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# ONLINE IDENTIFICATION AND CONTROL OF AEROSPACE VEHICLES USING RECURRENT NETWORKS

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## ABSTRACT

Methods for estimating the aerospace system parameters and controlling them through two neural networks are presented in this study. We equate the energy function of Hopfield neural network to integral square of errors in the system dynamics and extract the parameters of a system. Parameter convergence is proved. For control, we equate the equilibrium status of a 'modified' Hopfield neural network to the steady state Ricatti solution with the system parameters as inputs. Through these two networks, we present the online identification and control of an aircraft using its nonlinear dynamics.

## 1 INTRODUCTION

Recently, a lot of interest has been shown in the research of tailless aircraft, aircraft flying at high angles of attack and high speed vehicles flying at high mach numbers. Since the configurations are novel and the operating regimes are unexplored, force and moment derivatives are difficult to estimate with good fidelity by known codes. Due to the uncertainties associated with the modeling of the dynamics, control of these vehicles has become a difficult and interesting problem. In this study, we solve this problem by using a set of two recurrent networks. The first network can estimate or identify the parameters of linear and nonlinear systems online and is used to identify the stability derivatives of an aircraft. The input to the network is the states of the aircraft or a system. The second network calculates the time varying gain of an optimal controller; the inputs to the second network consist of the parameters of the system or the aircraft. These gains are used in the feedback control of the aircraft.

## 2 TWO RECURRENT NEURAL NETWORKS

### 2.1 HOPFIELD NETWORK

Consider the dynamical model of a Hopfield network. The synaptic weights $w_{11}, w_{12}, \cdots, w_{1n}, \cdots, w_{nn}$ represent conductance, and the respective outputs $v_1(t), v_2(t), \cdots, v_n(t)$ represent potentials; $n$ is the number of inputs and outputs. So, for some node $j$, define the dynamics of the network by the following system of first order differential equations

$$c_j \frac{du_j(t)}{dt} = -g_j u_j(t) + \sum_{i=1}^{n} w_{ji} f_i(u_i(t)) + i_j \qquad j = 1,2,\cdots,n \tag{1}$$

As $C = diag(c_1, c_2, \cdots, c_n), G = diag(g_1, g_2, \cdots, g_n)$, rewrite this equation in a vectorized form as

$$C\frac{dU}{dt} = -GU + WV + I \tag{2}$$

Assume that the nonlinear function $f(\cdot)$ relating the output $v_j(t)$ of neuron $j$ to its activation potential $u_j(t)$ is a continuous function and therefore differentiable and that $u_j(t) = f_j^{-1}(v_j(t))$ exists. According to Hopfield's work [5]-[7], the energy function of the recurrent network is defined by

$$E = -\frac{1}{2}\sum_{j=1}^{n}\sum_{i=1}^{n} w_{ji} v_i v_j + \sum_{j=1}^{n} g_j \int_0^{v_j} f_j^{-1}(v) dv - \sum_{j=1}^{n} i_j v_j \tag{3}$$

Then differentiate $E$ with respect to time $t$. For log sigmoid, tangent sigmoid and linear activation function the inverse output-input relation $f_j^{-1}(v_j)$ is a monotonically increasing function of the output $v_j(t)$. It can be shown that

$$\frac{dE}{dt} = -\sum_{j=1}^{n} c_j \left[\frac{d}{dv_j} f_j^{-1}(v_j)\right]\left(\frac{dv_j}{dt}\right)^2 \le 0 \tag{4}$$

Thus, we can say the model is stable in accordance with Lyapunov's theorem.

### 2.2 MODIFIED HOPFIELD NETWORK

The modified Hopfield network [1] is a variant of the classical Hopfield network. Its dynamical model is shown in Figure 1. Compared with classical Hopfield network, this modified version give us more variables in different locations, thus have more power to handle more complex problems. Since it is derived from Hopfield network, the

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modified Hopfield neural network keeps the characteristic of the former. The right side of the modified Hopfield neural networks is characterized by outputs $\phi_1(t), \phi_2(t), \cdots, \phi_m(t)$ that are transformed by nonlinear function $f$ from their states $u_1(t), u_2(t), \cdots, u_n(t)$; $m$ is the number of outputs of neurons in the right part. Conductance $w^r_{ji}$ connects the output of the $j$th neuron in the left part to the input of the $i$th neuron in the right part, which is shown in Figure 1. The superscript $r$ indicates the location of weights $w_{ji}$ in the right part of the network; $n$ is the number of outputs of neurons in the left part. So, by *Kirchoff's* law,

$$c_j \frac{dx_j(t)}{dt} = -a_j - \frac{x_j(t)}{R_j} - \sum_{i=1}^{n} w^l_{ji} f_i\left( \sum_{k=1}^{n} w^r_{ik} v_k - b_i \right) \qquad (5)$$

$j = 1,2,\cdots,n$

This is based on the assumption that their inverse exist

$$u_j(t) = f_j^{-1}(\phi_j(t)) \quad j = 1,2,\cdots m$$
$$x_j(t) = g_j^{-1}(v_j(t)) \quad j = 1,2,\cdots n \qquad (6)$$

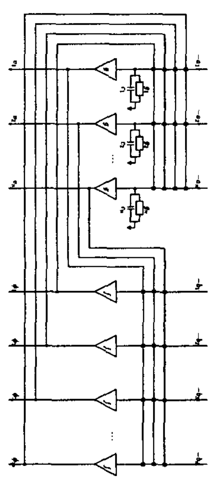

Figure 1 Modified Hopfield Neural Network

Matrix $C$ will be defined as before, and Eq. (5) can be rewritten as

$$C\frac{dX}{dt} = -A - GX - W^L f\left(W^R g(x) - B\right) \qquad (7)$$

It is easy to shown that this modified Hopfield neural network is also stable [1].

## 3 DYNAMIC SYSTEM PARAMETER IDENTIFICATION

This section describes a general nonlinear dynamic system and a linearization method for its nonlinearity. We use it and Hopfield neural network theory to compute the network's weights and biases. Convergence of the system parameters to their true values is proved.

### 3.1 DYNAMIC SYSTEM DESCRIPTION AND LINEARIZARION

Consider a general nonlinear dynamic system that can be described in state space form as

$$\dot{x} = F(A,x) + Bu \qquad (8)$$

where $x$ is an $n$x1 vector, $u$ is a $p$x1 control input, and $F(A,x)$ is a certain kind of nonlinear function that can be described as the following forms,

$$F(A,x) = \sum_{j=1}^{m} f_{ij}\left[a_{ij} h_{ij}(x)\right], \quad i = 1,2,\cdots,n \qquad (9)$$

$$A \equiv \left[a_{ij}\right]_{(n\times m)} \qquad (10)$$

$$B \equiv \left[b_{ij}\right]_{(n\times p)} \qquad (11)$$

In $F(A, x)$, each row can have different number of term, like $m_1, m_2, \cdots, m_n$. One can pick the longest row, which has $m_k$ terms. Make other rows also have $m_k$ terms by adding zeros at the back of each row. Here for simplicity but without losing generality, we suppose they all have equal number ($m$) of terms. To identify $A$ and $B$, which are matrices of parameters associated with the system, the key point is to get the parameters out of every term. In order to realize this transformation, we use a linearization.

We can linearize the nonlinear expression $F(A, x)$ about an equilibrium point $x^*$ as [11]

$$F(A,x) = \sum_{j=1}^{m} a_{ij} g_{ij}(x), \quad i = 1,2,\cdots,n \qquad (12)$$

where $g_{ij}(x) = f'_{ij}(a_{ij}h_{ij}(x))h'_{ij}(x)\big|_{x=x^*} \cdot (x - x^*)$.

Here prime denotes differentiation with respect to quantities within parenthesis evaluated at $x^*$.

Each row in $F(A, x)$ is now a sum of linear terms in $x$. Note, that all the unknown parameters appear as coefficients of the terms in $F(A, x)$. Now, we will relate $F(A,x)$ to the energy function and weights of the neural networks.

### 3.2 COMPUTATION OF WEIGHTS AND BIASES

The error dynamics between the plant and the model with unknown parameters are given by

$$e(A,B,x) = \dot{x} - F_s(A,x) - B_s u \qquad (13)$$

The subscript "s" denotes the system containing estimated parameters. The energy function of the neural network is defined as

$$E(A,B,x) = \frac{1}{2T}\int_0^T e^T e \; dt$$
$$= \frac{1}{2T}\int_0^T \left[\dot{x} - F_s(A,x) - B_s u\right]^T \cdot \left[\dot{x} - F_s(A,x) - B_s u\right] \; dt \qquad (14)$$

where $T$ is time period during which data are collected.

The equilibrium point for the energy function occurs when the partial derivatives $\partial E/\partial A_s$, $\partial E/\partial B_s$ are zero. The derivatives of the energy function $E$ with respect to parameters $a_{ij}$ and $b_{ij}$ are given by

$$\frac{\partial E}{\partial a_{ij}} = \frac{1}{T}\int_0^T\left[\dot{x}_i - (a_{i1}g_{i1} + a_{i2}g_{i2} + \cdots + a_{im}g_{im}) - \sum_{k=1}^{p} b_{ik}u_k\right]\cdot[-g_{ij}]dt \quad \text{(15)}$$

$$\frac{\partial E}{\partial b_{ij}} = \frac{1}{T}\int_0^T\left[\dot{x}_i - (a_{i1}g_{i1} + a_{i2}g_{i2} + \cdots + a_{im}g_{im}) - \sum_{k=1}^{p} b_{ik}u_k\right]\cdot[-u_k]dt \quad \text{(16)}$$

Define $V$ as [2-4]

$$V = \left[a_{11},\cdots a_{1m},\cdots,a_{n1},\cdots a_{nm},b_{11},\cdots b_{1p},\cdots b_{n1},\cdots b_{np}\right]^T \quad \text{(17)}$$

And $G_j = \left[g_{j1}, g_{j2}, \cdots, g_{jm}\right]^T$ (18)

Note that in terms of $G_j$, the linearized model representation becomes

$$\dot{x}_j = A_j^T G_j + B_j^T u \quad \text{(19)}$$

where $A_i$, $B_i$ represent the transpose of $i$th row of $A$ and $B$ respectively.

We can write the parameter identification formulation as

$$\frac{dE}{dV} = WV + I \quad \text{(20)}$$

where $W$ (weight) and $I$ (bias) are set as following,

$$W = \frac{1}{T}\int_0^T \begin{bmatrix} (G_1G_1^T) & 0 & \cdots & 0 & (G_1u^T) & 0 & \cdots & 0 \\ 0 & (G_2G_2^T) & \cdots & 0 & 0 & (G_2u^T) & \cdots & 0 \\ \vdots & & & \vdots & & \vdots & & \vdots \\ 0 & 0 & \cdots & (G_nG_n^T) & 0 & 0 & \cdots & (G_nu^T) \\ (uG_1^T) & 0 & \cdots & 0 & (uu^T) & 0 & \cdots & 0 \\ 0 & (uG_2^T) & \cdots & 0 & 0 & (uu^T) & \cdots & 0 \\ \vdots & & & \vdots & & \vdots & & \vdots \\ 0 & 0 & \cdots & (uG_n^T) & 0 & 0 & \cdots & (uu^T) \end{bmatrix} dt \quad \text{(21)}$$

$$I = -\frac{1}{T}\int_0^T\left[(\dot{x}_1G_1^T) \ (\dot{x}_2G_2^T) \ \cdots \ (\dot{x}_nG_n^T) \ (\dot{x}_1u^T) \ (\dot{x}_2u^T) \ \cdots \ (\dot{x}_nu^T)\right]^T dt \quad \text{(22)}$$

Now, we will relate this formulation to the dynamics of a Hopfield neural network. The network dynamics can be written in the following form [8]

$$\frac{dV}{dt} = -\mu\frac{dE}{dV} = -\mu(WV + I) \quad \text{(23)}$$

In order to find the expression for $\mu$, we choose tangent sigmoid as our nonlinear activation function, as

$$v_j = f(u_j) = \rho\left(1 - e^{-\lambda_j u_j}\right)/\left(1 + e^{-\lambda_j u_j}\right) \quad \text{(24)}$$

and rewrite Eq. (4) to get

$$dv_j/dt = -\lambda_i\left(\rho^2 - v_j^2\right)/\left(2\rho c_j\right)\cdot dE/dv_j \quad \text{(25)}$$

Comparing Eq. (23) with Eq. (25), we observe that

$$\mu_j = \lambda_i\left(\rho^2 - v_j^2\right)/\left(2\rho c_j\right) \quad \text{(26)}$$

and $\mu = \mathrm{diag}\left[\mu_1 \ \ \mu_2 \ \ \cdots \ \ \mu_{mn+np}\right]$.

### 3.3 CONVERGENCE ANALYSIS FOR TIME-INVARIANT PARAMETERS

Let us define identification error between true values and identified ones as

$$\begin{aligned} e_p = V_p - V_s &= \left[V_{Ap}^T \ \ V_{Bp}^T\right]^T - \left[V_{As}^T \ \ V_{Bs}^T\right]^T \\ &= \left[V_{Ap}^T - V_{As}^T \ \ V_{Bp}^T - V_{Bs}^T\right]^T \end{aligned} \quad \text{(27)}$$

where subscripts $A$ and $B$ denote correspondence with $A$ and $B$ matrices and $p$ and $s$ show correspondence with plant and 'estimated' values. We define a cost function as

$$J = \frac{1}{2T}\int_0^T e_p^T e_p \ dt = J_A + J_B \quad \text{(28)}$$

where

$$J_A = \frac{1}{2T}\int_0^T \left(V_{Ap} - V_{As}\right)^T\left(V_{Ap} - V_{As}\right) \ dt \quad \text{(29)}$$

$$J_B = \frac{1}{2T}\int_0^T \left(V_{Bp} - V_{Bs}\right)^T\left(V_{Bp} - V_{Bs}\right) \ dt \quad \text{(30)}$$

$T$ is the sampling period. Then the derivatives of the cost function with respect with time,

$$\frac{dJ_{Ai}}{dt} = \frac{1}{T}\int_0^T \left(V_{Aip} - V_{Ais}\right)^T \cdot\left\{\frac{\mu_i}{T}\int_0^T\left[(G_iG_i^T)\cdot\left(V_{Ais} - V_{Aip}\right) + (G_iu^T)\cdot\left(V_{Bis} - V_{Bip}\right)\right]dt\right\} dt \quad \text{(31)}$$

Similarly for $B$ matrix,

$$\frac{dJ_{Bi}}{dt} = \frac{1}{T}\int_0^T \left(V_{Bip} - V_{Bis}\right)^T \cdot\left\{\frac{\mu_{mn+i}}{T}\int_0^T\left[(uG_i^T)\cdot\left(V_{Ais} - V_{Aip}\right) + (uu^T)\cdot\left(V_{Bis} - V_{Bip}\right)\right]dt\right\} dt \quad \text{(32)}$$

And the combined row cost function thus is

$$\begin{aligned} \frac{dJ_i}{dt} &= \frac{dJ_{Ai}}{dt} + \frac{dJ_{Bi}}{dt} \\ &= -\frac{1}{T}\int_0^T (M + N + P + Q) \ dt \end{aligned} \quad \text{(33)}$$

Note, $M$, $N$, $P$ and $Q$ are all scalars. Each is expressed as

$$M = \left(V_{Aip} - V_{Ais}\right)^T \cdot \frac{\mu_i}{T}\int_0^T \left(G_iG_i^T\right)\cdot\left(V_{Aip} - V_{Ais}\right) \ dt \quad \text{(34)}$$

$$N = \left(V_{Bip} - V_{Bis}\right)^T \cdot \frac{\mu_{mn+i}}{T}\int_0^T \left(uu^T\right)\cdot\left(V_{Bip} - V_{Bis}\right) \ dt \quad \text{(35)}$$

$$P = \left(V_{Aip} - V_{Ais}\right)^T \cdot \frac{\mu_i}{T}\int_0^T \left(G_iu^T\right)\cdot\left(V_{Bip} - V_{Bis}\right) \ dt \quad \text{(36)}$$

$$Q = \left(V_{Bip} - V_{Bis}\right)^T \cdot \frac{\mu_{mn+i}}{T}\int_0^T \left(uG_i^T\right)\cdot\left(V_{Aip} - V_{Ais}\right) \ dt \quad \text{(37)}$$

Before the steps continue on, let us do some algebraic operations.

$$\begin{aligned} &\sqrt{\left(W_{1\times m}^T H_{m\times 1} H_{1\times m}^T W_{m\times 1}\right)_{1\times 1}\left(V_{1\times p}^T F_{p\times 1} F_{1\times p}^T V_{p\times 1}\right)_{1\times 1}} \\ &= W^T H F^T V = V^T F H^T W \end{aligned} \quad \text{(38)}$$

From the computation of Eq.(38), if we set

$$V_{Aip} - V_{Ais} = W, \ \frac{\sqrt{\mu_i}}{\sqrt{T}}\int_0^T G_i dt = H, \ V_{Bip} - V_{Bis} = V,$$

$$\frac{\sqrt{\mu_{mn+i}}}{\sqrt{T}}\int_0^T u dt = F$$

Then, $\sqrt{MN} = |P| = |Q|$ (39)

Since it is well known that $M + N - 2\sqrt{MN} \ge 0$, then

$$M + N + P + Q \ge M + N - (|P| + |Q|) \ge 0 \quad (40)$$

From Eq.(33), it is clear

$$dJ_i/dt \le 0 \quad (41)$$

Thus $$dJ/dt = \sum_{i=1}^{n} dJ_i/dt \le 0 \quad (42)$$

This means that the energy function of Hopfield neural networks for time-invariant identification acts like a Lyapunov function. Its dynamics will drive the identification error to zero.

## 4 SYSTEM PARAMETER ESTIMATION

Most of the time, all states are not available as the measurements are noisy. Let us consider a 9-variable nonlinear dynamic system [9-10]. Due to its nonlinearity and its over one hundred parameters, it is very difficult to estimate its parameters correctly.

$$\dot{x}(t) = Ax + Bu + F(x) + w(t) \quad (43)$$

where $x = [v \quad \alpha \quad q \quad \theta \quad \beta \quad p \quad r \quad \phi \quad \psi]^T$

$u = [\delta_{HR} \quad \delta_{HL} \quad \delta_{FR} \quad \delta_{FL} \quad \delta_C \quad \delta_R]^T$

$$F(x) = \begin{bmatrix} 0 \\ -p\cos\alpha\tan\beta - r\sin\alpha\tan\beta \\ c_{31}pr + c_{32}(r^2 - p^2) \\ q\cos\phi - r\sin\phi \\ p\sin\alpha - r\cos\alpha \\ c_{61}qp + c_{62}qr \\ c_{71}qp - c_{72}qr \\ q\tan\theta\sin\phi + r\tan\theta\cos\phi \\ q\cos^{-1}\theta\sin\phi + r\cos^{-1}\theta\cos\phi \end{bmatrix}$$

In order to make the problem more realistic, we use the estimated states based on measurements.

$$z(t) = x(t) + v(t) \quad (44)$$

We use an extended Kalman filter that contains a zero mean process noise $w(t)$ with an associated power spectral density. Both of these two noise sources are represented by gaussian noise $w(t)$ and $v(t)$ respectively have 0 mean and their magnitudes are 5% of the state $x$ and magnitude of measurement z' respectively. Following Kalman filter theory, it is easy to compute that

$$F(\hat{x}(t),t) = \left.\frac{\partial[Ax + F(x)]}{\partial x}\right|_{x=\hat{x}} \quad (45)$$

And $$H(\hat{x}(t),t) = I_{9\times 9} \quad (46)$$

A stochastic noise is used as input to stimulate the network. Because of the nonlinear structures, a batch method is also used, which means if the parameters do not converge after one iteration, the final values are picked up as the initial values for the next iteration. All parameters converge to their true values after 1 to 3 batches. The estimated $A$, $B$ and $C$ final values are provided below. Most of the errors between true values and the estimated ones are around 5%. Two parameters showed less than 10%, and five more than 10% errors. A deterministic case showed less than 3% error in all parameters. These exceptional are due to the noised added in both system and observer.

$$A = \begin{bmatrix} -0.0090 & 0.5301 & -0.2283 & -9.8 & -0.4638 & -0.0946 & -0.1397 & 0 & 0 \\ -0.0010 & -0.6928 & 1.0138 & 0 & 0.1236 & 0.0388 & 0.0051 & 0 & 0 \\ 0.0002 & 2.6757 & -0.5180 & 0 & 0.0088 & 0.0062 & 0.0302 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.0010 & 0.6743 & 0.4375 & 0.1502 & -0.6925 & 0.0853 & -0.1575 & 0 & 0 \\ 0.00002 & 1.1017 & 0.0427 & 0.0065 & -26 & -4.8231 & 0.5438 & 0 & 0 \\ 0.00001 & -1.6894 & 0.0984 & 0.0099 & 7.4 & -0.0366 & -0.8061 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.0950 & 0.0921 & 0.0448 & -0.0454 & -0.0692 & -0.1286 \\ -0.2821 & -0.2806 & -0.0078 & -0.0073 & 0.0060 & 0 \\ -24.9978 & -25.0001 & -0.5885 & -0.5919 & 3.5045 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0174 & -0.0155 & -0.3607 & 0.3591 & 0.0871 & -0.0493 \\ -0.2369 & 0.2402 & -9.7762 & 9.7819 & 0.2656 & -0.3714 \\ 0.3814 & -0.3798 & 0.1904 & -0.1905 & 0.5202 & -4.6001 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 1.0774 & 0 & 0 & 0.0224 & -0.7122 & 0 & 0 \\ 0 & 0 & 0.0136 & 0 & 0 & -1.4403 & 0.0312 & 0 & 0 \end{bmatrix}^T$$

## 5 OPTIMAL CONTROL USING RECURRENT NEURAL NETWORK

The second major and important component of this study is the control of this aircraft with the parameters identified online. This section will show how a modified Hopfield neural network can be programmed to suit the optimal control and its weights and biases of recurrent network are determined.

### 5.1 CONTROL THEORY FORMULATION

Consider a class of infinite-horizon nonlinear regulator problem of the form. Minimize

$$J = \frac{1}{2}\int_{t_0}^{\infty}\left(x^T Q(x)x + u^T R(x)u\right)dt \quad (47)$$

with respect to the state $x$ and control $u$ subject to the nonlinear differential constraint

$$\dot{x} = f(x) + g(x)u \quad (48)$$

where $x \in R^n$, $u \in R^m$, $f(x) \in C^k$, $g(x) \in C^k$, $Q(x) \in C^k$, $R(x) \in C^k$, $Q(x) = C(x)^T C(x) \ge 0$ and $R(x) > 0$ for all $x$. It is well-known that the nonlinear dynamics Eq.(48) can be represented by the following linear structure having state-dependent coefficient:

$$\dot{x} = A(x)x + B(x)u \quad (49)$$

where

$$f(x) = A(x)x \qquad B(x) = g(x) \quad (50)$$

The SDRE approach [1] for obtaining a sub-optimal solution of Eq.(47) , (48) is:

1. Use direct parameterization to bring the nonlinear dynamics to the form in Eq.(49).
2. Solve the state-dependent Ricatti equation
$$A^T(x)S + SA(x) - SB(x)R^{-1}B^T(x)S + Q(x) = 0 \quad (51)$$
to obtain $S \le 0$. Note that $S$ is a function of $x$.
3. Construct the nonlinear feedback controller via
$$u = -R^{-1}(x)B^T(x)S(x)x \quad (52)$$

Let the plant to be controlled be described by the linear equation with performance index defined in the time interval $[i, N]$. This procedure will finally lead to the control

$$u_k = -K_k x_k, \quad k < N \quad (53)$$

where the Kalman gain $K_k$ is given by

$$K_k = \left(B_k^T S_{k+1} B_k + R_k\right)^{-1} B_k^T S_{k+1} A_k \quad (54)$$

In terms of the Ricatti variable $S_k$, now

$$S_k = A_k^T S_{k+1}\left(A_k - B_k K_k\right) + Q_k \quad (55)$$

In the application where the control interval is finite, $S_k$ will be given. By alternative use of Eq.(54) and (55), we will get a series of $K_k$. The gain matrix $K_k$ will generally be time-varying even when the matrices $A_k$, $B_k$, $Q_k$ and $R_k$ are all constants.

### 5.2 MODIFIED HOPFIELD CONTROLLER GAIN IMPLEMENTATION

In order to get closed form expression for the converged values of the networks, we assume small signals and that work in the linear region of the amplifiers. These assumptions are reasonable because neural network signals are usually normalized. For those big signals log sigmoid and tangent sigmoid will limit their output magnitude and their effects are localized and will not be propagated.

Assume the nonlinear activation functions $f$ and $g$ uniformly take the form of a tangent sigmoid. Then, the equilibrium points are given by

$$X = \left[k_f k_g W^L W^R + G\right]^{-1}\left(k_f W^L B - A\right) \quad (56)$$
$$= \left[W^L W^R + G/k_f k_g\right]^{-1}\left(W^L B/k_g - A/k_f k_g\right)$$
$$V = k_g X = \left[W^L W^R + G/k_f k_g\right]^{-1}\left(W^L B - A/k_f\right) \quad (57)$$

The modified Hopfield networks contain both invariant and variable parameters. Invariant parameters are fixed in the neuron-computing model, while variable parameters can be modified. By comparing Eq.(54) and (55) with the stable output of the network Eq.(57), if we set $W^L = B_k^T S_{k+1}$, $W^R = B_k$, $-G/k_f k_g = R_k$, $B = A_k$ and $A$=0, the network will give us the Kalman sequence. As we know, it is not difficult for the circuits to achieve the multiplication of two signals.

The advantage of modified Hopfield neural network when it is applied to control problems, is that system parameters can be represented as inputs to the network. So, for nonlinear system, once we can find the system decomposition, which is similar to linear system, then the modified Hopfield neural network also can be used in implementing nonlinear optimal control.

### 5.3 CONTROL OF NONLINEAR AIRCRAFT

This example is the 9-variable aircraft dynamics, which is described as

$$\dot{x} = Ax + Bu + \begin{bmatrix} 0 \\ -x_6\cos x_2\tan x_5 - x_7\sin x_2\tan x_5 \\ c_{31}x_6x_7 + c_{32}(x_7^2 - x_6^2) \\ x_3\cos x_8 - x_7\sin x_8 \\ x_6\sin x_2 - x_7\cos x_2 \\ c_{61}x_3x_6 + c_{62}x_3x_7 \\ c_{71}x_3x_6 - c_{72}x_3x_7 \\ x_3\tan x_4\sin x_8 + x_7\tan x_4\cos x_8 \\ x_3(\cos x_4)^{-1}\sin x_8 + x_7(\cos x_4)^{-1}\cos x_8 \end{bmatrix} \quad (58)$$

To be used with this SDRE method, we need to transform this equation into a 'linear' form as

$$\dot{x} = A'(x)x + Bu \quad (59)$$

where

$$A'(x) = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} & a_{17} & a_{18} & a_{19} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} - \cos x_2\tan x_5 & a_{27} - \sin x_2\tan x_5 & a_{28} & a_{29} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} + c_{31}x_7 - c_{32}x_6 & a_{37} + c_{32}x_7 & a_{38} & a_{39} \\ a_{41} & a_{42} & a_{43} + \cos x_8 & a_{44} & a_{45} & a_{46} & a_{47} - \sin x_8 & a_{48} & a_{49} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} + \sin x_2 & a_{57} - \cos x_2 & a_{58} & a_{59} \\ a_{61} & a_{62} & a_{63} + c_{61}x_6 + c_{62}x_7 & a_{64} & a_{65} & a_{66} & a_{67} & a_{68} & a_{69} \\ a_{71} & a_{72} & a_{73} + c_{71}x_6 - c_{72}x_7 & a_{74} & a_{75} & a_{76} & a_{77} & a_{78} & a_{79} \\ a_{81} & a_{82} & a_{83} + \tan x_4\sin x_8 & a_{84} & a_{85} & a_{86} & a_{87} + \tan x_4\cos x_8 & a_{88} & a_{89} \\ a_{91} & a_{92} & a_{93} + \dfrac{\sin x_8}{\cos x_4} & a_{94} & a_{95} & a_{96} & a_{97} + \dfrac{\cos x_8}{\cos x_4} & a_{98} & a_{99} \end{bmatrix}$$

The matrices elements of $A$, $B$ and $C$ have all been identified. We assume

$Q = I_9$, $R = I_6$.

Its control and state trajectories are plotted in Figure 2 and Figure 3 separately.

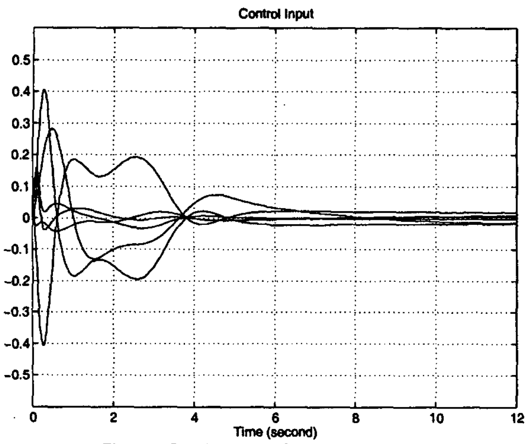


Figure 2 History of control with time

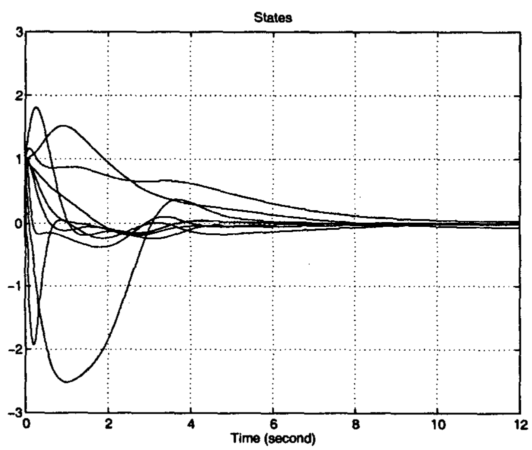


Figure 3 State trajectories with time

## 6 CONCLUSIONS

Our study shows that recurrent networks can be used successfully for online identification and control of complex nonlinear systems. Due to the parallelism of neural networks, our method may be attractive for implementation.

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