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# Adaptive Neural Network Control and Wireless Sensor Network Based Localization for UAV Formation

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# Adaptive Neural Network Control and Wireless Sensor Network-based Localization for UAV Formation

H. Wu and S. Jagannathan

*Abstract*— **We consider a team of unmanned aerial vehicles (UAV's) equipped with sensors and motes for wireless communication for the task of navigating to a desired location in a formation. First a neural network (NN)-based control scheme is presented that allows the UAVs to track a desired position and orientation with reference to the neighboring UAVs or obstacles in the environment. Second, we discuss a graph theory-based scheme for discovery, localization and cooperative control. The purpose of the NN cooperative controller is to achieve and maintain the desired formation shape in the presence of unmodeled dynamics and bounded unknown disturbances. Numerical results are included to illustrate the theoretical conclusions**.

## I. INTRODUCTION

A significant amount of research was done in the last few years in the area of formation of multiple unmanned aerial vehicles (UAVs), robots, undersea vehicles, autonomous agents and more since several tasks can be performed more efficiently and robustly using multiple robots, and UAVs. Multiple UAV formation flying has several advantages including, increased instrument resolution, reduced cost, reconfigurability, and overall system robustness.

In [1], authors control the range among the vehicles in the formation while avoiding obstacles using dynamic inversion with an adaptive neural network loop. The authors in [2] propose an algorithm to control the relative position and orientation of robots while following a planned trajectory by using feedback linearization of the relative kinematics where the unknown state of the leader is treated as an input. Many research works assume that vision is only available as sensory feedback where as other related works [3] consider that communication exists among the members of the formation so that each vehicle in the formation knows the state of the other vehicles.

Close formation of multiple aircrafts has been of interest to many due to drag reduction. However, close formation flying causes various problems including the nonlinear aerodynamic coupling effects. Among the papers which are concerned with such a formation scenario are [4] where the authors use PID control, and [5] where a linear quadratic regulator (LQG) controller is proposed, while in [6], an advanced scheme such as sliding mode control for the outer loop and an adaptive dynamic inversion inner loop are employed for close formation flying. On the other hand, in [7], a peak-seeking controller is considered in order to achieve drag reduction by selectively placing the follower relative to the leader. In [8], a

three-dimensional close formation flying is investigated by using a PID controller. By contrast, in [9], a novel pursuit guidance algorithm is utilized for formation flying of multiple UAVs using kinematics and by assuming that no communication exists among UAVs and with imaging data only available. Using imaging sensors, location information such as line of sight (LOS) angle and LOS rate are estimated on line using computer vision algorithms.

In this paper, the dynamics of the UAVs are considered in contrast with many kinematics-based formation control works [2,9,11] and an adaptive neural network-based backstepping approach is utilized to design a guidance algorithm in order to maintain the relative range and orientation of multiple UAV formations. Second, we extend a graph theory-based scheme for discovery, localization and cooperative control. Discovery allows the UAVs to form into an ad hoc mobile sensor network whereas localization allows each UAV to estimate its position and orientation relative to its neighbors and hence the formation shape. The purpose of the proposed NN cooperative controller is to achieve and maintain the desired formation shape.

## II. PROBLEM FORMATION

Figure 1 shows an example of basic idea of leader-following. In this example, leader-following method is applied to control relative orientation, range and bearing, $\psi_j, l_{ij}, \alpha_{ij}$ to their desired values.
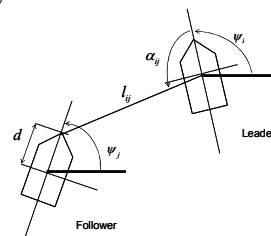


Figure 1. Basic idea of leader-following.

A formation consists of *N+1* UAV's and the leader follows the unknown trajectory relative to which the followers must track. Each follower can then estimate its desired relative range, orientation and bearing. The formation of UAVs is the topological relationship among UAVs, which can be described by relative range, orientation and bearing. Once the motion of the lead UAV is given, the formation is governed by local control laws based on the relative dynamics of each of the follower UAVs and the relative positions of the UAVs in formation.

### A. UAV Flat-Earth Dynamics and Kinematics

Taking the North-East-Down (NED) frame on the surface of the earth as an inertial reference frame, all the kinematics

and dynamics of the $i^{th}$ UAV are represented by using the following variables: $V_{B_i}$ denotes relative velocity of aircraft with respect to air mass, which including three components ($V_{Bx_i}, V_{By_i}, V_{Bz_i}$), $\omega_{B_i}$ represents absolute angular velocity of aircraft-body coordinate (ABC) frame, $\Phi_i$ denotes roll, pitch and yaw angles of the UAV, $P_{NED_i}$ represents the position of an UAV in NED coordinate frame [10]. The equations of motion of the $i^{th}$ UAV are given by

$$\dot{V}_{B_i} = -\Omega_{B_i} V_{B_i} + B_{B_i} g_o' + \left( F_{B_i} / m_i \right) \tag{1}$$

$$\dot{\omega}_{B_i} = -J_i^{-1} \Omega_{B_i} J_i \omega_{B_i} + J_i^{-1} T_{B_i} \tag{2}$$

$$\dot{\Phi}_i = \varepsilon_i(\Phi) \omega_{B_i} \tag{3}$$

$$\dot{P}_{NED_i} = B_{B_i}^T V_{B_i} \tag{4}$$

where $V_{B_i} = [U_i, V_i, W_i]^T$ is the velocity of $i^{th}$ UAV; $\omega_{B_i} = [P_i, Q_i, R_i]^T$ is the angular velocity of $i^{th}$ UAV; $\Phi_i = [\phi_i, \theta_i, \psi_i]^T$ denotes roll, pitch, and yaw of $i^{th}$ UAV; $P_{NED_i} = [P_{N_i}, P_{E_i}, h_i]^T$ is the position of the $i^{th}$ UAV in NED frame; $F_{B_i} = [F_{x_i}, F_{y_i}, F_{z_i}]^T$ is the control force input vector for $i^{th}$ UAV; $T_{B_i} = [L_i, M_i, N_i]^T$ is the control toque input for $i^{th}$ UAV; with

$$\Omega_{B_i} = \begin{bmatrix} 0 & -R_i & Q_i \\ R_i & 0 & -P_i \\ -Q_i & P_i & 0 \end{bmatrix}, \tag{5}$$

$$\varepsilon_i(\Phi) = \begin{bmatrix} 1 & \tan\theta_i \sin\phi_i & \tan\theta_i \cos\phi_i \\ 0 & \cos\phi_i & -\sin\phi_i \\ 0 & \dfrac{\sin\phi_i}{\cos\theta_i} & \dfrac{\cos\phi_i}{\cos\theta_i} \end{bmatrix}, \tag{6}$$

$$g_o' = [0 \quad 0 \quad 9.805]^T, \tag{7}$$

and

$$B_{B_i} = \begin{bmatrix} \cos\theta_i\cos\psi_i & \cos\theta_i\sin\psi_i & -\sin\theta_i \\ -\cos\phi_i\sin\psi_i+\sin\phi_i\sin\theta_i\cos\psi_i & \cos\phi_i\cos\psi_i+\sin\phi_i\sin\theta_i\sin\psi_i & \sin\phi_i\cos\theta_i \\ \sin\phi_i\sin\psi_i+\cos\phi_i\sin\theta_i\cos\psi_i & -\sin\phi_i\cos\psi_i+\cos\phi_i\sin\theta_i\sin\psi_i & \cos\phi_i\cos\theta_i \end{bmatrix} \tag{8}$$

Define $x_{1_i} = [\Phi_i^T \quad P_{NED_i}^T]^T, x_{2_i} = [v_{B_i}^T \quad w_{B_i}^T]^T$. Then the equations of motion above can be changed into following format with $m_i$ being the mass of UAV, and $J_i$ is the inertia matrix and in the presence of disturbances as

$$\dot{x}_{1_i} = g_{1i}(x_{1_i}) x_{2_i} + d_{1i} \tag{9}$$

$$\dot{x}_{2_i} = f_{2i}(x_{1_i}, x_{2_i}) + g_{2i} u_i + d_{2i} \tag{10}$$

where

$$g_{1i}(x_{1_i}) = \begin{bmatrix} 0 & \varepsilon_i(\Phi) \\ B_{B_i}^T & 0 \end{bmatrix}, f_{2i}(x_{1_i}, x_{2_i}) = \begin{bmatrix} -\Omega_{B_i} V_{B_i} + B_{B_i} g_o' \\ -J_i^{-1} \Omega_{B_i} J_i \end{bmatrix}, g_{2i} = \begin{bmatrix} M_i^{-1} & 0 \\ 0 & J_i^{-1} \end{bmatrix}, \tag{11}$$

$$M_i = \begin{bmatrix} m_i & 0 & 0 \\ 0 & m_i & 0 \\ 0 & 0 & m_i \end{bmatrix}, J_i = \begin{bmatrix} J_{x_i} & 0 & -J_{xz_i} \\ 0 & J_{y_i} & 0 \\ -J_{xz_i} & 0 & J_{z_i} \end{bmatrix}, \tag{12}$$

$$J_{x_i} = \int (y_i^2 + z_i^2) dm, J_{y_i} = \int (x_i^2 + z_i^2) dm, J_{z_i} = \int (x_i^2 + y_i^2) dm$$

$$J_{xz_i} = \int (x_i z_i) dm$$

and the control input vector is given by

$$u_i = [F_{B_i}^T, T_{B_i}^T]^T \tag{13}$$

Using the lead UAV kinematics, equations of motion for the follower UAV flying at the same altitude as that of the lead UAV can be defined as [2]

$$\dot{l}_{ij} = V_j \cos\gamma_i - V_i \cos\alpha_{ij} + d\omega_j \sin\gamma_i$$

$$\dot{\alpha}_{ij} = [V_i \sin\alpha_{ij} - V_j \sin\gamma_i + d\omega_j \cos\gamma_i - l_{ij}\omega_i] / l_{ij} \tag{14}$$

$$\dot{\psi}_j = \omega_j$$

where $\gamma_i = \psi_i + \alpha_{ij} - \psi_j$, and $V_j$ and $\omega_j$ represent the linear and angular velocities of the follower UAV. In order the avoid collisions, the separation, $l_{ij}$, must be greater than the length of the UAV denoted by $d$.

### B. Neural Network Controller Design for the Leader

In practical application, $g_{1i}(x_1), f_{2i}(x_{1i}, x_{2i}), g_{2i}$ are often unknown and vary with time. In this section, a neural network inner loop is designed with help of backstepping technique in order to approximate the unknown nonlinear terms and compensate them for the leader UAV. Then an outer tracking control loop is designed for the leader UAV. Indeed, similar control scheme can be employed for the follower UAV also. The leader will track its own trajectory $x_{d1_i}$. Here a two-layer NN is utilized to compensate the unknown nonlinear dynamics of the leader and the followers.

Step 1: Define $e_{1_i} = x_{1_i} - x_{d1_i}$ where $x_{1_i}$ and $x_{d1_i}$ are actual and desired states which are $6 \times 1$ vectors. Its derivative after substituting (9) is given by

$$\dot{e}_{1_i} = g_1(x_{1_i}) \cdot x_{2_i} - \dot{x}_{d1_i} + d_{1_i} \tag{15}$$

Define $e_{2_i} = x_{2_i} - x_{d2_i}$, and therefore $x_{2_i} = e_{2_i} + x_{d2_i}$. Substituting $x_{2_i}$ into equation (15) to get

$$\dot{e}_{1_i} = g_{1_i}(x_{1_i}) e_{2_i} + g_{1_i}(x_{1_i}) x_{d2_i} - \dot{x}_{d1_i} + d_{1_i} \tag{16}$$

By viewing $x_{2_i}$ as a virtual input for the $e_{1_i}$-subsystem in (16), there exists a desired feedback control law $x_{d2_i} = g_{1_i}^{-1}(x_{1_i})[\dot{x}_{d1_i} - k_{1_i} e_{1_i}]$, where $g_{1_i}^{-1}(x_{1_i})$ is a known matrix and $k_{1_i}$ is a diagonal matrix with positive elements.

Using $x_{d2_i}$ into (16), equation (16) can be written as

$$\dot{e}_{1_i} = g_{1_i}(x_{1_i}) \cdot e_{2_i} - k_{1_i} e_{1_i} + d_{1_i}$$

or

$$\dot{e}_{1_i} = -k_{1_i} e_{1_i} + g_{1_i}(x_{1_i}) \cdot e_{2_i} + d_{1_i} \tag{17}$$

Now considering $e_{2_i} = x_{2_i} - x_{d2_i}$, its derivative is given by

$$\dot{e}_{2_i} = \dot{x}_2 - \dot{x}_{d2_i} = f_{2_i}(x_{1_i}, x_{2_i}) + g_{2_i} u_i - \dot{x}_{d2_i} + d_{2_i} \quad (18)$$

where $g_{2_i}$ is known constant matrix, $f_{2_i}(x_{1_i}, x_{2_i})$ is unknown. Defining $F_{2_i}(.) = f_{2_i}(x_{1_i}, x_{2_i}) - \dot{x}_{d2_i}$, equation (18) is can be rewritten as

$$\dot{e}_{2_i} = F_{2_i}(.) + g_{2_i} u_i + d_{2_i}. \quad (19)$$

Now select a feedback control law

$$u_i = g_{2_i}^{-1}(-\hat{F}_{2_i}(.) - k_{2_i} e_{2_i}) \quad (20)$$

where $k_{2_i}$ is a diagonal matrix and $\hat{F}_{2_i}(.)$ is the neural network approximation value of $F_{2_i}(.)$. By employing a two-layer neural network $W_{1_i}^T \Phi_{1_i}(V_{1_i}^T z_{1_i})$ [12] to approximate $F_{2_i}(.)$, $F_{2_i}(.)$ can be expressed as $F_{2_i}(.) = W_{2_i}^T \Phi_i(V_{2_i}^T z_{2_i}) + \varepsilon_{2_i}$, where $z_{2_i} = [x_1^T, x_2^T, \dot{x}_{d2_i}^T]^T$ and $V_{2_i}$ and $W_{2_i}$ denote constant ideal weights, and $\varepsilon_{2_i}$ is the approximation error whose upper bound is given by known constant $\|\varepsilon_{2_i}\| \leq \varepsilon_{2N_i}$. It is important to note that matrices $W_{1_i} \in R^{n_1}$ and $V_{1_i} \in R^{4 \times n_1}$ represent target output and hidden layer weights, $\Phi_i(\cdot)$ represents the hidden layer activation function with $n_1$ denotes the number of the hidden layer nodes. For simplicity define $\Phi_i(z_{2_i}) = \Phi_i(V_{2_i}^T z_{2_i})$.

*Assumption 1 (Bounded Ideal Weights):* Let $W_{2_i}$ be the unknown output layer target weights for NN and assume that they are bounded above so that $\|W_{2_i}\| \leq W_{2M_i}$ where $W_{2M_i} \in R^+$ represents the bound on the unknown target weights when the Frobenius norm is used [13].

*Fact 1:* The activation functions are bounded above by known positive values so that $\|\Phi_i(\cdot)\| \leq \Phi_{iM}$.

Since $W_{2_i}$ is unknown, let $\hat{W}_{2_i}$ be the estimate of $W_{2_i}$. Define the weight estimation errors $\tilde{W}_{2_i} = W_{2_i} - \hat{W}_{2_i}$. Substitute equation (20) into equation (19), equation (21) can be rewritten as

$$\dot{e}_{2_i} = -k_{2_i} e_{2_i} + \tilde{F}_{2_i}(.) + d_{2_i} \quad (21)$$

where

$$\tilde{F}_{2_i}(\bullet) = F_{2_i}(.) - \hat{F}_{2_i}(.) = W_{2_i}^T \Phi_i + \varepsilon_{2_i} - \hat{W}_{2_i}^T \Phi_i = \tilde{W}_{2_i}^T \Phi_i + \varepsilon_{2_i}.$$

Consider following Lyapunov function

$$V_i = \frac{1}{2} e_{1_i}^T e_{1_i} + \frac{1}{2} tr\{\tilde{W}_{2_i}^T \Gamma_{2_i}^{-1} \tilde{W}_{2_i}\} + \frac{1}{2} e_{2_i}^T e_{2_i} \quad (22)$$

and weight adaptation law

$$\dot{\hat{W}}_{2_i} = \Gamma_{2_i} \Phi_i e_{2_i}^T - k_{\omega_i} \Gamma_{2_i} \|x_i\| \hat{W}_{2_i} \quad (23)$$

with $x_i = [e_{1_i}^T, e_{2_i}^T]^T$, constant positive definite diagonal matrix $\Gamma_{2_i}$, and $k_{\omega_i}$ scalar positive constant. Now the stability of the leader can be demonstrated. Using this result, the stability of the formation is inferred.

*Theorem 2.1:* Consider the $i^{th}$ UAV dynamics given in (9) and (10) and let the Assumption 1 and Fact 1 hold. Let the unknown disturbances be bounded by $\|d_{1_i}\| \leq d_{1m_i}$ and $\|d_{2_i}\| \leq d_{2m_i}$, respectively. Let the control input be given by (20) and NN weight tuning be provided by (23). The tracking errors, and the NN weights, $\hat{W}_{2_i}$, are bounded.

*Proof:* Consider the Lyapunov function (22) whose first derivative is given by

$$\dot{V}_i = e_{1_i}^T \dot{e}_{1_i} + tr\{\tilde{W}_{2_i}^T \Gamma_{2_i}^{-1} \dot{\tilde{W}}_{2_i}\} + e_{2_i}^T \dot{e}_{2_i} \quad (24)$$

Substituting equations (17), (21) and (23) into equation (24) and simplifying to get

$$\dot{V}_i = -x_i^T k_i x_i + tr\{k_{\omega_i} \|x_i\| \tilde{W}_{2_i}^T (W_{2_i} - \tilde{W}_{2_i})\} + x_i^T d_i \quad (25)$$

where $k_i = \begin{bmatrix} k_{1_i} & -g_{1_i}(.) \\ 0 & k_{2_i} \end{bmatrix}$, $d_i = \begin{bmatrix} 0 \\ \varepsilon_{2_i} \end{bmatrix}$ with $\|d_i\| \leq d_{M_i}$ using

Assumptions defined above. Hence

$$\dot{V}_i \leq -\Lambda_{\min_i} \|x_i\|^2 + tr\{k_{\omega_i} \|x_i\| (\tilde{W}_{2_i}^T W_{2_i} - \tilde{W}_{2_i}^T \tilde{W}_{2_i})\} + \|x_i\| d_{M_i} \quad (26)$$

where $\Lambda_{\min}$ is the smallest singular value of matrix $k_i$. The matrix $k_i$ can be shown to be positive definite if $k_{1_i}$ and $k_{2_i}$ are large enough. Using the following equality (Schwartz inequality) in (26)

$$tr\{\tilde{W}_{2_i}^T(W_{2_i} - \tilde{W}_{2_i})\} = <\tilde{W}_{2_i}, W_{2_i}>_F - \|\tilde{W}_{2_i}^T\|_F^2 \leq \|\tilde{W}_{2_i}\|_F \|W_{2_i}\|_F - \|\tilde{W}_{2_i}^T\|_F^2 \quad (27)$$

we have

$$\dot{V}_i \leq -\Lambda_{\min_i} \|x_i\|^2 + k_{\omega_i} \|x_i\| \|\tilde{W}_{2_i}\| (W_{2M_i} - \|\tilde{W}_{2_i}\|) + \|x_i\| d_{M_i} \quad (28)$$

$$= -\|x_i\| [\Lambda_{\min_i} \|x_i\| + k_{\omega_i} \|\tilde{W}_{2_i}\| (\|\tilde{W}_{2_i}\| - W_{2M_i}) - d_{M_i}]$$

which is negative as long as the term in the square bracket is positive. Completing the square yields

$$\Lambda_{\min_i} \|x_i\| + k_{\omega_i} \|\tilde{W}_{2_i}\| (\|\tilde{W}_{2_i}\| - W_{2M_i}) - d_{M_i}$$

$$= k_{\omega_i} (\|\tilde{W}_{2_i}\| - W_{2M_i}/2)^2 - k_{\omega_i} (W_{2M_i}^2/4) + \Lambda_{\min_i} \|x_i\| - d_{M_i}$$

which is guaranteed positive as long as

$$\|x_i\| > (k_\omega W_{2M_i}^2/4 + d_{M_i})/\Lambda_{\min_i} \quad (29)$$

or

$$\|\tilde{W}_{2_i}\| > \left(W_{2M_i} + \sqrt{W_{2M_i}^2 + (4 d_{M_i}/K_{\omega_i})}\right)/2. \quad (30)$$

Thus $\dot{V}$ is negative outside a compact set. The form of the right-hand side of (29) shows that the control gain $k_{1_i}$ and $k_{2_i}$ which are contained in $\Lambda_{\min_i}$, can be selected large enough so that

$$[k_{\omega_i} W_{2M_i}^2/4 + d_{M_i}]/\Lambda_{\min_i} < b_{x_i}.$$

Therefore, any trajectory $x_i(t)$ beginning in $U_{x_i}$ evolves completely within $U_{x_i}$. According to a standard Lyapunov extension [13], this demonstrates the UUB of both $x_i(t)$ and $\tilde{W}_{2_i}(t)$.

## C. Formation Control

The desired trajectories for the follower can be obtained by solving the kinematic equations (14) using input/output feedback linearization technique as

$$\omega_{ij} = \frac{\cos \gamma_1}{d}[\kappa_j l_{ij}(\alpha_{ijd} - \alpha_{ij}) - V_i \sin \alpha_{ij} + l_{ij}\omega_i + \rho_{ij}\sin \gamma_i]$$ (31)

$$V_j = \rho_{ij} - d\omega_j \tan \gamma_1$$

where $\rho_{ij} = (\kappa_i(l_{ijd} - l_{ij}) + V_i \cos \alpha_{ij})/\cos \gamma_i$, and $\kappa_i$ and $\kappa_j$ are positive constants. Equation (31) is used as a reference trajectory for follower UAVs and using (10), a controller can be designed for the follower UAVs..

In this section, we focus our attention on controlling the internal geometries of the formation. One scheme is referred to as separation bearing control ($l - \alpha$) where follower tracks one leader. By contrast, the separation-separation control ($l - l$) ensures a follower is tracking two leaders. In both cases, we assume the linear, angular velocities and orientation of leaders are known. Next, we will detail the $l - \alpha$ design. The $l - l$ control design is quite similar to the $l - \alpha$ design.

In Fig. 1, we show a system of two UAVs separated by a distance of $l_{ij}$ between the center of the $i^{th}$ UAV and the front point of the $j^{th}$. For these two UAVs system, we derive the following equations from (31)

$$\dot{l}_{ij} = v_j \cos \gamma_i - v_i \cos \alpha_{ij} + d\omega_j \sin \gamma_i$$

$$\dot{\alpha}_{ij} = (v_i \sin \alpha_{ij} - v_j \sin \gamma_i + d\omega_j \cos \gamma_i - l_{ij}\omega_i)/l_{ij}$$ (32)

$$\dot{\psi}_j = \omega_j$$

where $\gamma_i = \psi_i + \alpha_{ij} - \psi_j$, and $V_j$ and $\omega_j$ represent the linear and angular velocities of the follower UAV. In order the avoid collisions, the separation $l_{ij}$, must be greater than the length of the UAV denoted by $d$. Let $x_{ij} = [l_{ij}, \alpha_{ij}]^T$, above equations can be represented as

$$\dot{x}_{ij} = f_{ij}(x_{ij}) + g_{ij}(x_{ij})x_j,$$ (33)

where

$$f_{ij}(x_{ij}) = \begin{bmatrix} -v_i \cos \alpha_{ij} \\ \dfrac{v_i \sin \alpha_{ij}}{l_{ij}} - \omega_i \end{bmatrix}, g_{ij}(x_{ij}) = \begin{bmatrix} \cos \gamma_i & d \sin \gamma_i \\ -\dfrac{\sin \gamma_i}{l_{ij}} & \dfrac{d \cos \gamma_i}{l_{ij}} \end{bmatrix} x_j = \begin{bmatrix} v_j \\ \omega_j \end{bmatrix},$$

$v_j$ and $\omega_j$ are the linear, angular velocities of the follower UAV. We consider that $v_i$, $\omega_i$ and $\psi_i$ are known. So, $f_{ij}(x_{ij})$ and $g_{ij}(x_{ij})$ are both known function. Furthermore, $g_{ij}(x_{ij})$ is an invertible matrix. By viewing $x_j$ as virtual control input, we can use standard feedback linearization methods to generate a control law that gives exponentially convergent solutions in the internal shape variables $l_{ij}, \alpha_{ij}$ as

$$x_j = g_3^{-1}(x_{ij})(\dot{x}_{ijd} - f_3(x_{ij}) - k_{ij}e_{ij})$$

or

$$\omega_j = \frac{\cos \gamma_1}{d}[\kappa_j l_{ij}(\alpha_{ijd} - \alpha_{ij}) - V_i \sin \alpha_{ij} + l_{ij}\omega_i + \rho_{ij}\sin \gamma_i]$$ (34)

$$v_j = \rho_{ij} - d\omega_j \tan \gamma_1$$

where $\rho_{ij} = (\kappa_i(l_{ijd} - l_{ij}) + V_i \cos \psi_{ij})/\cos \gamma_i$, $\kappa_i$ and $\kappa_j$ are positive constants, and $e_{ij} = x_{ij} - x_{ijd}$. Equation (34) when applied to (32) guarantees the boundedness of the control input (34). Next, the follower dynamics are same as leader, given by equations (1),(2),(3),(4) or (9),(10).

We assume all UAVs fly at the same attitude and here in order to keep the notations the same, $\dot{x}_{2_j} = \dot{x}_j$ and $e_{2_j} = x_{2_j} - x_{d2_j}$. Then, the objective of the follower UAV controller is to keep: 1) all UAVs to fly at the same attitude; 2) pitch and roll of all UAVs zero; 3) all UAVs in a formation. In other words, it means that $v_{B_{jd}} = [v_j \cos \psi_j, v_j \sin \psi_j, W_j]^T$, $\omega_{B_{jd}} = [P_j, Q_j, \omega_j]^T$ $\Phi_{jd} = [0, 0, \psi_j]^T$, $P_{NEDd_j} = [P_{N_j}, P_{E_j}, h_{jd}]^T$ where $v_j$ and $\omega_j$ are derived from equation (34), $h_{jd}$ is the desired flying height. Combining $\Phi_{jd}$, $P_{NEDd_j}$ and equation(9), $W_j$, $P_j$, $Q_j$ can be calculated respectively. So $x_{d2_j} = [v_{B_{jd}}^T, \omega_{B_{jd}}^T]^T$ from (34).

Now considering $e_{2_j} = x_{2_j} - x_{d2_j}$, its derivative is given by

$$\dot{e}_{2_j} = \dot{x}_{2_j} - \dot{x}_{d2_j} = f_{2_j}(x_{1_j}, x_{2_j}) + g_{2_j}u_j - \dot{x}_{d2_j} + d_{2_j}$$ (35)

where $g_{2_j}$ is known constant matrix, $f_{2_j}(x_{1_j}, x_{2_j})$ is unknown.

Defining $F_{2_j}(\bullet) = f_{2_j}(x_{1_j}, x_{2_j}) - \dot{x}_{d2_j}$, equation (35) is can be rewritten as

$$\dot{e}_{2_j} = F_{2_j}(\bullet) + g_{2_j}u_j + d_{2_j}.$$ (36)

where $d_{2_j}$ is the unknown disturbances so that $\|d_{2_j}\| \le d_{2m_j}$

Now select a feedback control law

$$u_j = g_{2_j}^{-1}(-\hat{F}_{2_j}(\bullet) - k_{2_j}e_{2_j})$$ (37)

where $k_{2_j}$ is a positive definite diagonal matrix and $\hat{F}_{2_j}(\bullet)$ is the neural network approximation value of $F_{2_j}(\bullet)$. By employing a two-layer neural network $W_{2_j}^T \Phi_{2_j}(V_{2_j}^T z_{2_j})$, $F_{2_j}(\bullet)$ can be expressed as $F_{2_j}(\bullet) = W_{2_j}^T \Phi_j(V_{2_j}^T z_{2_j}) + \varepsilon_{2_j}$, where $z_{2_j} = [x_{1_j}^T, x_{2_j}^T, x_{d2_j}^T, \dot{x}_{d2_j}^T]^T$ and $V_{2_j}$ and $W_{2_j}$ denote constant ideal weights, and $\varepsilon_{2_j}$ is the approximation error whose upper bound is given by known constant $\|\varepsilon_{2_j}\| \le \varepsilon_{2N_j}$. Since $W_{2_j}$ is the target unknown weight matrix, let $\hat{W}_{2_j}$ be the estimate of $W_{2_j}$. Define the weight estimation errors as $\tilde{W}_{2_j} = W_{2_j} - \hat{W}_{2_j}$. Substitute equation (37) into equation (36), equation (36) can be rewritten as

$$\dot{e}_{2_j} = -k_{2_j}e_{2_j} + \tilde{F}_{2_j}(\bullet) + d_{2_j}$$ (38)

where $\tilde{F}_{2_j}(\bullet) = F_{2_j}(\bullet) - \hat{F}_{2_j}(\bullet) = W_{2_j}^T \Phi_j + \varepsilon_{2_j} - \hat{W}_{2_j}^T \Phi_j = \tilde{W}_{2_j}^T \Phi_j + \varepsilon_{2_j}$.

*Assumption 2 (Bounded Ideal Weights):* Let $W_{2_j}$ be the

unknown output layer target weights for NN and assume that they are bounded above so that $\|W_{2_j}\| \le W_{2M_j}$ where $W_{2M_j} \in R^+$ represents the bound on the unknown target weights when the Frobenius norm is used [13].

*Fact 2:* The activation functions are bounded above so that $\|\Phi_j(\cdot)\| \le \Phi_{jM}$ where $\Phi_{jM}$ is the upper bound.

Consider the following Lyapunov function

$$V_j = \frac{1}{2} e_{ij}^T e_{ij} + \frac{1}{2} tr\{\tilde{W}_{2_j}^T \Gamma_{2_j}^{-1} \tilde{W}_{2_j}\} + \frac{1}{2} e_{2_j}^T e_{2_j} \qquad (39)$$

and weight adaptation law

$$\dot{\hat{W}}_{2_j} = \Gamma_{2_j} \Phi_j e_{2_j}^T - k_{\omega_j} \Gamma_{2_j} \|x_{i_j}\| \hat{W}_{2_j} \qquad (40)$$

where $x_{i_j} = [e_{ij}^T, e_{2_j}^T]^T$, $\Gamma_{2_j}$ a constant positive definite diagonal matrix, and $k_{\omega_j}$ being a scalar positive constant. Now the stability of the tracking control can be demonstrated for the follower UAVs.

*Theorem 2.2:* Consider the follower UAVs and let $l-\alpha$ control method given in (34) is the preferred scheme. Let the Assumption 2 and Fact 2 hold for each follower UAV. Let the unknown disturbances for the UAVs be bounded by $\|d_{1_j}\| \le d_{1m_j}$ and $\|d_{2_j}\| \le d_{2m_j}$, respectively. Let the control input for each follower UAV be given by (37) and NN weight tuning be provided by (40). Let the control input for the leader UAV be given by (20) with the NN weight tuning be provided by (23). Let the desired trajectory and its derivatives of the leader be bounded. The follower separation and bearing errors, and NN weights, $\hat{W}_{2_j}$, are uniformly ultimately bounded.

*Theorem 2.3:* Consider the $i^{th}$ UAV dynamics given in (9) and (10) and let the Assumptions 1 and 2 hold. Let the unknown disturbances for each UAV be bounded. Let the control input for each UAV be given by (20) and NN weight tuning be provided by (23) for the leader UAV. Let the desired trajectory and its derivatives for the leader, for instance $i=1$, is bounded. Let the desired control input for the follower UAVs be provided by (37). Then the formation is stable.

*Proof:* The proof follows from Theorem 2.1 and 2.2.

## III. LOCALIZATION, DISCOVERY AND CONTROL

A group of UAVs can be modeled as a nonlinear interconnected system where the controller assignment for the UAVs can be represented as a graph. A directed edge from the leader to the followers denotes a controller for the followers while the leader is trying to track a trajectory. We have shown that the basic formation is stable that is relative distances and bearings reach their desired values with a bounded error. The shape vector consisting of separations and orientations determines the relative positions of the UAVs with respect to the leader. The position and orientation of the lead UAV can be used to describe the gross position and orientation of the group.

Then a group of $N$ UAVs is built on two networks: a physical network that captures the constraints on the

dynamics of the lead UAV and control of each follower UAV using a sensing and communication network, preferably wireless, that describes information flow, sensing and computational aspects across the group. The design of the graph is based on the task in hand. In this graph, nodes and edges represent UAVs and control policies, respectively. Any such graph can be described by its adjacency matrix [11]. The graph-theoretic approach provides a basis for the selection of the controllers for the follower UAVs since the selection depends upon the objective of tracking one or more leaders in the formation and the controller choice affects the stability of the group.

The network resulting from the formation is typically ad hoc because the leader(s) and the follower(s), along with the position of each UAV in the formation have to be determined on-line based on the task in hand and due to the presence of obstacles. This network is dependent upon the sensing and communication aspects. As a first step, a leader is elected similar to the case of multi-robot formations [11] followed by the discovery process in which the sensory information and physical network is used to establish a wireless network. The outcome of the leader election process must be communicated to the followers in order to construct an appropriate shape.

The optimal energy-delay sub-network routing protocol [12] allows the UAVs to communicate the information among the formation wirelessly using a multi-hop manner where an UAV in the formation is treated as a hop. Moreover, routing protocol allows the leader linear and angular velocities be communicated through the network to the followers whereas the separation errors can be measured. The energy-delay routing protocol can guarantee information transfer while minimizing energy and delay for real-time control purposes even for mobile ad hoc networks such as the case of UAV formation flying.

We envision four steps to establish the wireless ad hoc network. As mentioned earlier, leader election process is the first step. The discovery process is used as the second step where sensory information and physical network is used to establish a spanning tree. Since this is a multi-hop routing protocol, the communication network is created *on-demand* unlike in the literature where a spanning tree is utilized. Once a formation becomes stable, then a tree can be constructed until the shape changes. Then the third step will be assignment of the controllers online to each UAV based on the location of the UAV. Though previous section details the separation-bearing scheme, separation-separation scheme has to be employed for certain UAVs in the formation.

Using the wireless network, localization is used to combine local sensory information and routing from other UAVs in order to calculate relative position and orientation required for control assignment. Alternatively, range sensors provide relative separation and orientation information alone need to be communicated via a suitable routing protocol for generating suitable bearing control. Finally cooperative control allows the graph obtained from the network to be refined.

## IV. SIMULATION RESULTS

Consider three UAVs in a triangular formation which is shown in Figure 3. The desired separation and bearing are given by $\alpha_{12} = \alpha_{13} = \frac{2}{3}\pi, l_{12} = l_{13} = 30m, d = 0.15m$. In our simulation, the leader travels a circular trajectory at a height of 1000 meters. We use 200 neurons for each UAV to approximate its dynamics. The leader and the two followers will start from ground and fly to designated trajectory. During the whole process, three UAVs are controlled to maintain the formation defined in Figure 4. Table 1 presents the initial conditions of the UAVs. Mass of the UAV is 9307 Kgm. Moments of inertia is defined as $J_{xx} = 12874.84 \ kg - m^2$; $J_{yy} = 75673.60 \ kg - m^2$; $J_{zz} = 85552.09 \ kg - m^2$; $J_{xz} = 1331.41 \ kg - m^2$; The NN weight adaptation matrix gains are taken as $\Gamma_i = \Gamma_j = diag\{10\}$ and $k_{w_i} = k_{w_j} = 0.5$. The gains of the proportional controller are chosen as 8 and 100 respectively. Desired bearing is 120 degrees.

Table 1: Initial conditions

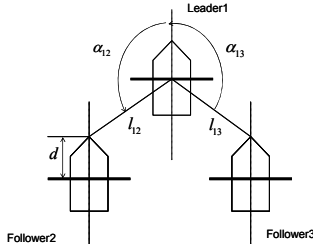| UAV | Initial Condition | | | | |
|---|---|---|---|---|---|
| | $x_0$ | $y_0$ | $z_0$ | $v_0$ | $\psi_0$ |
| | 0 | 0 | 0 | 0 | 0 |
| Follower1 | -50 | 0 | 0 | 0 | 0 |
| Follower2 | 50 | 0 | 0 | 0 | 0 |



Figure 2. Formation shape.

From Figure 3 (few figures included due to space requirements), we can observe that NN controller has a better performance in keeping desired relative range and bearing compared to a standard controller (not shown).

## V. CONCLUSIONS

In this paper, we consider a team of unmanned aerial vehicles (UAVs) equipped with sensors and motes for wireless communication for the task of navigating to a desired location in a formation. A neural network (NN)-based control scheme successfully compensates the unknown dynamics of the UAVs whereas a graph theory-based scheme provides discovery, localization and cooperative control. Numerical results demonstrate the theoretical conclusions.
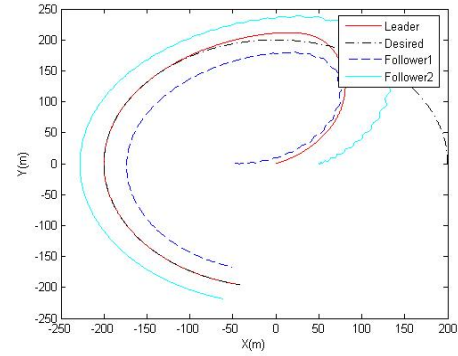


Figure 3. Top view of the trajectories with NN controller.

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