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# $\gamma'$-Realizability and Other Musings on Inverse Domination

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**Abstract**

We introduce and study $\gamma'$-realizable sequences. For a finite, simple graph $G$ containing no isolated vertices, $I \subseteq V(G)$ is said to be an *inverse dominating set* if $I$ dominates $G$ and $I$ is contained by the complement of some minimum dominating set $D$. Define a sequence of positive integers $(x_1, \ldots, x_n)$ to be *$\gamma'$-realizable* if there exists a graph $G$ having exactly $n$ distinct minimum dominating sets $D_1, \ldots, D_n$ where for each $i \in \{1, \ldots, n\}$, the minimum size of an inverse dominating set in $V(G) \setminus D_i$ is equal to $x_i$. In this work, we show which sequences having minimum entry 2 or less are $\gamma'$-realizable. We then detail a few observations and results arising during our investigations that may prove useful in future research.

**Keywords and phrases:** domination, inverse domination, graph realizations

# 1 Introduction

All graphs considered in this work will be finite, simple, and contain no isolated vertices. All sequences will be finite and contain as their entries positive integers only. For a guide to the labyrinth of topics and avenues of exploration one may encounter in the subject of domination theory, we refer our readers to [4], but for our needs, we can make do with the following basics. For a graph $G$, a set $D \subseteq V(G)$ is said to be a *dominating set* (or just *dominating*, for short) if for each vertex $v \notin D$, $v$ is adjacent to some vertex of $D$. The size of a minimum dominating set is called the *domination number* of $G$ and is denoted $\gamma(G)$. The concept of inverse domination is introduced in [5]. A set $I \subseteq V(G)$ is said to be an *inverse dominating set* if $I$ dominates $G$, and $I$ is contained in the complement of some minimum dominating set. The size of a minimum inverse dominating set (considered over all possible minimum dominating sets) is called the *inverse domination number* of $G$ and is denoted $\gamma'(G)$. A fundamental observation in domination theory is that for a graph $G$ containing no isolated vertices and having $D$ as a minimum dominating set, $V(G) \setminus D$ also dominates $G$. Therefore, for each minimum dominating set $D$ of $G$, the minimum size of an inverse dominating set $I \subseteq V(G) \setminus D$ is well-defined.

In this work, we introduce the idea of $\gamma'$-realizability. For a non-decreasing sequence of positive integers $\mathbf{x} = (x_1, \ldots, x_n)$, we say $\mathbf{x}$ is *$\gamma'$-realizable* if there exists a graph $G$ having exactly $n$ distinct minimum dominating sets $D_1, \ldots, D_n$ where for each $i \in \{1, \ldots, n\}$, the minimum size of an inverse dominating set in $V(G) \setminus D_i$ is equal to $x_i$. For the sake of clarity, we offer the following example. Consider the sequence $(2, 2, 3)$. As it turns out, this sequence is $\gamma'$-realizable and is realized by the path of length four, $P_5$. To see this, label its vertex set $V(P_5) = \{v_1, \ldots, v_5\}$ where $E(P_5) = \{v_1v_2, v_2v_3, v_3v_4, v_4v_5\}$, and note that $\gamma(P_5) = 2$. Furthermore, $P_5$ contains three minimum dominating sets which we designate as $D_1 = \{v_1, v_4\}$, $D_2 = \{v_2, v_5\}$, and $D_3 = \{v_2, v_4\}$. For each $i \in \{1, 2, 3\}$, let $I_i$ be a minimum inverse dominating set in $V(P_5) \setminus D_i$, and we have $I_1 = \{v_2, v_5\}$, $I_2 = \{v_1, v_4\}$, and $I_3 = \{v_1, v_3, v_5\}$. As $|I_1| = |I_2| = 2$ and $|I_3| = 3$, we have that $P_5$ realizes $(2, 2, 3)$.

Equipped with this definition, one may immediately ask which sequences are $\gamma'$-realizable. Although our assessments indicate that this question may be difficult to answer in complete generality, we make a beginning attempt in Section 2. There, we consider sequences whose smallest entry is a 1 or 2, and determine exactly which of those are $\gamma'$-realizable. We then turn our attention to a few related problems that have popped up along the way. In Section 3, we study the effect of edge-deletion on the parameter $\gamma'(G)$, ultimately showing that for any integer $k$, there exists a graph $G$ with $e \in E(G)$, where $\gamma'(G) - \gamma'(G - e) = k$. In Section 4, we offer a few questions for future study.

## 2 Sequences with Small Entries

For a sequence containing a 1 as an entry, it is quite easy to decide if that sequence is $\gamma'$-realizable.

**Theorem 2.1.** *A sequence containing a 1 is $\gamma'$-realizable if and only if it has length at least 2, and each other entry in the sequence is also a 1.*

*Proof.* Let $\mathbf{x}$ be a sequence with some entry being a 1. Since $\gamma(G) \leq \gamma'(G)$, for $\mathbf{x}$ to be realized by a graph $G$, we must have $\gamma(G) = \gamma'(G) = 1$. In other words, there are two vertices of $G$, call them $v_1$ and $v_2$, that dominate all of $G$. Thus $\mathbf{x}$ must have length at least two. Any minimum dominating set $D$ of $G$ will have at least one of $v_1, v_2$ in $V(G) \setminus D$, so any entry of $\mathbf{x}$ must be a 1. Now supposing that $\mathbf{x}$ is a sequence of length $n \geq 2$ whose every entry is a 1, we have that $\mathbf{x}$ is realized by the complete graph $K_n$. □

For the rest of this section, we will operate under the assumption that any sequence mentioned contains no 1's as entries.

**Theorem 2.2.** *Let $m \geq 2$ and let $\mathbf{x} = (m, \ldots, m)$ be a sequence length $n$. Then $\mathbf{x}$ is $\gamma'$-realizable.*

*Proof.* We construct a graph $G$ realizing $\mathbf{x}$ in the following fashion. Start with a copy of the complete graph $K_{mn}$, designate by $A$ its set of vertices, and partition $A$ into $n$ sets of $m$ vertices each. Label these disjoint sets $D_1, \ldots, D_n$, and for each $i \in \{1, \ldots, n\}$, label the vertices of $D_i$ as being $a_{i,1}, \ldots, a_{i,m}$. When our construction is finished, these $D_i$ will form the $n$ minimum dominating sets that $G$ requires.

For the next step in the construction of our graph $G$, place in $G$ a set $B$ of $mn$ independent vertices labeled $b_0, b_1, \ldots, b_{mn-1}$. We establish adjacencies between the vertices of $A$ and $B$ with the following recursive relation. Beginning with the vertices of $D_1$, let $a_{1,1}$ be adjacent to each of $\{b_0, \ldots, b_{n-1}\}$, $a_{1,2}$ be adjacent to each of $\{b_n, \ldots, b_{2n-1}\}$, $\ldots$, $a_{1,m}$ be adjacent to each of $\{b_{(m-1)n}, \ldots, b_{mn-1}\}$. For $i \in \{2, \ldots, n\}$, supposing vertex $a_{i-1,j}$ is adjacent to each of $\{b_\alpha, \ldots, b_{\alpha+n-1}\}$ for some positive integer $\alpha$, let $a_{i,j}$ be adjacent to each of $\{b_{\alpha+1}, \ldots, b_{\alpha+n}\}$, where each subscript is computed modulo $mn$.

For $i \in \{1, \ldots, n\}$, we now have that $D_i$ is a minimum dominating set, and since the $D_i$ are disjoint from each other, we have the existence of an inverse dominating set of size $m$ in each $V(G) \setminus D_i$. Our only issue of concern is that our construction did not accidentally result in $G$ having an $(n+1)^{th}$ minimum dominating set. We allay this concern by observing that each vertex of $A$ dominates exactly $n$ vertices of $B$, and if $m$ vertices of $A$ were chosen without each of them being members of the same set $D_i$, then there must be some vertex of $B$ dominated by two of the selected vertices. This guarantees that $G$ does in fact realize $\mathbf{x}$. □

We will now focus solely on sequences whose minimum entry is a 2, and through the next four lemmas establish exactly which of those sequences are $\gamma'$-realizable.

**Lemma 2.3.** *Let $\mathbf{x}$ be a sequence of length greater than two which has exactly one of its entries being a 2. Then $\mathbf{x}$ is not $\gamma'$-realizable.*

*Proof.* Let $\mathbf{x}$ be as described and suppose to the contrary that $\mathbf{x}$ is $\gamma'$-realizable. Let $G$ be a graph realizing $\mathbf{x}$ and note that $G$ cannot have a minimum dominating set of size one as that would force each entry of $\mathbf{x}$ to be a 1. It follows that there exists some minimum dominating set $D$ of size two along with an inverse dominating set $I \subseteq V(G) \setminus D$ with $I$ of size two as well. However, $I$ is also a minimum dominating set with $D$ as an inverse dominating set in its complement, and we have that $\mathbf{x}$ must have at least two of its entries being a 2. □



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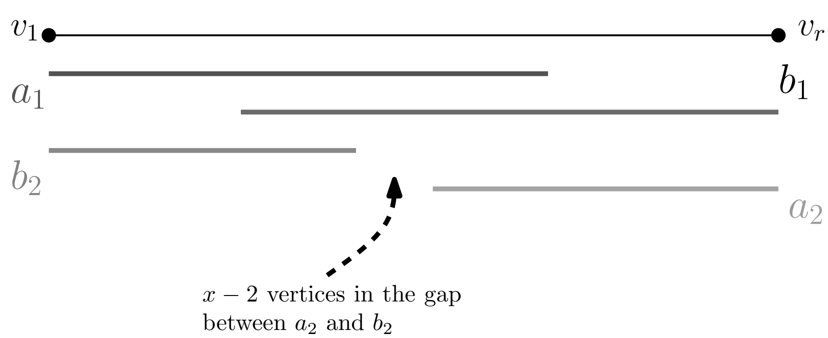
**Lemma 2.4.** *Let* $\mathbf{x}$ *be a sequence that contains at least one* 2 *as an entry and at least three entries that are each greater than* 2*. Then* $\mathbf{x}$ *is not* $\gamma'$*-realizable.*

*Proof.* Again, let $\mathbf{x}$ be as described and suppose to the contrary that $G$ is a graph realizing $\mathbf{x}$. We must have $\gamma(G) = 2$, and by Lemma 2.3, $G$ must have two disjoint minimum dominating sets which we will designate $\{a_1, a_2\}$ and $\{b_1, b_2\}$. Let $x_1, x_2, x_3$ be three entries of $\mathbf{x}$ that are each greater than two and label the minimum dominating sets corresponding to the inverse dominating sets of these sizes as being $D_1, D_2, D_3$. Since each of these $D_1, D_2, D_3$ cannot have an inverse dominating set of size two in their complement, we must have each of $D_1, D_2, D_3$ intersecting both $\{a_1, a_2\}$ and $\{b_1, b_2\}$ and also we must have that no two of $D_1, D_2, D_3$ can be disjoint. Without loss of generality, assume that $D_1 = \{a_1, b_1\}$ and $D_2 = \{a_1, b_2\}$. There is then no way of constructing $D_3$ to satisfy the above conditions. □

**Lemma 2.5.** *For* $n \geq 3$*, let* $\mathbf{x}$ *be a sequence of length* $n$ *that contains* $n - 1$ *2's as entries and one entry* $x$ *for any* $x \geq 3$*. Then* $\mathbf{x}$ *is* $\gamma'$*-realizable.*

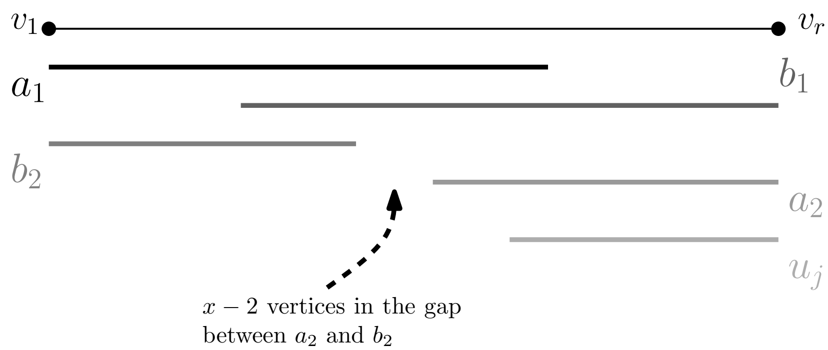
*Proof.* First, we consider the case $n = 3$, and create a graph $G$ to realize $(2, 2, x)$. Begin with a copy of $K_4$ with vertex set $\{a_1, a_2, b_1, b_2\}$. Now place a number of independent vertices $v_1, \ldots, v_r$ for some large $r$. We will place edges to make each of $a_1, a_2, b_1, b_2$ adjacent to various $v_i$, but instead of just listing the adjacencies, it is easier to depict them visually. In the diagram below, suppose all of the $v_i$ are situated on a line segment $\overline{v_1 v_r}$. The bars labeled $a_1, a_2, b_1, b_2$ will be used to indicate which of these $v_i$ are adjacent to the respective $a_1, a_2, b_1, b_2$, with the stipulation that if a vertex $v_i$ lies on the portion of the line segment above a bar, it is adjacent to the vertex corresponding to that bar. As $r$ can be made arbitrarily large, we can make those portions of the line segment above the overlap of two different bars have as many $v_i$ vertices as we want, but as long as there are $x - 2$ vertices not adjacent to either of $a_2$ and $b_2$, we have constructed the desired graph $G$.




In the end, $G$ has the following three minimum dominating sets and corresponding inverse dominating sets: $D_1 = \{a_1, a_2\}$ with $I_1 = \{b_1, b_2\}$, $D_2 = \{b_1, b_2\}$ with $I_2 = \{a_1, a_2\}$, and $D_3 = \{a_1, b_1\}$ with $I_3$ equaling the union of $\{a_2, b_2\}$ and the set of $x - 2$ $v_i$'s not adjacent to either or $a_2$ or $b_2$.

For $n > 3$, we start with the graph $G$ constructed above and place independent vertices $u_1, \ldots u_{n-3}$, making all of them adjacent to each of $a_1, a_2, b_1, b_2$. Our goal will be to place edges between the $u_j$'s and $v_i$'s so that each $\{a_1, u_j\}$ is a minimum dominating set. Note also that each $\{a_1, u_j\}$ has $\{b_1, b_2\}$ as a corresponding minimum inverse dominating set. The only precaution we need to take is to make sure that we do not interfere with $\{a_1, b_1\}$ having a corresponding minimum inverse dominating set of size $x$. This is easily done by making each of the $u_j$ adjacent to the same $v_i$'s, that is, if the vertices in $u_j$ are adjacent to $v_z, v_{z+1}, \ldots, v_r$ then $u_{j+1}$ is adjacent to $v_z, v_{z+1}, \ldots, v_r$, and $u_{j+2}$ is adjacent to $v_z, v_{z+1}, \ldots, v_r$, and so on – enough to ensure that $\{a_1, u_j\}$

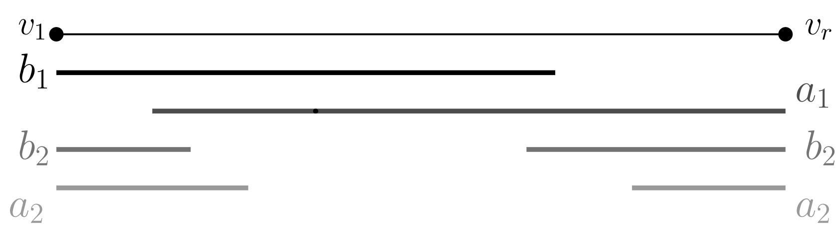
is dominating, but at the same time a proper subset of those $v_i$'s that are adjacent to $a_2$. Our diagram will now look like the one given below.




This completes the proof. □

**Lemma 2.6.** *For $n \geq 4$, let $\mathbf{x}$ be a sequence of length $n$ that contains $n-2$ 2's along with entries $x$ and $y$ for any $x, y \geq 3$. Then $\mathbf{x}$ is $\gamma'$-realizable.*

*Proof.* We begin by considering the sequence $(2, 2, x, y)$ and proceed in a fashion similar to Lemma 2.5. Form a graph $G$ by starting with a copy of $K_4$ whose vertices are labeled $a_1, a_2, b_1, b_2$. When our construction is complete, the four minimum dominating sets of $G$ will be $\{a_1, a_2\}$, $\{b_1, b_2\}$, $\{a_1, b_1\}$, and $\{a_1, b_2\}$. The first two of these four sets of vertices are disjoint and therefore have corresponding minimum inverse dominating sets of size two, while the last two of these four will end up having corresponding minimum inverse dominating sets of size $x$ and $y$, respectively. As in the proof of Lemma 2.5, for some large $r$, place independent vertices $v_1, \ldots, v_r$, and place edges between these vertices and $a_1, a_2, b_1, b_2$ as stipulated by the figure below. We just need to arrange the vertices $v_1, \ldots, v_r$ on the line segment so that there are $x-2$ $v_i$'s in the gap between $a_2$ and $b_2$ and $y-2$ $v_i$'s in the gap between $a_2$ and $b_1$.




For $n > 4$, we start with the graph $G$ constructed above and place independent vertices $u_1, \ldots u_{n-4}$, making all of them adjacent to each of $a_1, a_2, b_1, b_2$. As in the previous lemma, we again want to place edges between the $u_j$'s and $v_i$'s so that each $\{a_1, u_j\}$ is a minimum dominating set. Note that each $\{a_1, u_j\}$ has $\{b_1, b_2\}$ as a corresponding minimum inverse dominating set. Here we only need to take care that $\{a_1, b_1\}$ still has a corresponding minimum inverse dominating set of size $x$ and that $\{a_1, b_2\}$ has a corresponding minimum inverse dominating set of size $y$. To do this, just make each of the $u_j$ adjacent to the same $v_i$'s as given in the figure below.



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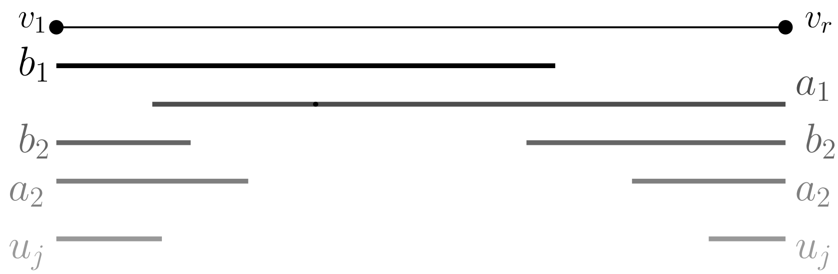



□

Summarizing the results of the previous lemmas, we have the main result of this section.

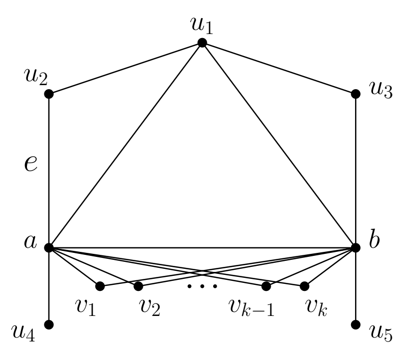
**Theorem 2.7.** *Let* $\mathbf{x}$ *be a sequence having minimum entry 2. Then* $\mathbf{x}$ *is* $\gamma'$*-realizable if and only if* $\mathbf{x}$ *consists of a single entry or* $\mathbf{x}$ *contains at least two 2's and at most two entries that are not 2's.*

## 3 Inverse Domination and Edge-Deletion

In this section we detail the behavior of the inverse domination number of a graph with regard to edge-deletion. It is shown that for any integer $k$, there exists a graph $G$ and $e \in E(G)$ such that $\gamma'(G) - \gamma'(G-e) = k$. The interesting (and perhaps unexpected) result is that, unlike $\gamma(G)$ or say, the independence number $\alpha(G)$, we have that $\gamma'(G)$ can actually increase as a result of the deletion of an edge.

**Theorem 3.1.** *For any* $k \geq 0$*, there exists a graph* $G$ *and* $e \in E(G)$ *such that* $\gamma'(G) - \gamma'(G-e) = k$.

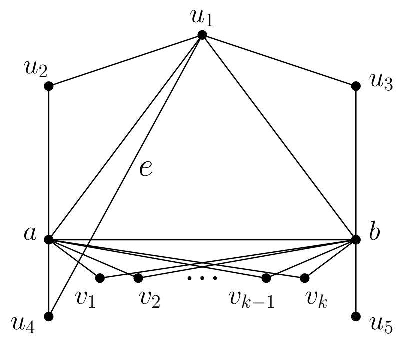
*Proof.* Consider the graph $G$ given below, noting that each of the vertices $v_1, \ldots, v_k$ is adjacent to both $a$ and $b$.




It is clear that $\gamma(G) = 2$ with $\{a, b\}$ being the only minimum dominating set of $G$. It is also straightforward to see that the only minimum inverse dominating set is $\{u_1, u_4, u_5, v_1, \ldots, v_k\}$, and so $\gamma'(G) = k + 3$. Letting $e = au_2$, we have that $\{a, b\}$ is no longer dominating in $G - e$. This means that $\gamma(G - e) = 3$, and in fact $\{a, u_1, u_5\}$ and $\{b, u_2, u_4\}$ are disjoint minimum dominating sets in $G - e$. This gives us $\gamma'(G - e) = 3$, and thus $\gamma'(G) - \gamma'(G - e) = k$. □

**Theorem 3.2.** *For any* $k \geq 0$*, there exists a graph* $H$ *and* $e \in E(H)$ *such that* $\gamma'(H-e) - \gamma'(H) = k$.

*Proof.* Let $G$ be the graph given in the proof of the previous theorem, and form $H$ by placing an edge between vertices $u_1$ and $u_4$. This graph is given in the figure below.




We have that $\gamma(H) = 2$, but in contrast to $G$, there are two minimum dominating sets of $H$, which we denote $D_1 = \{a, b\}$ and $D_2 = \{b, u_1\}$. The smallest size of an inverse dominating set of $H$ is 3, and is evidenced by $I = \{a, u_3, u_5\}$. Letting $e = u_1u_4$, we have $H - e = G$, and from the observations in the proof of the previous theorem, $\gamma'(H-e) = k+3$. So $\gamma'(H-e) - \gamma'(H) = k$. □

# 4 Further Work

In this section we collect a few problems that seem (to the authors at least) like fertile ground for future investigation. The most immediate is the question below.

**Question 1.** *Determine in general which sequences are $\gamma'$-realizable.*

It may also be worthwhile to examine which $\gamma'$-realizable sequences have a unique graph $G$ leading to their realization. However, to make this question interesting, we must first elaborate on what we mean by the word "unique". We do this with a toy example. Consider the sequence $\mathbf{x}$ of length one whose only entry is a 2, which is realized by the path of length 2, $P_3$. There are other graphs that realize $\mathbf{x}$ as well, and in fact, an infinite family of them can be constructed by, for any positive integer $n$, starting with $P_3$ and then placing $n$ independent vertices and making each of them adjacent to all three of the vertices of $P_3$. However, any graph realizing $\mathbf{x}$ must contain $P_3$ as a subgraph. To see this, note that a graph $G$ with no isolated vertices and not containing subgraph $P_3$ must simply be a collection of independent edges (or in other words, a matching) and will have more than one minimum dominating set. Thus $G$ cannot realize $\mathbf{x}$. Along these lines, we define a sequence $\mathbf{x}$ to be *uniquely $\gamma'$-realizable* if $\mathbf{x}$ is $\gamma'$-realizable, and there exists a graph $G$ realizing $\mathbf{x}$ such that any graph realizing $\mathbf{x}$ contains $G$ as a subgraph. Our question is now the following.

**Question 2.** *Determine which sequences are uniquely $\gamma'$-realizable.*

The most widely known open problem in the realm of inverse domination concerns the relationship between the inverse domination number and vertex independence number of a graph $G$. Define the *independence number* $\alpha(G)$ to be the maximum size of an independent set of vertices in $G$.

**Question 3.** *For an arbitrary graph $G$ containing no isolated vertices, is it true that $\gamma'(G) \leq \alpha(G)$?*

This question is answered in the affirmative as a theorem in [5], although the proof given in [5] was later found to have an irreparable hole. In the years since, Question 3 has been presented as a conjecture in [1], and shown to be true for various families of graphs (most notably, see [2]). In general, however, it is still open.

In Section 3, we showed the existence of a graph where the deletion of an edge causes the inverse domination number to decrease. With this in mind, formally define a graph $G$ to be *$\gamma'$-reducible* if there exists $e \in E(G)$ such that $\gamma'(G) > \gamma'(G-e)$.

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**Question 4.** *For a $\gamma'$-reducible graph $G$, is it true that $\gamma'(G) \leq \alpha(G)$?*

We present Question 4 because its resolution could offer a new avenue of attack for Question 3. Of course, if a $\gamma'$-reducible graph $G$ is produced to answer Question 4 in the negative, that same $G$ answers Question 3 in the negative as well. So let's suppose that through some amount of ingenuity and luck, Question 4 is proven to have a positive answer. Assume that Question 3 possesses a negative answer – that is, there exists some graph $G$ satisfying $\alpha(G) < \gamma'(G)$. Furthermore, we may assume that this graph $G$ is minimum and also that it is connected.

Since $G$ is minimum, for every $e \in E(G)$, the graph $G'$ formed by deleting $e$ does not satisfy $\alpha(G') < \gamma'(G')$. This could occur for two reasons. Either $\alpha(G') \geq \gamma'(G')$, in which case $e$ must be critical to the independence number of $G$, or $\gamma'(G')$ is undefined as $G'$ may have an isolated vertex. However, it turns out that the latter option cannot occur. To see this, first note that $G$ cannot be a star as any star has its independence and inverse domination numbers being equal. Letting $e = ab$ be a pendant edge of $G$, we must then have that $G$ contains the path $P_4$ as a subgraph, whose edges we will label $ab, bc, cd$. The edge $bc$ must be critical to $\alpha(G)$. Therefore, any maximum independent set $I$ of $G - bc$ must have $b, c \in I$. However, $I' = I \setminus \{b\} \cup \{a\}$ is also an independent set with $|I| = |I'|$. This contradiction completes the claim that $\alpha(G') \geq \gamma'(G')$ and guarantees that each edge of $G$ is critical.

The reason we call attention to this line of inquiry is that so-called $\alpha$-critical graphs – that is, graphs $G$ satisfying $\alpha(G) < \alpha(G - e)$ for every $e \in E(G)$ – have been extensively studied (see [3], [6], and many others). Perhaps some already established properties of such graphs could be used to eventually resolve Question 3.

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