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An Optimal Dynamic Inversion Approach for Controlling a Class of One-Dimensional Nonlinear Distributed Parameter Systems

Radhakant Padhi and S. N. Balakrishnan

Abstract— **Combining the principles of dynamic inversion and optimization theory, a new approach is presented for stable control of a class of one-dimensional nonlinear distributed parameter systems, assuming the availability a continuous actuator in the spatial domain. Unlike the existing approximate-then-design and design-then-approximate techniques, here there is no need of any approximation either of the system dynamics or of the resulting controller. Rather, the control synthesis approach is fairly straight-forward and simple. The controller formulation has more elegance because we can prove the convergence of the controller to its steady state value. To demonstrate the potential of the proposed technique, a real-life temperature control problem for a heat transfer application is solved. It has been demonstrated that a desired temperature profile can be achieved starting from any arbitrary initial temperature profile.**

I. INTRODUCTION

Control of distributed parameter systems (DPS) has been studied both from mathematical as well as engineering point of view. An interesting brief historical perspective of the control of such systems can be found in [11]. There exist infinite-dimensional operator theory based methods for the control of distributed parameter systems. While there are many advantages, these operator theory based approaches are mainly limited to linear systems [8] and some limited class of problems like spatially invariant systems [3]. Moreover for the purpose of implementation, the infinite-dimensional control solution needs to be approximated (*e.g.* truncating an infinite series, reducing the size of feedback gain matrix *etc.*) and hence is not completely free from errors. Such a control design approach is known as "design-then-approximate".

Another control design approach is "approximate-then-design". Here, the partial differential equations describing the system dynamics are first approximated to yield a finite dimensional approximate model. This approximate system is then used for controller synthesis. In this approach, it is relatively easy to design controllers using various concepts of finite-dimensional control design. An interested reader can refer to [6] for discussions on the relative merits and limitations of the two approaches.

An "approximate-then-design" approach to deal with the infinite dimensional systems is to have a finite dimensional approximation of the system using a set of orthogonal basis functions via Galerkin projection [10]. This technique normally leads to high order lumped system representations to adequately represent the properties of the original system, if arbitrary orthogonal functions are used as the basis functions. For this reason, in recent literature attention is being increasingly focused to come up with reduced-order approximations. One such powerful technique is Proper Orthogonal Decomposition (POD). Out of numerous literatures published on this topic and its use in control system design (both for linear and nonlinear DPS), we cite [4], [6], [7], [10], [14], [15] for reference. However, there are a few important drawbacks in the POD approach: (i) the technique is problem dependent and not generic; (ii) there is no guarantee that the snap-shots will capture all dominant modes of the system and, more important, (iii) it is very difficult to have a set of 'good' snap-shot solutions for the closed-loop system prior to the control design. This is a serious limiting factor if one has to apply this technique for the closed-loop control design. Because of this reason, some attempts are being made in recent literature to adaptively redesign the basis functions, and hence the controller, in an iterative manner. An interested reader can see [1], [2], [15] for a few ideas in this regard.

Even though the "design-then-approximate" and "approximate-then-design" approaches have been used in practice for designing the controllers for DPS, and attempts are being made to generalize and refine the techniques, a *fundamentally different* technique is presented in this paper, which is applicable for a class of one-dimensional nonlinear distributed parameter systems. This has been done by combining the ideas of dynamic inversion [9], [12], [16] and optimization theory [5]. The formulation assumes a continuous controller in the spatial domain. In addition to the above important advantage, the formulation has elegance in the sense that we can prove the convergence of the controller to its steady state value.

To demonstrate the potential advantages, we have applied the technique to a real-life heat transfer application

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to achieve a desired temperature profile. Numerical results are very promising. We have simulated the system from a number of random initial conditions to validate the claim. Also note that even though a constant temperature profile (with respect to time) is chosen for the numerical studies (because of practical significance), the formulation presented is capable of tracking 'time-varying' target profiles. Moreover, even though we have chosen an exponential target profile (again for practical significance), the formulation presented can track any 'arbitrary' smooth trajectory that satisfies the spatial boundary conditions.

II. PROBLEM DESCRIPTION

A. System Dynamics

In this paper the following system dynamics is considered

$$\dot{x} = f\left(x, x', x'', \dots\right) + g\left(x, x', x'', \dots\right)u \tag{1}$$

where the state $x(t, y)$ and controller $u(t, y)$ are continuous functions of time $t \geq 0$ and spatial variable $y \in [0, L]$. $\dot{x}$ represents $\partial x / \partial t$ and $x', x'', \dots$ represent $\partial x / \partial y, \partial^2 x / \partial y^2, \dots$ respectively. We assume that appropriate boundary conditions (e.g. Dirichlet, Neumann etc.) are available to make the system dynamics description (1) complete. Note that here both $x(t, y)$ and $u(t, y)$ are scalar functions. The control variable appears linearly, and hence, the system dynamics is in the control affine form. Furthermore, we assume that $g\left(x, x', x'', \dots\right) \neq 0 \quad \forall t, y$. In this paper, we do not take into account those situations where control action enters the system dynamics through the boundary actions (i.e. boundary control problems are not considered).

B. Goal for the Control Design

The controller should make sure that the state variable $x(t, y) \to x^*(t, y)$ as $t \to \infty$ for all $y \in [0, L]$, where $x^*(t, y)$ is a known (possibly time-varying) profile in the domain $[0, L]$. It is assumed that $x^*(t, y)$ is continuous and smooth in $y \quad \forall t$ and satisfies the spatial boundary conditions, which simplifies our task. This is because we do not have the control action at the boundary, and hence, it will be difficult to guarantee $x(t, y) \to x^*(t, y)$ at the boundary, unless $x^*(t, y)$ itself satisfies the boundary condition.

III. CONTROL SYNTHESIS

First, let us define an output (an integral error) term as

$$z(t) = \frac{1}{2}\int_0^L \left[x(t, y) - x^*(t, y)\right]^2 dy \tag{2}$$

Note that when $z(t) \to 0$, $x(t, y) \to x^*(t, y)$ everywhere in $y \in [0, L]$. Next, following the principle of dynamic inversion [9], [12], [16], we attempt to design a controller such that the following first-order equation is satisfied

$$\dot{z} + k\, z = 0 \tag{3}$$

where, $k > 0$ serves as a gain; an appropriate value of it has to be chosen by the control designer. To have a better physical interpretation, one may choose it as $k = 1/\tau$, where $\tau > 0$ serves as a "time constant" for the error $z(t)$ to decay. Using the definition of $z(t)$ in (2), (3) leads us to

$$\int_0^L \left(x - x^*\right)\left(\dot{x} - \dot{x}^*\right)dy = -\frac{k}{2}\int_0^L \left(x - x^*\right)^2 dy \tag{4}$$

Substituting for $\dot{x}$ from (1) in (4) and simplifying we get

$$\int_0^L \left(x - x^*\right) g\left(x, x', x'', \dots\right) u\, dy = \gamma \tag{5}$$

where $\gamma \triangleq -\int_0^L \left(x - x^*\right)\left[f\left(x, x', x'', \dots\right) - \dot{x}^*\right]dy - \frac{k}{2}\int_0^L \left(x - x^*\right)^2 dy$

Note that the value for $u(t, y)$ satisfying (5) will eventually guarantee that $z(t) \to 0$ as $t \to \infty$. However, since (5) is in the form of an integral, no unique solution can be obtained for $u(t, y)$ from it. To obtain a unique solution, however, we have the freedom of putting an additional goal. We take advantage of this fact and aim to obtain a solution for $u(t, y)$ that will not only satisfy (5), but at the same time, will also minimize the cost function

$$J = \frac{1}{2}\int_0^L r(y)\left[u(t, y)\right]^2 dy \tag{6}$$

In other words, we wish to minimize the cost function in (6), subjected to the constraint in (5). An implication of choosing this cost function is that we wish to achieve our objective with minimum control effort. In (6), $r(y) > 0 \quad \forall y \in [0, L]$ is the weighting function (which needs to be chosen by the control designer). This weighting function gives the designer the flexibility of putting relative importance of the control magnitude at different spatial locations. The choice of $r(y) = c > 0$ (a positive constant) means the control magnitude is given equal importance at all spatial locations.

Next, following the technique for constrained optimization [5], we formulate the augmented cost function

$$\bar{J} = \frac{1}{2}\int_0^L r u^2 dy + \lambda\left[\int_0^L \left(x - x^*\right) g\, u\, dy - \gamma\right] \tag{7}$$

where λ is a Lagrange multiplier, which is a free variable needed to convert the constrained optimization problem to a free optimization problem. In (7), there are two free variables, namely u and λ and $\bar{J}$ has to be minimized by appropriate selection of these variables.

It is well-known [5] that the necessary condition of optimality is given by

$$\delta \bar{J} = 0 \tag{8}$$

where $\delta \bar{J}$ represents the "first variation" of $\bar{J}$. However, we know that

$$
\begin{aligned}
\delta \bar{J} &= \int_0^L [ru]\delta u\, dy + \lambda\left[\int_0^L \left(x-x^*\right)g\, \delta u\, dy\right] \\
&\quad + \delta\lambda\left[\int_0^L \left(x-x^*\right)g\, u\, dy - \gamma\right] \\
&= \int_0^L \left[ru + \lambda\left(x-x^*\right)g\right]\delta u\, dy \\
&\quad + \delta\lambda\left[\int_0^L \left(x-x^*\right)g\, u\, dy - \gamma\right]
\end{aligned} \tag{9}
$$

From (8) and (9), we obtain

$$\int_0^L \left[ru + \lambda\left(x-x^*\right)g\right]\delta u\, dy + \delta\lambda\left[\left(x-x^*\right)g\, u\, dy - \gamma\right] = 0 \tag{10}$$

Since (10) must be satisfied for all variations δu and $\delta\lambda$, the following equations should be satisfied simultaneously:

$$ru + \lambda\left(x-x^*\right)g = 0 \tag{11}$$

$$\int_0^L \left(x-x^*\right)g\, u\, dy = \gamma \tag{12}$$

Note that (12) is nothing but (5a). From (11) u can be expressed as

$$u = -\left(\lambda/r\right)\left(x-x^*\right)g \tag{13}$$

Substituting this expression for u from (13) in (12), λ can be expressed as

$$\lambda = \frac{-\gamma}{\int_0^L \left[\left(x-x^*\right)^2 g^2/r(y)\right]dy} \tag{14}$$

Substituting the expression for λ from (14) back in (13), we finally obtain

$$u = \frac{\gamma\left(x-x^*\right)g}{r(y)\int_0^L \left[\left(x-x^*\right)^2 g^2/r(y)\right]dy} \tag{15}$$

Note that as a special case, if $r(y)$ is a constant $c > 0$ and $g\left(x, x', x'', \ldots\right) = \beta \in \mathbb{R}$, then the expression in (15) simplifies to

$$u = \frac{\gamma\left(x-x^*\right)}{\beta\int_0^L \left(x-x^*\right)^2 dy} \tag{16}$$

IV. CONVERGENCE ANALYSIS

Note that when $x(t,y) = x^*(t,y)$ (i.e. perfect tracking occurs), there is some computational difficulty in the sense that zero appears in the denominator of (15) and (16). This leads to the impression that as $x(t,y) \to x^*(t,y)$, it leads to singularity in the control solution, i.e. $u \to \infty$. Even though this seems to be intuitively obvious, it does not happen. To see this we intend to show that when $x(t,y) \to x^*(t,y)$, $u(t,y) \to u^*(t,y)$, where $u^*(t,y)$ is defined as the control required to keep $x(t,y)$ at $x^*(t,y)$.

Note that when $x(t,y) = x^*(t,y)$ and $u(t,y) = u^*(t,y)$, from (1) we can write

$$\dot{x}^* = f^* + g^* u^* \tag{17}$$

where $f^* \triangleq f\left(x^*, x^{*\prime}, x^{*\prime\prime}, \cdots\right)$, $g^* \triangleq g\left(x^*, x^{*\prime}, x^{*\prime\prime}, \cdots\right)$

From (17a-b), we can write the control solution as

$$u^*(t,y) = -\frac{1}{g^*}\left[f^* - \dot{x}^*\right] \tag{18}$$

Note that the solution $u^*(t,y)$ in (18) will always be of finite magnitude, since by our assumption of the class of DPS, $g^* = g(x^*, x^{*\prime}, x^{*\prime\prime}, \cdots)$ is always bounded away from zero. Equation (18) helps us to verify the convergence of the actual controller, which is carried out here.

First we notice that at any point $y_0 \in (0,L)$, the control solution in (15) leads to

$$u(t,y_0) = \frac{-\left[x(y_0) - x^*(y_0)\right]g(y_0)\left\{\int_0^L \left(\begin{array}{c}\left[x(y) - x^*(y)\right] \\ \times\left[f(y) - \dot{x}^*(y)\right]\end{array}\right)dy + \frac{k}{2}\int_0^L \left[x(y) - x^*(y)\right]^2 dy\right\}}{r(y_0)\int_0^L \frac{\left[x(y) - x^*(y)\right]^2\left[g(y)\right]^2}{r(y)}dy} \tag{19}$$

We want to analyze this solution for the case when $x(t,y) \to x^*(t,y)$ $\forall y \in [0,L]$. Without loss of generality, we analyze the case in the limit when $x(t,y) \to x^*(t,y)$ for $y \in \left[y_0 - \varepsilon/2,\ y_0 + \varepsilon/2\right] \subset [0,L]$, $\varepsilon \to 0$ and $x(t,y) = x^*(t,y)$ everywhere else. In such a limiting case, let us denote $u(t,y_0)$ as $\bar{u}(t,y_0)$. Then, in the limit $\bar{u}(t,y_0)$ can be shown to be equal to $u^*(t,y_0)$ as in (20).

$$
\begin{aligned}
\bar{u}(t,y_0) &= \lim_{\substack{x(t,y)\to x^*(t,y)\\ \varepsilon\to 0}} \frac{-\left[x(t,y_0) - x^*(t,y_0)\right]g(t,y_0)\left\{\int_{y_0-\frac{\varepsilon}{2}}^{y_0+\frac{\varepsilon}{2}}\left[x(t,y) - x^*(t,y)\right]\left[f(t,y) - \dot{x}^*(t,y)\right]dy + \frac{k}{2}\int_{y_0-\frac{\varepsilon}{2}}^{y_0+\frac{\varepsilon}{2}}\left[x(t,y) - x^*(t,y)\right]^2 dy\right\}}{r(y)\int_{y_0-\frac{\varepsilon}{2}}^{y_0+\frac{\varepsilon}{2}}\frac{\left[x(t,y) - x^*(t,y)\right]^2\left[g(t,y)\right]^2}{r(y)}dy} \\
&= \lim_{\substack{x(t,y)\to x^*(t,y)\\ \varepsilon\to 0}} \frac{-\left[x(t,y_0) - x^*(t,y_0)\right]g(t,y_0)\left\{\left[x(t,y_0) - x^*(t,y_0)\right]\left[f(t,y_0) - \dot{x}^*(t,y_0)\right]\varepsilon + \frac{k}{2}\left[x(t,y_0) - x^*(t,y_0)\right]^2\varepsilon\right\}}{r(y_0)\frac{\left[x(t,y_0) - x^*(t,y_0)\right]^2\left[g(t,y_0)\right]^2}{r(y_0)}\varepsilon} \\
&= \frac{-1}{g(t,y_0)}\left[f(t,y_0) - \dot{x}^*(t,y_0)\right] = u^*(t,y_0)
\end{aligned} \tag{20}
$$

The above analysis is true $\forall y_0 \in (0,L)$. Hence $u(t,y) \to u^*(t,y)$ as $x(t,y) \to x^*(t,y)$ $\forall y \in [0,L]$. Note that combining the results in (15) and (18), we can finally write the control solution as

$$u(t,y) = \begin{cases} -\dfrac{1}{g^*}\left[f^* - \dot{x}^*\right], & \text{if } x(t,y) = x^*(t,y) \ \forall y \in [0,L] \\ \dfrac{\gamma(x-x^*)g}{r(y)\int_0^L \dfrac{(x-x^*)^2 g^2}{r(y)} dy}, & \text{otherwise} \end{cases} \quad (21)$$

V. A MOTIVATING NONLINEAR PROBLEM

A. Mathematical Model

A real-life nonlinear heat transfer problem is selected to demonstrate the theoretical developments presented in Section III. The problem is to achieve a desired temperature profile along a fin of a heat exchanger. The schematic of the problem is depicted in Figure 1.

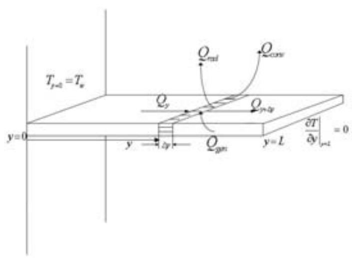

Figure 1: Pictorial representation of the physics of the problem

First we develop a mathematical model from the principles of heat transfer [13]. Using the law of conservation of energy in an infinitesimal volume at a distance y having length Δy, we write

$$Q_y + Q_{gen} = Q_{y+\Delta y} + Q_{conv} + Q_{rad} + Q_{chg} \quad (22)$$

where Q_y is the rate of heat conducted in, Q_{gen} is the rate of heat generated, $Q_{y+\Delta y}$ is the rate of heat conducted out, Q_{conv} is the rate of heat convected out, Q_{rad} is the rate of heat radiated out and Q_{chg} is the rate of heat change. Next, from the laws of physics for heat transfer [13], we can write the following expressions

$$Q_y = -kA(\partial T/\partial y) \quad (23a)$$

$$Q_{gen} = S\, A\Delta y \quad (23b)$$

$$Q_{conv} = h\, P\, \Delta y\left(T - T_{\infty_1}\right) \quad (23c)$$

$$Q_{rad} = \varepsilon\, \sigma\, P\, \Delta y\left(T^4 - T_{\infty_2}^4\right) \quad (23d)$$

$$Q_{chg} = \rho C\, A\Delta y(\partial T/\partial t) \quad (23e)$$

In (23a-e), $T(t,y)$ represents the temperature (this is the state $x(t,y)$ in the context of discussion in Section III), which is a function of both time t and spatial location y; $S(t,y)$ is the rate of heat generation per unit volume (this is the control u in the context of discussion in Section III) for this problem. The meanings of various parameters and their numerical values used are given in Table 1.

Table 1: Definitions and numerical values of the parameters

Parameter	Meaning	Numerical value
k	Thermal conductivity	$180\, W/\left(m^oC\right)$
A	Cross sectional area	$2\, cm^2$
P	Perimeter	$9\, cm$
h	Convective heat transfer coefficient	$5\, W/\left(m^2\, {}^0C\right)$
T_{∞_1}	Temperature of the medium in the immediate surrounding of the surface	$30\, °C$
T_{∞_2}	Temperature at a far away place in the direction normal to the surface	$-40\, °C$
ε	Emissivity of the material	0.2
σ	Stefan-Boltzmann constant	$5.669\times10^{-8}\, W/m^2$
ρ	Density of the material	$2700\, kg/m^3$
C	Specific heat of the material	$860\, J/\left(kg°C\right)$

The values representing of the properties of the material were chosen assuming the material to be Aluminum. The area A and perimeter P have been computed assuming a fin of dimension $40cm\times4cm\times0.5cm$. Note that we have made a one-dimensional approximation for the dynamics, assuming uniform temperature in the other two dimensions being arrived at instantaneously.

Using Taylor series expansion and considering a small $\Delta y \to 0$, we can write

$$Q_{y+\Delta y} \approx Q_y + \left(\partial Q_y/\partial y\right)\Delta y \quad (24)$$

Using (23-24) in (22), and simplifying it leads to

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C}\left(\frac{\partial^2 T}{\partial y^2}\right) - \frac{P}{A\rho C}\left[h\left(T - T_{\infty_1}\right) + \varepsilon\,\sigma\left(T^4 - T_{\infty_2}^4\right)\right] + \left(\frac{1}{\rho C}\right)S \quad (25)$$

Next, for convenience we define $\alpha_1 \triangleq (k/\rho C)$, $\alpha_2 \triangleq -(Ph)/(A\rho C)$, $\alpha_3 \triangleq -(P\varepsilon\sigma)/(A\rho C)$ and $\beta \triangleq 1/(\rho C)$. Using these definitions, (25) can be written as

$$\frac{\partial T}{\partial t} = \alpha_1\left(\frac{\partial^2 T}{\partial y^2}\right) + \alpha_2\left(T - T_{\infty_1}\right) + \alpha_3\left(T^4 - T_{\infty_2}^4\right) + \beta\, S \quad (26)$$

Along with (26), we consider the following boundary conditions

$$T_{y=0} = T_w, \quad \left.(\partial T/\partial y)\right|_{y=L} = 0 \quad (27)$$

where T_w is the wall temperature. Insulated boundary condition at the tip has been used with the assumption that either there is some physical insulation at the tip or the heat loss at the tip due to convection and radiation is negligible (mainly because of its low surface area).

The goal for the controller is to make sure that the actual temperature profile $T(t,y) \to T^*(y)$, where we chose $T^*(y)$ to be a constant (with respect to time) temperature profile, as follows

$$T^*(y) = T_w + (T_w - T_{tip})^{-\zeta y} \tag{28}$$

In (28) we chose the wall temperature $T_w = 150\ ^0C$, fin tip temperature $T_{tip} = 130\ ^0C$ and the decaying parameter $\zeta = 20$. The selection such a $T^*(y)$ from (28) was motivated by the fact that it leads to a smooth continuous temperature profile across the spatial dimension y.

In our simulation studies, we selected the control gain as $k = 1/\tau$, $\tau = 30\, sec$. $r(y)$ was assumed to be a constant $c > 0$, and hence, we were able to use the simplified formula for the control in (16) (instead of (15)). Because of this, a numerical value for $r(y)$ was not necessary for the simulation studies.

B. Analysis of Numerical Results

First we chose an initial condition (profile) for the temperature as obtained from the expression $T(0,y) = T_m + x(0,y)$, where $T_m = 150\ ^0C$ (a constant value) serves as the mean temperature and $x(0,y)$ represents the deviation from T_m. Taking $A = 50$ we computed $x(0,y)$ as $x(0,y) = (A/2) + (A/2)\cos(-\pi + 2\pi y/L)$. Applying the controller as synthesized in (21), we simulated the system in (26-27) from time $t = t_0 = 0$ to $t = t_f = 5\,\text{min}$. The simulation results obtained are as in Figures 2-5. We can see from Figure 2 that the goal of tracking $T^*(y)$ is met without any problem. To see the error of tracking effect more clearly we have plotted the deviation profile $\left[T(y) - T^*(y)\right]$ in Figure 4, which shows that the deviation profile approaches to zero for all $y \in [0,L]$ with time.

The associated control (rate of energy input) profile $S(t,y)$ obtained is as in Figure 3. This figure shows that the required control magnitude is not much in the entire spatial domain $[0,L]$ and for all time $t \in \left[t_0, t_f\right]$ (hence the question of control saturation may not arise in implementation). It is important to note that even as $T(t,y) \to T^*(y)$, there is no control singularity. In fact the control profile develops (converges) towards the steady-state control profile given in (17). To see this effect clearly, we have plotted the $S(t_f, y)$ and $S^*(y)$ in Figure 5. The closeness of the two plots justifies this claim.

To demonstrate that similar results (as in Figures 2-5) will be obtained for any arbitrary initial condition of the temperature profile $T(0,y)$, next we considered a number of random profiles for $T(0,y)$ and carried out the simulation studies. We generated the 'random profiles' using the relationship $T(0,y) = T_m + x(0,y)$, where $x(0,y)$ was generated using the concept of "Fourier Series", such that it satisfies $\|x(0,y)\|^2 \le k_1 \|x\|^2_{\max}$, $\|x'(0,y)\|^2 \le k_2 \|x'\|^2_{\max}$ and $\|x''(0,y)\|^2 \le k_3 \|x''\|^2_{\max}$. Note that constants $k_1, k_2, k_3 > 0$ are judiciously selected so that it allows sufficient flexibility to generate a large number of smooth profiles and yet does not lead to too much (unrealistic) waviness in the profiles. The values for $\|x\|_{\max}$, $\|x'\|_{\max}$ and $\|x''\|_{\max}$ were computed using an envelope profile $x_{env}(y) = A\sin(\pi y/L)$. The norm used is the L_2 norm defined by $\|x\| \triangleq \left(\int_0^L x^2(y)\, dy\right)^{1/2}$. We selected the value of parameters $A = 50$, $k_1 = 2$ and $k_2 = k_3 = 10$. For more details about the generation of these random profiles, the reader is referred to [14]. The results obtained are similar in the sense that the objective of $T(t,y) \to T^*(y)$ is met. We also noticed that the control (rate of energy input) magnitude is not high and, more important, the control profile develops (converges) towards the steady-state control profile. Due to space limitations, these results are not included here.

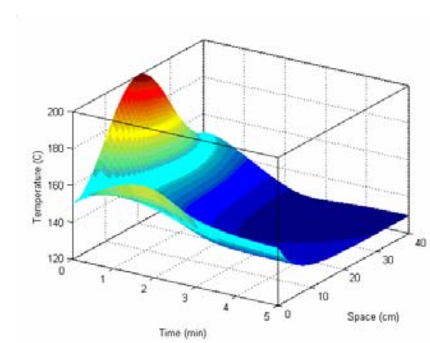

Figure 2: Evolution of the temperature (state) profile from a sinusoidal initial condition

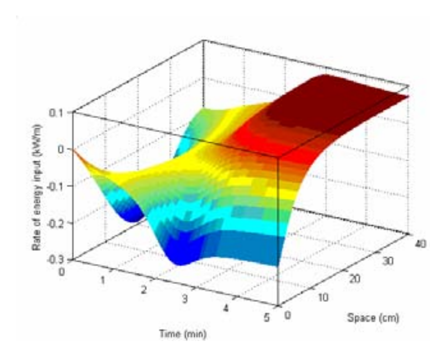

Figure 3: Rate of energy input (control) for the evolution of temperature profile in Figure 4

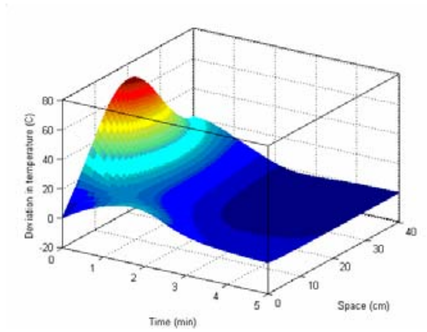

Figure 4: Evolution of the deviation of the temperature profile in Figure 4 from the desired final temperature profile

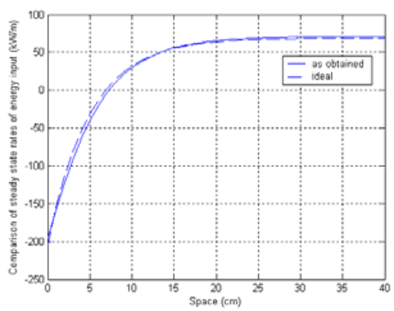

Figure 5: Comparison of the steady state rate of energy input profile obtained with the ideal case

VI. CONCLUSIONS

Combining the principles of dynamic inversion and optimization theory, we have presented a general control synthesis technique. The technique is fairly straightforward and is applicable to a class of one-dimensional nonlinear distributed parameter systems. The convergence of the controller to its steady state value has been proved (and shown from numerical simulations as well). An important novelty of the technique is that it is independent of the "design-then-approximate" or "approximate-then-design" philosophies reported in the literature, and hence, there is no need of any approximation either of the system dynamics or the controller obtained. For implementing the controller, one may still have to discretize it (which will depend on the size of the spatial grid); however that is not the focal point of this paper. Note that the technique presented in this paper can easily be implemented on-line since we essentially obtain a *closed form solution* for the controller in state feedback form. In addition, since the control is in a state feedback form, it retains the benefits of a state feedback controller (like noise suppression). To demonstrate the potential of the proposed techniques, a real-life temperature control problem for a heat transfer application has been solved. It has been verified that a desired temperature profile can be achieved starting from any 'random' initial temperature profile.

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REFERENCES

[1] Annaswamy A., Choi J. J., Sahoo D., Active Closed Loop Control of Supersonic Impinging Jet Flows Using POD Models, *Proceedings of the 41st IEEE Conference on Decision and Control, Las Vegas, 2002.*

[2] Arian E., Fahl M. and Sachs E. W., Trust-region Proper Orthogonal Decomposition for Flow Control, *NASA/CR-2000-210124, ICASE Report No. 2000-25.*

[3] Bameih B., The Structure of Optimal Controllers of Spatially-invariant Distributed Parameter Systems. *Proceedings of the Conference on Decision & Control, 1997,* 1056-1061.

[4] Banks H. T., Rosario R. C. H and Smith R. C., Reduced-Order Model Feedback Control Design: Numerical Implementation in a Thin Shell Model, *IEEE Transactions on Automatic Control, Vol. 45, 2000, 1312-1324.*

[5] Bryson A. E. and Ho Y. C., Applied Optimal Control, *London: Taylor and Francis, 1975.*

[6] Burns J.A. and King, B.B., Optimal sensor location for robust control of distributed parameter systems. *Proceedings of the Conference on Decision and Control, 1994,* 3967- 3972.

[7] Christofides P. D., Nonlinear and Robust Control of PDE Systems – Methods and Applications to Transport-Reaction Processes, *Birkhäuser , Boston, 2000.*

[8] Curtain R. F. and Zwart H. J., An Introduction to Infinite Dimensional Linear Systems Theory, *Springer-Verlag, New York, 1995.*

[9] Enns, D., Bugajski, D., Hendrick, R. and Stein, G., Dynamic Inversion: An Evolving Methodology for Flight Control Design, *International Journal of Control, Vol.59, No.1,1994, pp.71-91.*

[10] Holmes P., Lumley J. L. and Berkooz G., Turbulence, Coherent Structures, Dynamical Systems and Symmetry, *Cambridge University Press, 1996, 87-154.*

[11] Lasiecka I., Control of Systems Governed by Partial Differential Equations: A Historical Perspective, *Proceedings of the 34th Conference on Decision and control, 1995, 2792-2796.*

[12] Lane, S. H. and Stengel, R. F., Flight Control Using Non-Linear Inverse Dynamics, *Automatica, Vol.24, No.4, 1988, pp.471-483.*

[13] Miller A. F., Basic Heat and Mass Transfer, *Richard D. Irwin Inc., MA, 1995.*

[14] Padhi R. and Balakrishnan S. N., Proper Orthogonal Decomposition Based Feedback Optimal Control Synthesis of Distributed Parameter Systems Using Neural Networks, *Proceedings of the American Control Conference, 2002, pp. 4389-4394.*

[15] Ravindran S. S., Adaptive Reduced-Order Controllers for a Thermal Flow System Using Proper Orthogonal Decomposition, *SIAM Journal on Scientific Computing, Vol.23, No.6, 2002, pp.1924-1942.*

[16] Slotine, J-J. E. and Li, W., Applied Nonlinear Control, *Prentice Hall, 1991.*