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Transmit Precoding for MIMO Systems with Partial CSI and Discrete-Constellation Inputs

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Abstract—In this paper, we consider the transmit linear precoding problem for MIMO systems with discrete-constellation inputs. We assume that the receiver has perfect channel state information (CSI) and the transmitter only has partial CSI, namely, the channel covariance information. We first consider MIMO systems over frequency-flat fading channels. We design the optimal linear precoder based on direct maximization of mutual information over the MIMO channels with discrete-constellation inputs. It turns out that the optimal linear precoder is a non-diagonal non-unitary matrix. Then, we consider MIMO systems over frequency selective fading channels via extending our method to MIMO-OFDM systems. To keep reasonable computational complexity of solving the linear precoding matrix, we propose a sub-optimal approach to restrict the precoding matrix as a block-diagonal matrix. This approach has near-optimal performance when we integrate it with a properly chosen interleaver. Numerical examples show that for MIMO systems over frequency flat fading channels, our proposed optimal linear precoder enjoys 6-9 dB gain compared to the same system without linear precoder. For MIMO-OFDM systems, our reduced-complexity sub-optimal linear precoder captures 3-6 dB gain compared to the same system with no precoding. Moreover, for those MIMO systems employing a linear precoder designed based on Gaussian inputs with gap approximation technique for discrete-constellation inputs, significant loss may occur when the signal-to-noise ratio is larger than 0 dB.

I. INTRODUCTION

Transmit linear precoding has been a very active research topic in the last few years, see [1]-[10] and the references therein. The existing precoder design methods may be classified into two groups: (i) diversity oriented designs; and (ii) transmission rate oriented designs. The first group usually employs the pairwise error probability analysis technique, which has been used in the space-time trellis coding [11], to maximize the diversity order through the rank criterion developed in [11]. This approach can achieve the steepest asymptotic slope (highest diversity order) on the error probability versus SNR curve, however, it may not obtain the highest possible coding gain [12]. The second group often utilizes the Gaussian-input channel capacity (ergodic capacity and/or outage capacity) as design criteria to optimize the precoders. However, Gaussian inputs are too idealistic to be implemented in practical communication systems, and replacing Gaussian inputs by realistic discrete-constellation inputs for the designed precoders will often lead to significant performance degradation [13].

Very recently, linear precoding methods were investigated in [14]-[17] based on mutual information with discrete-

constellation inputs. In [14], optimal precoder design rules were derived for binary phase-shift keying (BPSK) signaling for bit-interleaved coded modulation (BICM) [18] on additive white Gaussian noise (AWGN) channels. However, it was pointed out in [14] that the derived design rules are somewhat cumbersome to be extended to higher-order constellations, and developing a general optimal scheme is a challenging topic for further research. Both [15] and [16] employed unitary matrices to maximize diversity order for block-fading channels to improve outage performance. It was shown that the unitary matrices can achieve 1-3 dB diversity gain at 10^{-3} outage level when the outer code rate is 0.75, and there is no diversity gain if the outer code rate is equal to or less than 0.5 for all SNRs [16]. In [17], a generalized linear precoder was proposed for maximizing the mutual information of vector Gaussian channels with finite discrete inputs, where the channel state information was assumed to be perfectly available at both the transmitter and receiver.

In this paper, we study the design of optimal linear precoder for spatially correlated MIMO systems with discrete-constellation inputs, where the receiver knows perfect channel state information (CSI) and the transmitter only knows partial CSI.

II. GAUSSIAN INPUT-BASED RESULTS REVISITED

Provided a MIMO system over frequency flat fading with N_t transmit and N_r receive antennas, the baseband complex system model is given by

$$\mathbf{y} = \mathbf{H}\mathbf{G}\mathbf{x} + \mathbf{v} \quad (1)$$

where $\mathbf{y} \in \mathbb{C}^{N_r \times 1}$ is the received signal vector; $\mathbf{x} \in \mathbb{C}^{N_t \times 1}$ is the transmitted signal vector, assumed to have independent and identically distributed (i.i.d.) unit-energy entries, i.e., $E\{\mathbf{x}\mathbf{x}^h\} = \mathbf{I}$ with \mathbf{I} being identity matrix and $(\cdot)^h$ denoting transpose conjugate; $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ is the zero-mean channel matrix; $\mathbf{G} \in \mathbb{C}^{N_t \times N_t}$ is the linear precoder with an average power constraint $\text{trace}(\mathbf{G}\mathbf{G}^h) \leq N_t$; and $\mathbf{v} \in \mathbb{C}^{N_r \times 1}$ is the channel noise vector, assumed i.i.d. complex Gaussian $\sim \mathcal{CN}(\mathbf{0}, \sigma^2\mathbf{I})$.

A. Capacity-Achieving Linear Precoder for Gaussian Inputs

When the channel matrix \mathbf{H} is perfectly known at the receiver, and the channel covariance $\mathbf{\Psi}_{TX} \triangleq E\{\mathbf{H}^h\mathbf{H}\}$ is known at the transmitter, then the capacity-achieving inputs \mathbf{x}

are zero-mean Gaussian distributed [19], [20], and the channel capacity is given by

$$C_{\text{cap}} = \max_{\mathbf{G}: \text{trace}(\mathbf{G}\mathbf{G}^h) \leq N_t} E \left[\log_2 \det \left(\mathbf{I} + \frac{1}{\sigma^2 N_t} \mathbf{H}\mathbf{G}\mathbf{G}^h \mathbf{H}^h \right) \right] \quad \text{bits/s/Hz.} \quad (2)$$

The solution \mathbf{G} to (2) is well-known given by [21]-[23]

$$\mathbf{G} = \mathbf{U}\mathbf{D}^{1/2} \quad (3)$$

where \mathbf{U} is the unitary matrix obtained via the singular value decomposition (SVD) of the channel covariance matrix $\Psi_{TX} = \mathbf{U}\Sigma\mathbf{U}^h$ with Σ being a diagonal matrix; and $\mathbf{D} = \text{diag}\{d_1, d_2, \dots, d_{N_t}\}$ is the optimal power allocation function obtained by iterative algorithms [22], [23].

It is noted that the columns of \mathbf{U} correspond to the spatial directions of the linear precoder \mathbf{G} , and the diagonal elements of \mathbf{D} signify the transmit powers allocated onto each of these spatial directions. Apparently, the spatial directions are the same as the eigenvectors of the channel covariance matrix.

B. Gap Approximation Technique for Finite Discrete Inputs

Although Gaussian inputs are capacity-achieving signaling, they can never be realized in practice. The inputs are usually drawn from finite discrete constellations such as quadrature amplitude modulation (QAM), pulse amplitude modulation (PAM) and/or phase shift keying (PSK) modulation, which may significantly depart from the Gaussian signaling. However, very limited study has been done to date for linear precoders that maximize the mutual information over MIMO fading channels with finite discrete inputs.

In the literature, a common approach is to utilize the Gaussian-input channel capacity formula as the design criteria to optimize the precoders, then replace the theoretical Gaussian inputs by practical finite constellation inputs for transmission. The reasons of assuming Gaussian inputs for precoder optimizations are two folds: first, it is more convenient in mathematics due to the elegant capacity formula; second, it was reported in [24] for fixed channels and in [25] for fading channels that there is a constant power gap between the Shannon capacity (with Gaussian inputs) and the spectral efficiency of realistic constellations such as QAM and PAM constellations. This constant gap is often referred to as ‘‘gap approximation’’ [26], [27]. This constant gap is indeed correct in the context of [24], [25], however, discrepancies may occur if it is directly employed as a basis for designing power allocation policies and linear precoders [13], [17] when the inputs are taken from discrete constellations.

C. Difference Between Gaussian and Discrete Inputs

Although the precoder structure $\mathbf{G} = \mathbf{U}\mathbf{D}^{1/2}$, *i.e.*, the product of optimal transmit directional matrix and power allocation diagonal matrix, is optimal for Gaussian inputs. This structure is no longer optimal to maximize the mutual information for channels with discrete-constellation inputs. We will show that an optimal precoder for discrete-constellation inputs is a general non-unitary non-diagonal matrix, which is solved via an iterative algorithm.

III. MUTUAL INFORMATION FOR DISCRETE INPUTS

We consider the MIMO system described in (1) with \mathbf{x} drawn from conventional equiprobable discrete constellations such as M -ary QAM, PSK or PAM, etc. Where M is the number of points in the signal constellation. The mutual information between \mathbf{x} and \mathbf{y} with \mathbf{H} and \mathbf{G} known at the receiver, per data symbol interval, is $\mathcal{I}(\mathbf{x}; \mathbf{y} | \mathbf{H}, \mathbf{G})$ given by (4) at the top of next page, where $\|\cdot\|$ denotes Euclidean norm of a vector, \mathbf{x} contains N_t symbols, which are independently taken from the M -ary signal constellation.

The proof of (4) can be done by extending the mutual information of a discrete memoryless channel to the case of continuous-valued output [28, Page 33].

The objective is to develop an algorithm for solving the linear precoder matrix \mathbf{G} , under power constraint $\text{trace}(\mathbf{G}\mathbf{G}^h) = N_t$, by maximizing the mutual information $\mathcal{I}(\mathbf{x}; \mathbf{y} | \mathbf{H}, \mathbf{G})$ with the transmitter knowing Ψ_{TX} .

The above constrained maximization problem can be written as the unconstrained maximization of

$$J = \mathcal{I}(\mathbf{x}; \mathbf{y} | \mathbf{H}, \mathbf{G}) + \lambda \left[\text{trace}(\mathbf{G}\mathbf{G}^h) - N_t \right] \quad (5)$$

where λ is a Lagrange multiplier, which is chosen to satisfy the power constraint, and the solution is stated in the following proposition.

Proposition: The optimal linear precoder \mathbf{G} , which maximizes the mutual information given by (4), satisfies the following equation

$$\frac{\log_2 e}{\sigma^2} \cdot E_{\mathbf{H}} \{ \mathbf{H}^h \mathbf{H} \mathbf{G} \Sigma_e \} + \lambda \cdot \mathbf{G} = 0 \quad (6)$$

where Σ_e is the minimum mean square error (MMSE) matrix given by (7) at the top of next page.

Proof: This proposition can be proved by employing the techniques developed in [13], [29], [30] for derivatives of mutual information, and the techniques developed in [31] for complex-valued matrix differentiation. Details are omitted for brevity.

From (6) and (7), we can easily conclude that \mathbf{G} is very likely to be a general $N_t \times N_t$ matrix to maximize the mutual information under the power constraint.

It is pointed out that it is very difficult to find a closed-form solution (if any) to (6) for \mathbf{G} , because Σ_e is also a function of \mathbf{G} . We developed an iterative algorithm employing gradient descent method [32] to solve (6) via utilizing the partial derivative $\frac{\partial}{\partial \mathbf{G}} \mathcal{I}(\mathbf{x}; \mathbf{y} | \mathbf{H}, \mathbf{G})$, which is equal to the first term at the left hand side of (6). For a practical wireless system, N_t is usually not larger than 4, therefore, the computational complexity of the iterative algorithm is reasonable. Furthermore, the computation of the MMSE matrix Σ_e involves multidimensional integration with integration kernel $\exp\left(-\frac{\|\mathbf{v}\|^2}{2\sigma^2}\right)$, where $\mathbf{v} \in \mathbb{C}^{N_r \times 1}$ is an AWGN vector. The integration can be computed by using Gauss-Hermite quadrature rules [33], which is directly employed for the case when $N_r = 1$.

$$\mathcal{I}(\mathbf{x}; \mathbf{y} | \mathbf{H}, \mathbf{G}) = N_t \log_2 M - \frac{1}{M^{N_t}} \sum_{m=1}^{M^{N_t}} E_{\mathbf{H}} \left\{ E_{\mathbf{V}} \left[\log_2 \sum_{k=1}^{M^{N_t}} \exp \left(\frac{-\|\mathbf{H}\mathbf{G}(\mathbf{x}_m - \mathbf{x}_k) + \mathbf{v}\|^2 + \|\mathbf{v}\|^2}{\sigma^2} \right) \right] \right\} \quad (4)$$

$$\begin{aligned} \Sigma_e &= E \left\{ [\mathbf{x} - E(\mathbf{x} | \mathbf{y}, \mathbf{H}, \mathbf{G})] [\mathbf{x} - E(\mathbf{x} | \mathbf{y}, \mathbf{H}, \mathbf{G})]^h \right\} \\ &= \mathbf{I}_{N_t} - \frac{1}{(\pi\sigma^2)^{N_r} M^{N_t}} \int_{\mathbf{y}} \frac{\left[\sum_{l=1}^{M^{N_t}} \mathbf{x}_l \exp \left(-\frac{\|\mathbf{y} - \mathbf{H}\mathbf{G}\mathbf{x}_l\|^2}{\sigma^2} \right) \right] \left[\sum_{k=1}^{M^{N_t}} \mathbf{x}_k^h \exp \left(-\frac{\|\mathbf{y} - \mathbf{H}\mathbf{G}\mathbf{x}_k\|^2}{\sigma^2} \right) \right]}{\sum_{m=1}^{M^{N_t}} \exp \left(-\frac{\|\mathbf{y} - \mathbf{H}\mathbf{G}\mathbf{x}_m\|^2}{\sigma^2} \right)} d\mathbf{y}. \end{aligned} \quad (7)$$

However, when N_r is larger than 1, we extend the Gauss-Hermit quadrature rules into multidimensional cases with some algebraic manipulations. Details are omitted for brevity.

IV. EXTENSION TO MIMO-OFDM SYSTEMS

In this section, we extend our linear precoding method, which is proposed for MIMO systems over frequency flat fading, to MIMO-OFDM systems [34], which are able to handle frequency selective fading efficiently.

Consider a baseband MIMO-OFDM system described by

$$\mathbf{Y} = \mathbf{H}\mathbf{G}\mathbf{X} + \mathbf{V} \quad (8)$$

where $\mathbf{X} = [\mathbf{x}_1^t \ \mathbf{x}_2^t \ \cdots \ \mathbf{x}_K^t]^t \in \mathbb{C}^{N_t K \times 1}$ with $\mathbf{x}_k \in \mathbb{C}^{N_t \times 1}$ being the k -th time transmitted information data, $\mathbf{Y} = [\mathbf{y}_1^t \ \mathbf{y}_2^t \ \cdots \ \mathbf{y}_K^t]^t \in \mathbb{C}^{N_r K \times 1}$ with $\mathbf{y}_k \in \mathbb{C}^{N_r \times 1}$ being the k -th time received signal, $\mathbf{V} = [\mathbf{v}_1^t \ \mathbf{v}_2^t \ \cdots \ \mathbf{v}_K^t]^t \in \mathbb{C}^{N_r K \times 1}$ is the channel noise vector, assumed i.i.d. complex Gaussian $\sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I})$, $\mathbf{H} = \text{block-diag} \{ \mathbf{H}_1 \ \mathbf{H}_2 \ \cdots \ \mathbf{H}_K \} \in \mathbb{C}^{N_r K \times N_t K}$ with $\mathbf{H}_k \in \mathbb{C}^{N_r \times N_t}$ being the k -th tone channel matrix, $\mathbf{G} \in \mathbb{C}^{N_t K \times N_t K}$ is the linear precoder with average power constraint $\text{trace}(\mathbf{G}\mathbf{G}^h) \leq N_t K$, and K is the data symbol block length per antenna. The superscript t denotes transpose operation.

We partition \mathbf{H} into N_g groups via a properly chosen interleaver, each group has a block-diagonal matrix $\mathbb{H}_k \in \mathbb{C}^{K_{\mathbb{G}} \times K_{\mathbb{G}}}$, $k = 1, 2, \dots, N_g$. Then we find a local optimal linear precoder \mathbb{G}_k for the channel matrix \mathbb{H}_k with a given discrete constellation. The mutual information is then given by the summation

$$\mathcal{I}(\mathbf{X}; \mathbf{Y} | \mathbb{H}, \mathbb{G}) = \sum_{k=1}^{N_g} \mathcal{I}(\mathbf{X}_k; \mathbf{Y}_k | \mathbb{H}_k, \mathbb{G}_k). \quad (9)$$

Instead of solving a huge matrix \mathbb{G} , we are now solving N_g smaller matrices \mathbb{G}_k , therefore, the computational complexity can be reduced to a reasonable level, and the cost is that we get a sub-optimal solution. Our preliminary results show that if we carefully design the interleaver to place the sub-channel matrices \mathbb{H}_k into the N_g groups, and if the size of each matrix \mathbb{G}_k is properly chosen, the sub-optimal solution can be a near optimal solution.

The choice of the matrix size $K_{\mathbb{G}}$ is important for tradeoff between mutual information and computational complexity. The larger the $K_{\mathbb{G}}$, the larger the mutual information we can

possibly obtain through the sub-precoders \mathbb{G}_k , however, the higher computational complexity we will have to cope with.

The interleaver is another important design issue for maximizing the mutual information with low-complexity constraint. Our extensive research indicates that if the interleaved and grouped sub-channel matrices \mathbb{H}_k have similar statistics, then the mutual information have higher chance to be maximized.

Finally, we would like to state that the N_g sub-precoders \mathbb{G}_k are the solutions to the following equations:

$$\begin{aligned} \frac{\log_2 e}{K\sigma^2} \cdot E \{ \mathbb{H}_k^h \mathbb{H}_k \mathbb{G}_k \Sigma_{e,k} \} + \lambda_k \cdot \mathbb{G}_k &= 0 \\ k &= 1, 2, \dots, N_g \end{aligned} \quad (10)$$

where $\Sigma_{e,k}$ is the MMSE matrix corresponding the \mathbb{H}_k . It is defined similarly to Σ_e in (7).

It is noted that solving \mathbb{G}_k of (10) is similar to solving \mathbf{G} of (6). Details are omitted for brevity.

V. NUMERICAL EXAMPLES

In this section, we consider two examples. The first example shows that the optimal linear precoder provides large performance gain on the mutual information for MIMO systems over frequency flat fading with discrete-constellation inputs. The second example shows that the proposed technique is readily applicable to MIMO-OFDM systems which convert the time-domain frequency-selective fading into frequency-domain nonselective fading.

Example 1: Consider a 4×4 MIMO system over frequency-flat Rayleigh fading channels. The receive antennas are assumed uncorrelated. It is further assumed that the transmitter has a broadside truncated Gaussian power azimuth spectrum with a 2° root-mean-square spread. Hence, the transmit correlations Ψ_{TX} are approximately given by $(\Psi_{TX})_{i,j} \approx \exp(-0.05(i-j)^2)$ as discussed in [21], [35].

For this 4×4 system, we performed the following precoding schemes: 1) optimal precoding with Gaussian inputs using the algorithm presented in [22], [23] to obtain $\mathbf{G} = \mathbf{U}\mathbf{D}^{1/2}$; 2) statistical waterfilling [8] for Gaussian inputs; 3) our proposed optimal precoding for BPSK inputs; 4) maximum diversity precoding [4]; 5) taking the statistical waterfilling [8] for BPSK inputs; 6) taking the Gaussian optimal precoder $\mathbf{G} = \mathbf{U}\mathbf{D}^{1/2}$ for BPSK inputs; 7) statistically parallelizing the channel with $\mathbf{G} = \mathbf{U}$ for BPSK inputs. The mutual information, along with channel capacity, are depicted in Fig. 1. For comparison purpose, we also plotted the mutual information

for the MIMO fading channel with no precoding for Gaussian and BPSK inputs.

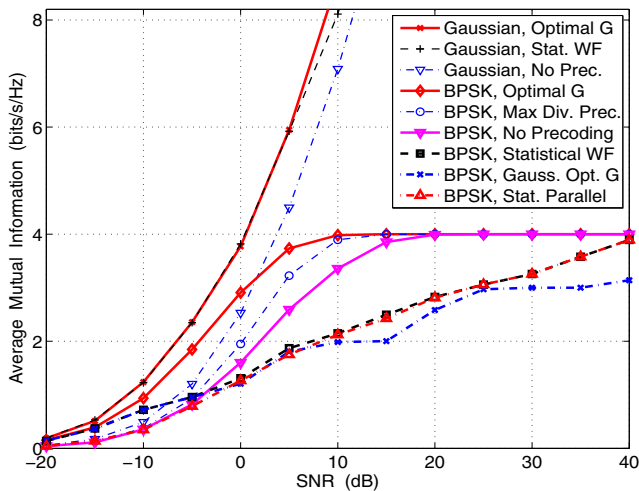


Fig. 1. Mutual information of the 4×4 MIMO channel with Gaussian and BPSK inputs.

Furthermore, we also performed the linear precoding schemes for QPSK inputs for this 4×4 system. The mutual information and channel capacity are depicted in Fig. 2.

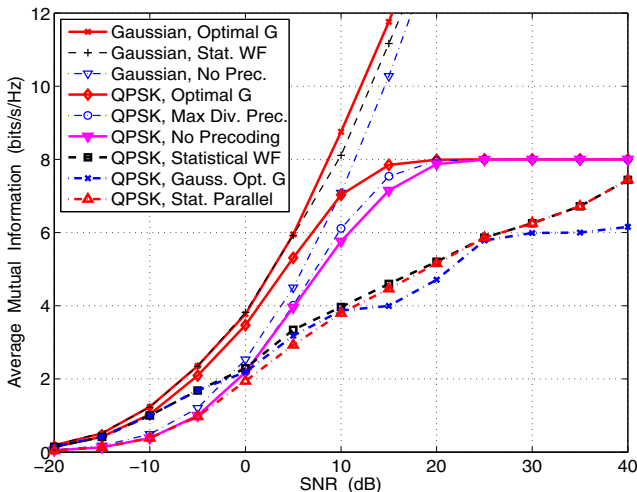


Fig. 2. Mutual information of the 4×4 MIMO channel with Gaussian and QPSK inputs.

From Figs. 1 and 2, we have the following observations:

- 1) Compared to the original system with no precoding, employing the Gaussian-input-based optimal precoder and/or statistical waterfiling scheme for discrete-constellation inputs will lead to significant loss in the mutual information for outer channel-coding rate higher than 0.28. The higher the outer coding rate, the larger the loss. For example, the loss is 25 dB when the outer coding rate is 0.9, this is a huge loss.
- 2) Compared to the original system with no precoding, our proposed linear precoding provides 6~9 (or 5~7) dB gain for the outer channel-coding rate ranging from 0.01 to 0.95, for BPSK (or QPSK) inputs.

- 3) Our proposed linear precoder achieves higher mutual information for BPSK and QPSK inputs than Gaussian inputs with no precoding for outer channel-coding rate ranging from 0.01 to 0.8. This is significant.
- 4) Maximum diversity precoding provides 2~4 (or 0.5~3) dB gain compared to the original system with no precoding, for BPSK (or QPSK) inputs when the outer coding rate is between 0.5 and 0.95.
- 5) The optimal linear precoder structure, $\mathbf{G} = \mathbf{U}\mathbf{D}^{1/2}$ for Gaussian inputs is no longer optimal for discrete-constellation inputs. This structure may result in large loss in the mutual information.
- 6) For Gaussian inputs, the statistical waterfiling [8] has almost optimal capacity performance [21]-[23] when the signal-to-noise ratio (SNR) is lower than 5 dB, however, it becomes sub-optimal when SNR is larger than 5 dB.

Example 2: we now consider a 2×2 MIMO system over typical urban frequency selective Rayleigh fading channel [36], it has $L = 5$ taps, which have inter-tap correlations given by [36]. The transmit antenna correlations are given by $\begin{bmatrix} 1 & 0.8 \\ 0.8 & 1 \end{bmatrix}$. We assume slowly time-varying with improved Jakes' (or Clarke's) model [37] to generate Rayleigh fading. We set $K = 64$ to form the MIMO-OFDM channel matrix \mathbb{H} . We choose $K_G = 8$ to group the block-diagonal channel matrix via an interleaver. Therefore, we have $N_g = \frac{N_t K}{K_G} = 16$ groups sub-channel matrices \mathbb{H}_k .

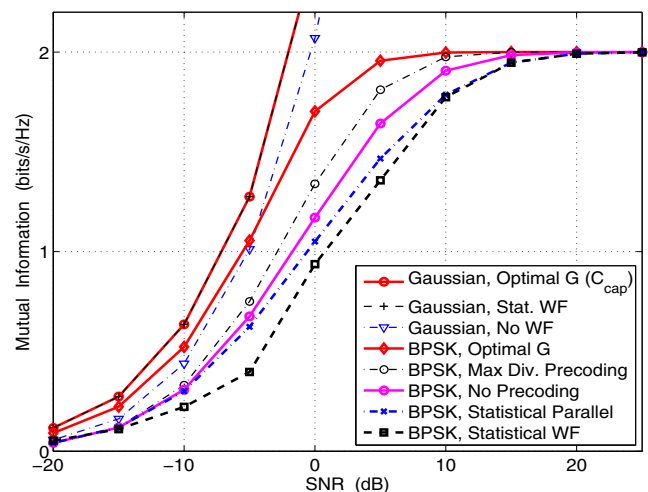


Fig. 3. Mutual information of the MIMO-OFDM system over frequency selective Rayleigh fading channel with inter-tap correlation, BPSK inputs.

Fig. 3 depicts the mutual information of the MIMO-OFDM system with various linear precoding schemes. From this figure, we can conclude the following: 1) although our proposed block-diagonal-matrix linear precoder is a sub-optimal solution for MIMO-OFDM systems with discrete-constellation inputs, the performance gain is 3~6 dB compared to the MIMO-OFDM system with no precoding, when the outer channel-coding rate is ranging from 0.01 to 0.95; 2) our proposed linear precoder outperforms the maximum diversity precoder by 3~4 dB for outer coding rates from 0.01 to

0.98; 3) our proposed linear precoder outperforms the MIMO-OFDM system with Gaussian inputs when the outer coding rate is less than 0.5; 4) the statistical waterfilling scheme leads to performance loss compared to the MIMO-OFDM system with no precoding, for BPSK inputs.

VI. CONCLUSION

In this paper, we have proposed a new linear precoding scheme which directly maximizes the mutual information of MIMO systems with discrete-constellation inputs. Under the assumption that the receiver has perfect channel state information and the transmitter knows only the channel covariance information, we have shown that the optimal linear precoder is generally a non-unitary matrix. We have demonstrated that the Gaussian-input-based optimal linear precoder structure is no longer optimal for discrete-constellation inputs. Our numerical examples have shown that for MIMO systems over frequency flat fading channels, our proposed optimal linear precoder achieves 6-9 dB gain compared to the same system without linear precoder. For MIMO-OFDM systems, our reduced-complexity sub-optimal linear precoder captures 3-6 dB gain compared to the same system with no precoding. Furthermore, for those MIMO systems applying a linear precoder, which was designed with gap approximation technique, to discrete-constellation input case, huge loss in the mutual information may occur when the signal-to-noise ratio is larger than 0 dB.

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