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# AC Impedance Measurement by Line-to-Line Injected Current

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**Abstract**—Naval ship as well as aerospace power systems are incorporating a greater degree of power electronic switching sources and loads. Although these components provide exceptional performance, they are prone to instability due to their high efficiency and constant power characteristics which lead to a negative impedance nature. When designing these systems, system integrators must consider the impedance versus frequency at an interface (which designates source and load). Stability criteria have been developed in terms of source and load impedance for both dc and ac systems and it is often helpful to have techniques for impedance measurement. For dc systems, the measurement techniques have been well established. This paper introduces a new method of impedance measurement for three-phase ac systems. By injecting a line-to-line (unbalanced without zero sequence) current between two lines of the ac system, all impedance information may be determined. Since the current injection is line-to-line, the measurement device may be used for ac or dc interfaces. Simulation and laboratory measurements demonstrate the effectiveness of this new technique.

**Keywords**- Power electronic, impedance measurement, stability

## I. INTRODUCTION

Power electronic based systems are prone to negative impedance instability due to the constant-power nature of the individual components [1-5]. Previous research has shown that the instabilities can be avoided in some systems by modification of the power electronic controls [2]. Other research has defined admittance space criteria based on a dc interface which can be used to design system components [3]. Recent research has shown that stability criteria for ac systems can be developed based on the  $q-d$  impedances of the source and load (defined at a particular system interface) [4]. Along these lines, a method of ac impedance measurement was proposed which is based on series injection of perturbation voltages [5]. Three-phase shunt current injection using various disturbance sources has also been recently explored [6]. This paper proposes a new line-to-line current injection method to extract ac impedance, which is much simpler than the three-phase injection method [6]. Simulation and laboratory results are used to validate the proposed method using a line-to-line chopper circuit.

## II. STABILITY ANALYSIS AND IMPEDANCE

This analysis starts with a negative feedback system as in Figure 1.

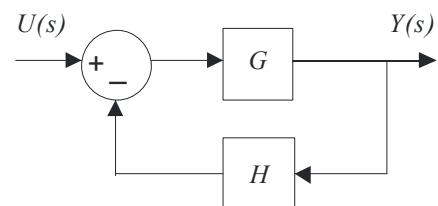


Figure 1. General negative feedback system.

The closed-loop transfer function can be expressed by

$$\frac{Y(s)}{U(s)} = \frac{G}{1+GH} \quad (1)$$

It is well known that the stability of the system can be observed by the Nyquist contour of the open-loop transfer function  $GH$ . Similarly, a power system can be simplified as shown in Figure 2. If the power source is taken as the input and the load voltage as the output, the transfer function will be

$$\frac{v}{v_s} = \frac{1}{1+Z_s/Z_l} = \frac{1}{1+Z_s Y_l} \quad (2)$$

By examining the Nyquist contour of the product of the source impedance  $Z_s$  and load admittance  $Y_l$  at an operating point, the stability of system can be easily determined.

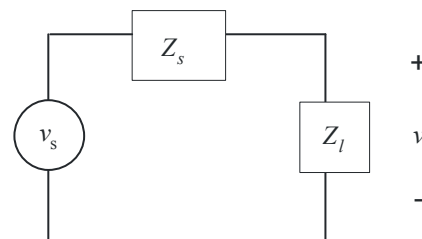


Figure 2. Simplified power system.

In 1976, Middlebrook first stated the criterion that a system is stable if the Nyquist contour of  $Z_s Y_l$  remains within the unit

circle [1]. Since then, considerable research on the impedance/admittance method has been conducted for dc systems. Extraction of impedance is becoming increasingly important in many applications. Methods of extracting impedance in dc systems have been well established [5]. For these systems, small disturbance signals with varying frequencies are injected into the system and the pertinent currents and voltages are calculated and processed to obtain the desired impedance or admittance over the frequency range of interest. The disturbance injection may be a shunt current [6] or a series voltage [5].

For ac systems, the stability criterion can be stated based on the  $q$ - $d$  model impedance matrix [4]. It is possible to determine the matrix entries for a particular frequency by solving the following equation at steady state with small signal injection.

$$\begin{bmatrix} \Delta v_q \\ \Delta v_d \end{bmatrix} = \begin{bmatrix} Z_{qq} & Z_{qd} \\ Z_{dq} & Z_{dd} \end{bmatrix} \begin{bmatrix} \Delta i_q \\ \Delta i_d \end{bmatrix} \quad (3)$$

In the next section, the techniques that have been applied to extract impedance in ac systems are described.

### III. TECHNIQUES OF AC IMPEDANCE EXTRACTION

In this paper, shunt currents were chosen for the injection disturbance. In [5, 6], the quadrature amplitude modulation (QAM) of the fundamental, also known as the suppressed-carrier injection technique, was used for impedance measurement. With this method, a set of three-phase modulated currents are injected to the system with the form

$$i_{ainj} = I_m \cos(\omega_s t) \cos(\omega_e t - \phi_{inj}) \quad (4)$$

$$i_{binj} = I_m \cos(\omega_s t) \cos(\omega_e t - \frac{2\pi}{3} - \phi_{inj}) \quad (5)$$

$$i_{cinj} = I_m \cos(\omega_s t) \cos(\omega_e t + \frac{2\pi}{3} - \phi_{inj}) \quad (6)$$

The injection signals are sinusoidal in  $q$ - $d$  synchronous reference frames with frequency  $\omega_s$ , which are similar to the small-signal injection used in dc systems. In the synchronous reference frame, the injected  $q$ - $d$  currents are

$$i_{qinj} = I_m \cos(\omega_s t) \cos(\phi_{inj}) \quad (7)$$

$$i_{dinj} = I_m \cos(\omega_s t) \sin(\phi_{inj}) \quad (8)$$

When the currents are injected, in steady-state, the source or load impedances are related to the disturbance of voltages and currents in the arbitrary reference frame by (3) at the injected frequency.

For a three-phase system with no zero-sequence current, the impedances matrix consists of four unknowns,  $Z_{qq}$ ,  $Z_{qd}$ ,  $Z_{dq}$ , and  $Z_{dd}$ . Therefore, at least two sets of measurements are needed in order to solve the linear system of equations. For the two measurements, the full set of equations is

$$V_{q1} = Z_{qq} I_{q1} + Z_{qd} I_{d1} \quad (9)$$

$$V_{d1} = Z_{dq} I_{q1} + Z_{dd} I_{d1} \quad (10)$$

$$V_{q2} = Z_{qq} I_{q2} + Z_{qd} I_{d2} \quad (11)$$

$$V_{d2} = Z_{dq} I_{q2} + Z_{dd} I_{d2} \quad (12)$$

where the capital  $I$  and  $V$  are the measured currents and voltages at the injection frequency  $\omega_s$ , respectively. The equations have a unique solution for the impedance matrix only when the two measurements are linearly independent.

In previous research [6], two different injection angles  $\phi_{inj}$  are used for each injection frequency of interest, resulting in injected currents of

$$i_{q1inj} = I_m \cos(\omega_s t) \cos(\phi_{1inj}) \quad (13)$$

$$i_{d1inj} = I_m \cos(\omega_s t) \sin(\phi_{1inj}) \quad (14)$$

$$i_{q2inj} = I_m \cos(\omega_s t) \cos(\phi_{2inj}) \quad (15)$$

$$i_{d2inj} = I_m \cos(\omega_s t) \sin(\phi_{2inj}) \quad (16)$$

where  $I_m$  is the desired injection current amplitude,  $\phi_{1inj}$  and  $\phi_{2inj}$  are different injection angles and  $c$  is the injection radian frequency which will be swept over a range of interest. For the same injection frequency, the current amplitude  $I_m$  is kept the same for the two different angles. To illustrate this process, the two sets of injected currents in the  $q$ - $d$  plane are shown in Figure 3. It can be seen that the two current vectors are linearly independent, and thus are suitable to be used to solve for the impedance matrix.

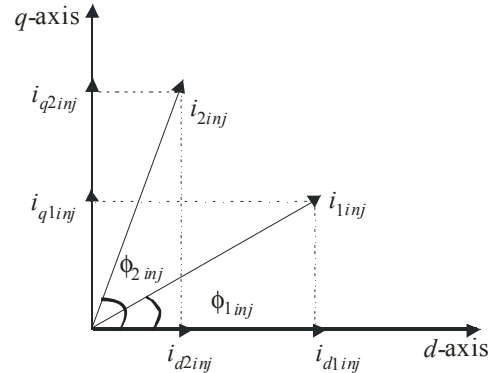


Figure 3. Vector Diagram of injected currents in three-phase injection method.

It is instructive to look at other methods of obtaining two independent measurements. The injected signals can be pure sinusoidal in  $a$ - $b$ - $c$  variables instead of modulated signals as in (4-6). In order to obtain pure sinusoidal injected signals in the  $q$ - $d$  reference frame, two measurements need to be taken at different injection frequencies; one at  $f_s + f_e$  and the other one at  $|f_s - f_e|$  with the phase sequence reversed, yielding

$$\begin{aligned}
i_{a1inj} &= I_m \cos(\omega_s t + \omega_e t) \\
i_{b1inj} &= I_m \cos\left(\omega_s t + \omega_e t - \frac{2\pi}{3}\right) \\
i_{c1inj} &= I_m \cos\left(\omega_s t + \omega_e t + \frac{2\pi}{3}\right) \\
i_{a2inj} &= I_m \cos(\omega_s t - \omega_e t) \\
i_{b2inj} &= I_m \cos\left(\omega_s t - \omega_e t + \frac{2\pi}{3}\right) \\
i_{c2inj} &= I_m \cos\left(\omega_s t - \omega_e t - \frac{2\pi}{3}\right)
\end{aligned} \tag{17}$$

Another way of obtaining two independent measurements is to allow the injection signal to be unbalanced as

$$\begin{aligned}
i_{a1inj} &= I_m \cos(\omega_s t + \omega_e t) \\
i_{b1inj} &= I_m \cos(\omega_s t + \omega_e t) \\
i_{c1inj} &= -2I_m \cos(\omega_s t + \omega_e t) \\
i_{a2inj} &= \frac{1}{2}I_m \cos(\omega_s t + \omega_e t) \\
i_{b2inj} &= I_m \cos\left(\omega_s t + \omega_e t - \frac{2\pi}{3}\right) \\
i_{c2inj} &= -\frac{\sqrt{3}}{2}I_m \sin(\omega_s t + \omega_e t)
\end{aligned} \tag{18}$$

Alternatively, other unbalanced injection currents, without zero-sequence current, can be injected to extract the impedance.

As the above examples illustrated, any two sets of injected signals that are linearly independent can be used to obtain the  $q$ - $d$  impedance matrix. So, besides three-phase injection method described in [5, 6], there are a lot of different injected current forms.

The following injection currents represent another example of unbalanced injected currents which might be the easiest when it comes to hardware implementation because one phase current is set to zero.

$$\begin{aligned}
i_{a1inj} &= 0 \\
i_{b1inj} &= -I_m \cos(\omega_s t + \omega_e t) \\
i_{c1inj} &= I_m \cos(\omega_s t + \omega_e t) \\
i_{a2inj} &= 0 \\
i_{b2inj} &= -I_m \cos(\omega_s t - \omega_e t) \\
i_{c2inj} &= I_m \cos(\omega_s t - \omega_e t)
\end{aligned} \tag{19}$$

In the synchronous reference frame, the injected currents include terms at the injected frequency,  $\omega_s + 2\omega_e$  and  $|\omega_s - 2\omega_e|$ . Regarding only the terms at injection frequency as important, the injected current in  $q$ - $d$  frame can be expressed as

$$\begin{aligned}
i_{q1inj} &= \frac{1}{\sqrt{3}}I_m \sin(\omega_s t) + \dots \\
i_{d1inj} &= \frac{1}{\sqrt{3}}I_m \cos(\omega_s t) + \dots \\
i_{q2inj} &= -\frac{1}{\sqrt{3}}I_m \sin(\omega_s t) + \dots \\
i_{d2inj} &= \frac{1}{\sqrt{3}}I_m \cos(\omega_s t) + \dots
\end{aligned} \tag{20}$$

In this method, magnitude of  $q$ - and  $d$ -axis terms of injected current are maintained while the injected signals are symmetrical about the  $d$ -axis. This is illustrated graphically in Figure 4. It can also be seen that the two current vectors are linearly independent, and thus can be used to obtain the desired impedance.

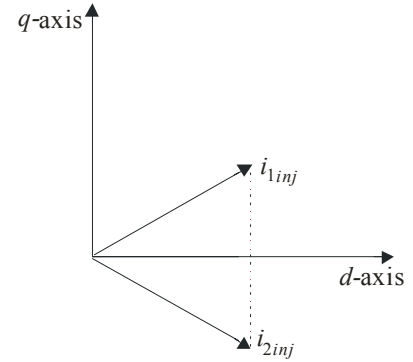


Figure 4. Vector Diagram of injected currents in line-to-line injection method.

Figure 5 shows a detailed flow chart for measuring a set of impedances. It involves injecting currents of (19) at  $f_s + f_e$  and  $|f_s - f_e|$  and capturing steady-state data for each measurement frequency. The impedances are then calculated and the process is repeated for all injection frequencies of interest.

#### IV. PRACTICAL MEASUREMENT CIRCUITS

The previous section described several current forms that can be injected to determine the source and load impedance (and thereby admittance). As a practical matter, the equations (19) give the simplest way to implement current injection. Since only phase  $b$  and  $c$  are needed and they have opposite signs, a line-to-line injection signal can be used as shown in Figure 6a. The practical measurement can be accomplished in several ways. Two such circuits are shown in Figure 6b and Figure 6c.

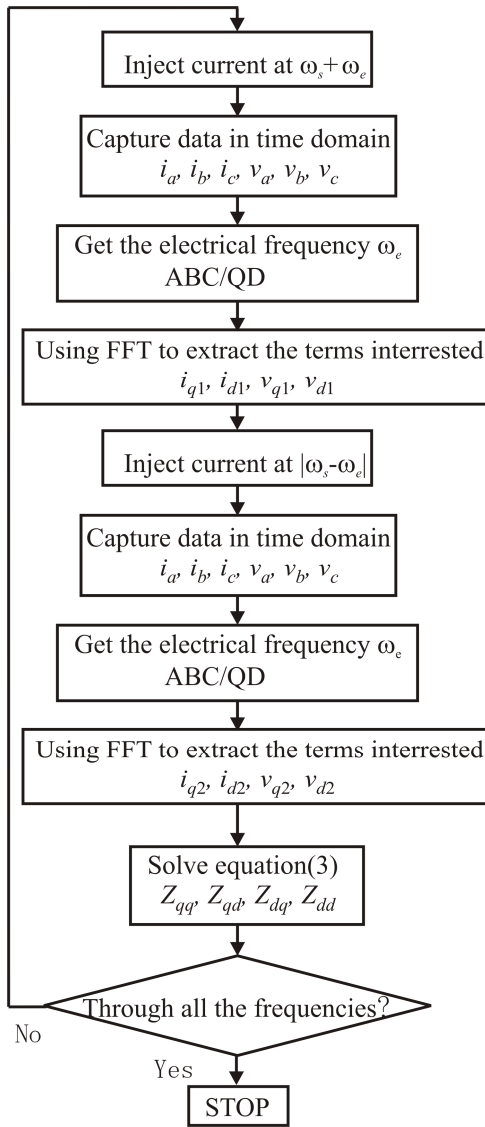


Figure 5. Impedance calculation algorithm.

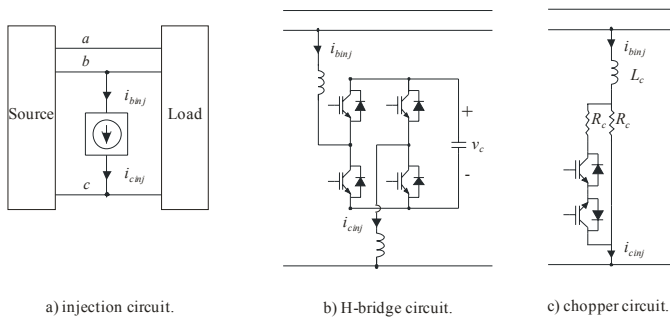


Figure 6. Practical current injection techniques.

### A. H-bridge circuit

Figure 6b shows how the current injection can be accomplished with an H-bridge circuit. When applying this circuit for current injection, a current-regulated PWM method such as hysteresis modulation or delta modulation [7] can be used to ensure the appropriate currents. The advantage of the active H-bridge converter is accurate control of the injected currents. However, this control means that the switching frequency of the inverter must be several times the highest frequency component of the injected current. For this reason, the H-bridge circuit is recommended for low-voltage low-power systems where higher-frequency transistors can be found.

### B. Chopper Circuit

The second impedance measurement circuit introduced above is the line-to-line chopper topology. This circuit is effectively an R-L load on the system where the load resistance can be changed by switching the power transistors. The transistors are switched using square-waves with fixed 50% duty cycle, the clock frequency is set to  $|\omega_s \pm 2\omega_e|$ .

The chopper circuit can be viewed in the synchronous  $q-d$  reference frame as a switched R-L load with varying square-wave impedance. The relationship between square-wave impedance and desired injected current is shown in the Appendix.

The primary advantage of the chopper circuit is that the switching frequency is set to  $|\omega_s \pm 2\omega_e|$  and is very low compared to the H-bridge circuit. Therefore, this circuit is more suitable for medium-voltage systems where the transistor switching frequency is limited. The primary disadvantage is that the injected current will have considerable harmonics. However, these are physically reduced by the system inductance and mathematically removed by the FFT process.

Compared with the three-phase injection method [6], the proposed method has several advantages. It has a simpler hardware configuration, and the generation of fixed duty cycle square-waves is much easier than the generation of PWM sinusoidal waves. In addition, for very-low-frequency impedance measurement, such as 0.1Hz, the proposed method uses a switching frequency around  $2f_e \pm 0.1$  Hz, while the three-phase injection method requires near-zero switching frequencies, which may be problematic in hardware implementation.

## V. COMPUTER SIMULATION RESULTS

A line-to-line chopper circuit injection was simulated for the system shown in Figure 7 using ACSL [9]. A fixed-frequency (60 Hz) ac power source with input inductors is used to represent a utility grid or a synchronous generator. The source is connected to an R-L load through a R-C filter. The injection circuit was placed after the R-C filter and before the R-L load. Therefore, the source impedance measured in this

system is composed of source R-L circuit and R-C filter. The load impedance is simple R-L load circuit. The frequency was swept from 48-Hz to 4995 Hz and the source impedance and load admittance was extracted for each frequency. Figure 8 and Figure 9 show the source impedances and load admittances determined from simulation using ideal current injection and chopper circuit current injection. A Jacobian analysis was also applied in ACSL to obtain the Thevenin  $q-d$  impedance matrix [9]. This is a frequency domain calculation about the current point in state space by numerical perturbation. The Thevenin  $q-d$  impedance matrix was then used to compare with simulation results from the injected signal.

Herein, the solid line denotes the Thevenin  $q-d$  impedance and the + markers correspond to the points predicted from the injection methods. As can be seen, the effectiveness of injection methods was validated in the simulation.

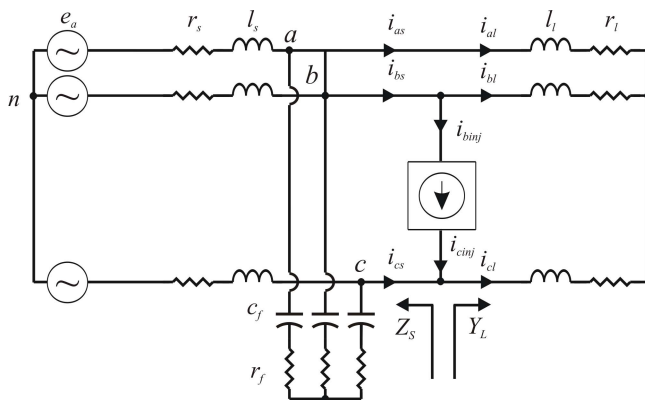


Figure 7. Example System for chopper injection.

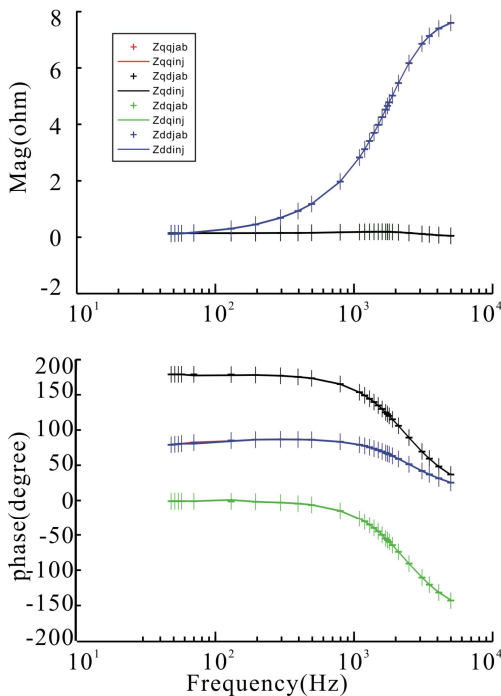


Figure 8. Simulation results with chopper circuit injection (Source).

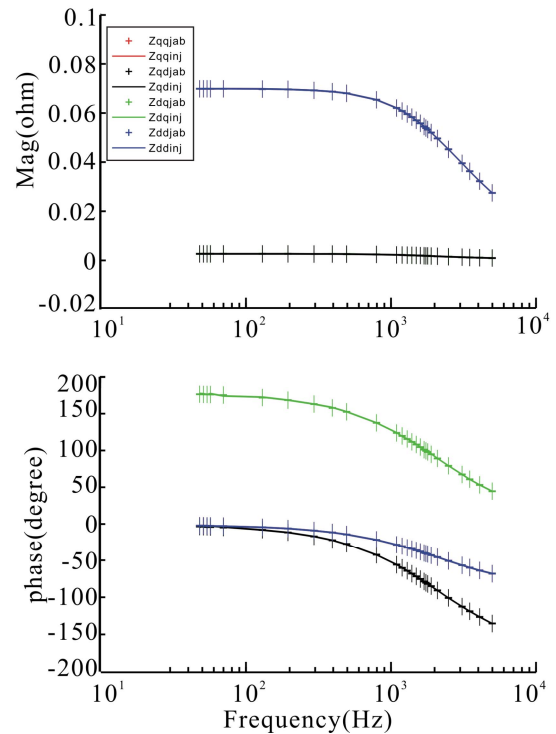


Figure 9. Simulation results with chopper circuit injection (Load).

## VI. LABORATORY VALIDATION

In order to validate the proposed method, a line-to-line chopper injection circuit was built in the laboratory. The system was the same as in the simulation. The chopper circuit was operating with  $R_c = 200\Omega$  and  $L_c = 1\text{ mH}$ .

In the lab, the injection frequencies are swept from 50 Hz to 4,995 Hz. To avoid the effects of system harmonics on the injection signal measurement, all the frequencies that are multiples of system fundamental frequency were skipped during the test. The data acquisition device sampling frequency and resolution was high enough such that the small injection signal can be extracted without degradation of accuracy.

The measurement results are plotted in Figure 10. Each passive component was using an Elgar-SW5250A [10] for consideration of parasitic elements at higher frequencies. The actual impedance was obtained mathematically for each measured component, and the resulting impedance is shown as the solid line. The x's are from the line-to-line injection measurements using the chopper circuit. The data is displayed in magnitude and angle as well as real and imaginary part. The magnitude and phase angle of the source impedance and load admittance is illustrated in Figure 10a and 10b respectively. The angles of  $Z_{qd}$  and  $Z_{dq}$  were ignored because of sensitivity relating to small magnitudes. Figures 10c and 10d express the real part and imaginary part of the impedance respectively. From this study, it can be seen that the measured impedances are very close to their actual values.

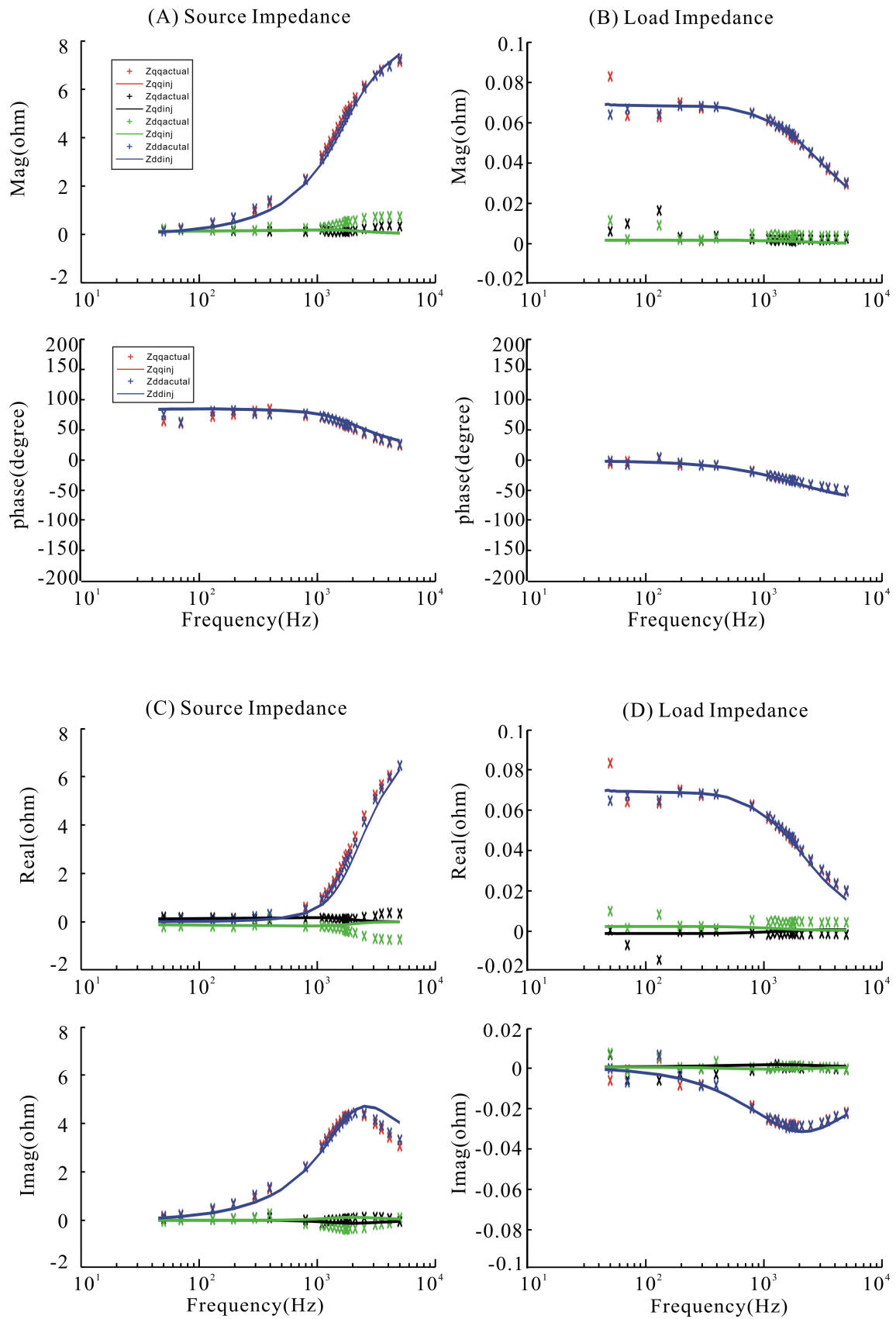


Figure 10. Laboratory results with chopper circuit injection.

## VII. CONCLUSION

This paper has proposed a new method to implement small-signal injection for extracting the impedance or admittance of ac power system sources and loads. The implementation involves a chopper circuit which is relatively simple compared to other measurement techniques. Simulation and laboratory measurements were used to verify the proposed method.

## VIII. APPENDIX

The switching waveform for the chopper transistor command is a square wave which can be expressed by the sum of a series of the harmonics of the switching frequency

$$sw = d + 2 \sum_{k=1}^{\infty} \frac{\sin(k\pi d)}{k\pi} \cos(k\omega t) \quad (21)$$

where  $d$  is the duty cycle,  $k$  is the harmonic number,  $\omega$  is the switching frequency, which was  $|\omega_s \pm 2\omega_e|$ .

The coefficient of each harmonic is  $\frac{2\sin(k\pi d)}{k\pi}$  with an offset of  $d$ ; the value of which will decrease as  $k$  increases. It can be seen the term at the switching frequency is dominant. When duty cycle  $d$  is set to 50%, this term can have maximum amplitude.

The equivalent resistance of the chopper circuit in Figure 6c is

$$R = sw \frac{R_{ch}}{2} + (1-sw)R_{ch} = \frac{R_{ch}}{2} (2-sw) \quad (22)$$

Taking inverse of equivalent resistance, the equivalent admittance is

$$Y = \frac{1}{R} = \frac{2}{R_{ch}(2-sw)} = \frac{1+sw}{R_{ch}} \quad (23)$$

Considering the dominant terms of the waveform, the injected current becomes

$$\begin{aligned} i_{inj} &= v_{bc} Y = V_{ll} \cos(\omega_e t) \frac{1+d+2\frac{\sin(\pi d)}{\pi} \cos(\omega_s t \pm 2\omega_e t)}{R_{ch}} \\ &= \frac{V_{ll}}{R_{ch}} (1+d) \cos(\omega_e t) + \frac{V_{ll}}{R_{ch}} \frac{\sin(\pi d)}{\pi} \cos(\omega_s t \pm 3\omega_e t) \dots \\ &\quad + \frac{V_{ll}}{R_{ch}} \frac{\sin(\pi d)}{\pi} \cos(\omega_s t \pm \omega_e t) \end{aligned} \quad (24)$$

The last term of (24) is exactly the desired injection current defined above in (19). Although the expression of (24) has more terms than needed, these terms will be

mathematically removed by the FFT technique during data processing.

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