or both small compared to the characteristic impedance of the transmission line, maximum coupling is shown to be given by approximately $X_{MAX} = \frac{C_m}{C_{22}+C_m}$. A similar equation was found for the case where the load impedances are matched. Experiments show these equations predict the maximum value of coupling well.**

Keywords- Crosstalk; Transmission line; Coupling

I. INTRODUCTION

Crosstalk between circuits may cause failure or malfunction if it is not well controlled. It is useful to estimate the maximum crosstalk between circuits early in the design stage to avoid potential problems after manufacture. Traditionally, a full-wave solver has been used to get an exact solution for crosstalk, but this method is time consuming and impractical in many cases. Moreover, full-wave simulation has to be repeated for every small change in the design and it is difficult to link simulation results to specific characteristics of the circuit configuration.

An exact value of crosstalk at all frequencies usually is not needed for preliminary evaluation of a circuit configuration. A ballpark approximation of maximum crosstalk levels is adequate [1]. The aim of the research presented here is to develop simple formulas to estimate the maximum value of crosstalk between culprit and victim circuits at high frequencies, where the wavelength under consideration is small compared to the length of the circuit. These estimates may be calculated easily and give the user a feel for the important contributors to crosstalk. This paper presents experimental results and simple closed form formulas to estimate maximum crosstalk at high frequencies for coupled transmission lines.

II. DERIVATION

A lumped-element model can be applied to simplify calculations when circuit dimensions are much smaller than the wavelength of the signals of interest. An equivalent model is shown in Figure 1. The crosstalk (coupling) is defined as $X = \frac{V_2}{V_1}$ and may be calculated from this equivalent circuit model.

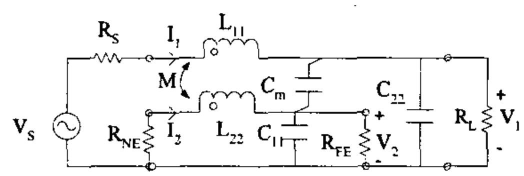

Figure 1. Lumped model of inductive and capacitive coupling.

The lumped-element model is not valid at high frequencies where the circuit length is greater than a quarter-wavelength. At these frequencies, multi-conductor transmission line models should be used. In this case, voltage is not constant across the length of the circuit. While the coupling might be defined as for the lumped element model, calculations can be simplified by defining coupling as $X = \frac{V_{2MAX}}{V_{1MAX}}$, where V_{2MAX} is the maximum voltage along the victim circuit and V_{1MAX} is the maximum voltage along the culprit circuit. Using this approximation for crosstalk, simple formulas can be derived from multi-conductor transmission line theory to estimate maximum levels of crosstalk at high frequencies between two circuits that share the same wiring harness.

Assumptions were made to bound the complexity of the problem. Assumptions include:

- The transmission lines are uniform across their length (i.e. geometry does not change)

- The coupling media is air (The permittivity and/or permeability of wire insulation or other nearby material is ignored)
- The transmission lines are lossless (The resistance of the wire and conductivity of the coupling media are ignored)

The voltage and current of two coupled circuits are related. Assuming the transmission line is lossless, their relationship can be expressed as [2][3]:

$$\frac{\partial}{\partial z}\bar{V} = -j\omega \bar{\bar{L}}\bar{I}, \qquad (1)$$

$$\frac{\partial}{\partial z}\bar{I} = -j\omega \bar{\bar{C}}\bar{V}. \qquad (2)$$

where $\bar{V} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$, V_1 and V_2 are the voltages on the culprit and victim circuit respectively; $\bar{I} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$, where I_1 and I_2 are the currents through the culprit and victim circuit respectively, $\bar{L} = \begin{bmatrix} L_{11} & L_m \\ L_m & L_{22} \end{bmatrix}$, where L_{11} is the self-inductance per unit length (PUL) of the culprit circuit, L_{22} is the self-inductance PUL of the victim circuit, and Lm is the mutual inductance PUL between the two circuits, and $\bar{C} = \begin{bmatrix} C_{11}+C_m & -C_m \\ -C_m & C_{22}+C_m \end{bmatrix}$, where C_{11} is the self-capacitance PUL of the culprit circuit, C_{22} is the self-capacitance PUL of the victim circuit, and Cm is the mutual capacitance PUL between two circuits.

Taking the partial derivative of (1) and (2) with respect to z and substituting appropriately for $\frac{\partial}{\partial z}\overline{V}$ and $\frac{\partial}{\partial z}\overline{I}$ gives:

$$\frac{\partial^2}{\partial z^2}\bar{V} = -\omega^2 \bar{\bar{L}}\bar{\bar{C}}\bar{V}, \qquad (3a)$$

$$\frac{\partial^2}{\partial z^2}\bar{I} = -\omega^2 \bar{\bar{C}}\bar{\bar{L}}\bar{I}. \qquad (3b)$$

Because the media is homogeneous, both $\bar{\bar{L}}\bar{\bar{C}}$ and $\bar{\bar{C}}\bar{\bar{L}}$ yield a diagonal matrix, and $\bar{\bar{L}}\bar{\bar{C}} = \bar{\bar{C}}\bar{\bar{L}} = \frac{1}{v_p^2}\bar{\bar{U}}$ where $\bar{\bar{U}}$ is the unit matrix and v_p is the phase velocity of wave propagation. In free space, $v_p = 3\times10^8$ m/s.

If one defines the term $\beta^2 \stackrel{\Delta}{=} \omega^2\mu\varepsilon$, then the solution to equation (3) can be expressed as:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_1^+ e^{-j\beta z} \\ V_2^+ e^{-j\beta z} \end{bmatrix} + \begin{bmatrix} V_1^- e^{j\beta z} \\ V_2^- e^{j\beta z} \end{bmatrix},$$

or

$$\overline{V} = \overline{V^+}e^{-j\beta z} + \overline{V^-}e^{j\beta z} \qquad (4)$$

where $\overline{V^+}$ is the incident voltage wave (from source to load) and $\overline{V^-}$ is the reflected voltage wave (from load to source).

Substituting (1) into (4), we get the general solution for the current as

$$\begin{aligned}\overline{I} &= \frac{1}{v_p}\bar{\bar{L}}^{-1}(\overline{V^+}e^{-j\beta z} - \overline{V^-}e^{j\beta z}) \\ &= v_p\bar{\bar{C}}\left(\overline{V^+}e^{-j\beta z} - \overline{V^-}e^{j\beta z}\right)\end{aligned} \qquad (5)$$

Boundary conditions must be applied at both source and load ends to get the specific solution for this problem. At the load end ($z = 0$), we have

$$\overline{V}(z=0) = \bar{\bar{Z}}_{Load}\overline{I}(z=0), \qquad (6)$$

where z represents position along the transmission line, $\overline{\overline{Z_{Load}}} = \begin{bmatrix} R_L & 0 \\ 0 & R_{FE} \end{bmatrix}$, and R_L and R_{FE} are the load impedance of the culprit and the far-end impedance of victim circuit respectively (see Fig. 1). At the source end, where $z = -l$, we have

$$\overline{V}(z=-l) = \overline{V_S} - \overline{\overline{Z_S}}\overline{I}(z=-l), \qquad (7)$$

where $\overline{\overline{Z_S}} = \begin{bmatrix} R_S & 0 \\ 0 & R_{NE} \end{bmatrix}$, and R_S and R_{NE} are the source impedance of the culprit circuit and near-end impedance of the victim circuit respectively.

The problem can be simplified by assuming rough values of termination impedances. First let us assume that both the culprit and victim circuit are shorted at the far ends. This is a reasonable approximation when far-end termination impedances are much less than the characteristic impedance of the transmission lines (typically about 300 ohms). In this case

$$\overline{V}(z=0) = \overline{V^+} + \overline{V^-} = 0. \qquad (8)$$

Substituting (8) back into (4) and (5), we get the simplified voltage and current equations as:

$$\overline{V} = -2j\sin(\beta z)\overline{V^+} \quad (9)$$

and

$$\overline{I} = 2\cos(\beta z)v_p\overline{\overline{C}}\overline{V^+}. \quad (10)$$

Therefore the maximum voltage on the culprit circuit is $2V_1^+$, and the maximum voltage on the victim circuit is $2V_2^+$. Applying boundary condition at the source end, where $z=-l$, we get

$$\overline{V}(z=-l) = \overline{V_S} - \overline{\overline{Z_S}}\overline{I}(z=-l).$$

Substituting this into (9) and (10)

$$\overline{V_s} = 2j\sin(\beta l)\overline{V^+} + 2\cos(\beta l)v_p\overline{\overline{Z_S}}\overline{\overline{C}}\overline{V^+},$$

where $\overline{V_S} = \begin{bmatrix} V_S \\ 0 \end{bmatrix}$, and Vs is the source voltage of the culprit circuit.

If we define $\overline{\overline{A}} = 2j\sin(\beta l) + 2\cos(\beta z)v_p\overline{\overline{Z_S}}\overline{\overline{C}}$, then

$$\overline{V_s} = \overline{\overline{A}}\overline{V^+}$$

where ,

$$\overline{\overline{A}} = \begin{bmatrix} 2v_p\cos(\beta l)R_S(C_{11}+C_m) + 2j\sin(\beta l) & -2v_p\cos(\beta l)R_SC_m \\ -2v_p\cos(\beta l)R_{NE}C_m & 2v_p\cos(\beta l)R_{NE}(C_{22}+C_m) + 2j\sin(\beta l) \end{bmatrix}$$

Thus

$$\overline{V_S} = \begin{bmatrix} V_S \\ 0 \end{bmatrix} = \begin{bmatrix} 2v_p\cos(\beta l)R_S(C_{11}+C_m) + 2j\sin(\beta l) & -2v_p\cos(\beta l)R_SC_m \\ -2v_p\cos(\beta l)R_{NE}C_m & 2v_p\cos(\beta l)R_{NE}(C_{22}+C_m) + 2j\sin(\beta l) \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

For a victim circuit operating at high frequencies, and therefore susceptible to high frequency noise, it is reasonable to assume the near end impedance of the victim circuit is around the characteristic impedance of the transmission line and to ignore the extreme case, i.e. $R_{NE}=0$ or $R_{NE}=\infty$. Then, solving for V_1 and V_2, the maximum coupling between the culprit and victim circuit is given by:

$$X_{Coupling} = \left|\frac{V_{2MAX}}{V_{1MAX}}\right| = \left|\frac{V_2^+}{V_1^+}\right| = \left|\frac{C_m}{(C_{22}+C_m) + j\dfrac{\tan(\beta l)}{v_pR_{NE}}}\right|$$

Maximum coupling occurs when $\sin(\beta l)=0$, thus minimizing the divisor. Peak coupling occurs at frequencies

$$f = \frac{kv_p}{2l}, \text{k} = 1, 2, \ldots$$

with value

$$X_{MAX} = \frac{C_m}{C_{22}+C_m}. \quad (11)$$

Similar derivations can be made when both the culprit and victim circuits are terminated with high impedance or are matched at the far ends. Without detailed derivation, the results are given as:

- When both the culprit and victim circuit are terminated with high impedance at the far end, the maximum coupling is given by approximately

$$X_{MAX} = \frac{C_m}{C_{22}+C_m}.$$

- When both circuits are matched at the far-end, the maximum coupling is given by approximately

$$X_{MAX} = \frac{C_m}{\dfrac{1}{v_pR_{NE}} + C_{22} + C_m}.$$

In real applications, the impedance of the culprit circuit should be well controlled to meet functionality requirements. If we assume that the culprit circuit is matched at the source end and open or shorted at the load end then, ignoring feedback coupling from the victim to the culprit, we can get the maximum voltage along the culprit circuit as:

$$V_{1MAX} = V_S. \quad (12)$$

When the culprit and victim circuit are right next to each other (strong coupling case, i.e. $C_{11} \ll C_m$ and $C_{22} \ll C_m$), the energy may couple back from the victim circuit to the culprit circuit and be absorbed by the source impedance of the culprit circuit because it is matched at the source end. Thus the maximum level of coupling could be given approximately by

$$X_{MAX} = \frac{C_m}{C_{22}+C_m}$$

regardless of the source and load impedance. In this case, the maximum voltage along the victim circuit is given by:

$$V_{2MAX} = V_{1MAX}X_{MAX} = V_SX_{MAX}. \quad (13)$$

III. VALIDATION

Experiments were performed to validate the equations derived above. Fig. 2 shows the experimental setup. Culprit and victim circuits were created by suspending two wires over a ground plane. Both circuits used the ground plane as the current return path. The source end of the culprit circuit was connected to port 1 of a network analyzer and the far end of the victim circuit was connected to port 2 of the network analyzer. S_{21} was measured to calculate crosstalk between the two circuits. The radius of the wires was 0.4 mm. The height of the wires over the ground plane was about 4.5 cm. The length of the wires was 87 cm. The distance between the circuits was varied from 2 mm (when they were placed right next to one another, separated only by their insulation) to 9 cm. For brevity, only some of the experimental results are discussed in the following text.

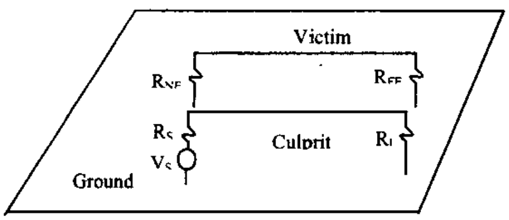

Figure 2. Experimental setup

The characteristic impedance of the culprit circuit was measured using a TDR and found to be 297 ohms. In the experiment, the source impedance of the culprit was set to 290 ohms, approximately matching the characteristic impedance of the transmission line.

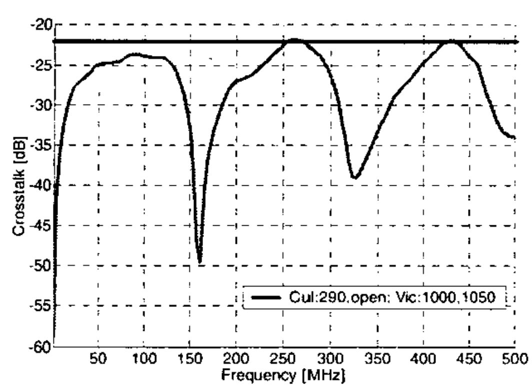

Figure 3. Measured and maximum-predicted crosstalk when the culprit and victim are 9 cm apart and 4.5 cm above the ground plane. The culprit circuit uses a 290 ohm source impedance and an open-circuit load. The victim uses a 1000 ohm near-end impedance and 1050 far-end (load) impedance. The red dashed line shows the measured value of crosstalk. The solid black line shows the maximum predicted value of crosstalk.

Fig's 3 and 4 show the measured crosstalk and the maximum value of crosstalk predicted by our simple equations when the two circuits were 9cm apart. Figure 5 shows the measured crosstalk and the maximum value of crosstalk predicted by our simple equations when the two circuits were right next to each other (approximately 2 mm apart). The derived values of maximum coupling are close (within 6dB) to values measured experimentally. It should be noted that the value of coupling, as defined here, may be slightly different than the value measured, as coupling was defined to be the ratio between the maximum voltages over the length of the line – not the ratio of voltages at the terminations. Of course, only the voltage at the end of the lines could be measured.

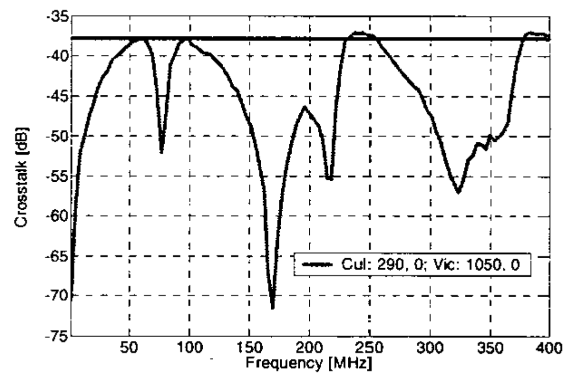

Figure 4. Measured and maximum-predicted crosstalk when the culprit and victim are 9 cm apart and 1.5 cm above the ground plane. The culprit circuit uses a 290 ohm source impedance and a short-cicuit load. The victim uses a 1050 ohm near-end impedance and short-circuit far-end load. The red dashed line shows the measured value of crosstalk. The solid black line shows the maximum predicted value of crosstalk.

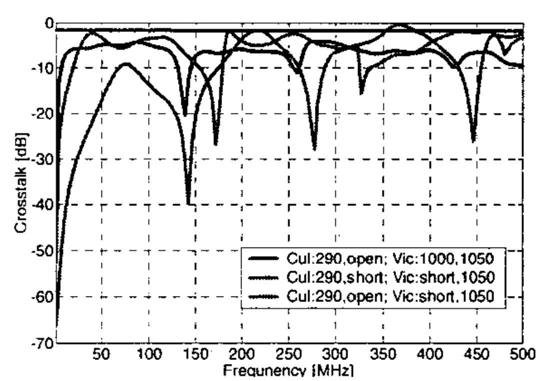

Figure 5. Measured and maximum-predicted crosstalk when the culprit and victim were right next to each other and 4.5 cm above the ground plane. The source and load impedances for the culprit and victim circuits are given in the figure legend. The solid black line shows the maximum predicted value of crosstalk.

IV. SUMMARY

Formulas were derived using multi-conductor transmission line theory to estimate the maximum coupling between two circuits at frequencies where the length of the circuit is greater than a quarter of the wavelength. The developed formulas were validated experimentally. The experimental results agree well with calculations. However, the formulas only work for some configurations, for example when both the culprit and victim circuits are terminated with a high or low impedance at the far (load) end. Currently, we are working on the development of formulas to estimate the high-frequency crosstalk for the general case, using any combination of source- and load-impedances.

REFERENCES

[1] S. Ranganathan; D. G. Beetner.; R. Wiese.; T. H. Hubing, "An Expert System Architecture to Detect System-level Automotive EMC Problems", Proceedings of the 2002 IEEE EMC Symposium, pp. 976 – 981, Minneapolis, August, 2002.

[2] D. M. Pozar, "Microwave Engineering", 2nd Edtion, John Wiley & Sons, Inc., 1998

[3] C. R. Paul, "Introduction to Electromagnetic Compatibility", John Wiley & Sons, Inc., 1992