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NONLINEAR TRACKING CONTROL OF BRUSHLESS DC MOTORS FOR HIGH-PERFORMANCE APPLICATIONS

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ABSTRACT

The tracking control problem associated with Brushless DC Motors (BLDCM) for high performance applications is considered. To guarantee their high dynamic performance operation in motion control systems, the magnetic saturation and reluctance variation effects are accounted for in the BLDCM mathematical model. The trajectory tracking control problem is addressed in the context of the transformation theory of nonlinear systems. A nonlinear control law is implemented, which is shown to compensate for the nonlinearities of BLDCM. A case study is presented in which a direct drive inverted pendulum actuated by a BLDCM is chosen to investigate the effectiveness of the control law. The effectiveness of the proposed control in compensating for modeling errors, external disturbances, and measurement errors is demonstrated.

I. Introduction

The control problem of Brushless DC Motors (BLDCM) for high performance applications is considered. This study has been motivated by the increasing interest in adopting BLDCM for high performance applications. In recent years, brushless motors have become a viable choice for industrial applications, specially those related to robotics, numerically controlled machine tools, electric propulsion, etc., e.g.[1,9]. This increasing interest has been the consequence of the advantages of brushless motors compared to their brushed counter parts.

To guarantee the high performance of BLDCM in motion control applications, its mathematical model must include the effects of magnetic saturation as well as reluctance variations. Such a model constitutes a highly nonlinear and coupled dynamical system. Another class of brushless motors which has gained considerable attention in the motion control industry is the Switched Reluctance Motor (SRM). The detailed modeling and control of SRM has been studied by numerous researchers, e.g.[5,6]. However, SRM constitutes a different dynamical system from BLDCM, since the mutual inductances associated with the phase windings of SRM are usually neglected whereas in a BLDCM the mutual inductances play a significant role. This introduces a major difficulty in terms of the mathematical model when magnetic saturation is present and also in terms of construction of commutation strategies. The proposed approach in this paper eliminates the need for the derivation of explicit commutation strategies by representing the BLDCM mathematical model in a rotating frame.

Based on the transformation theory of nonlinear systems[4], a nonlinear control law is proposed and examined through computer simulations. A case study is presented in which a direct drive inverted pendulum actuated by a BLDCM, whose model has been experimentally evaluated and verfied, is considered.

II. BLDCM Mathematical Model

A BLDCM consists of a permanent magnet rotor, a position sensor mounted on the rotor, and a means to provide signals to the phase windings of the motor; see figure 1. The signals from the signal generator are synchronized with the output of the position sensor to provide the electronic commutation. The armature windings of a typical motor are 3-phase, Y-connected, sinusoidally distributed, and are located on the stator.

The governing differential equations describing the dynamic behavior of BLDCM may be written as

$$\underline{V}(t) = \underline{R}\,\underline{I}(t) + \frac{d\underline{\Lambda}(\underline{I},\theta)}{dt} \qquad (1)$$

where $\underline{V}(t)$ and $\underline{I}(t)$ are the input voltage, and current vectors, repectively. $\underline{R} = \mathrm{diag}\{R\}$is the resistance matrix. The motion of the rotor and the load attached may be described by

$$J\frac{d\omega}{dt} = T(\underline{I},\theta) - T_L \qquad (2)$$

where J is inertia, ω is the angular velocity of the rotor, T_L represents the load torque, θ is the angular displacement of the rotor, and T represents the torque generated by the motor. In the absence of magnetic saturation, the fulx linkage vector, $\underline{\Lambda}$, is expressed by

$$\underline{\Lambda}(\underline{I},\theta) = \underline{L}(\theta)\,\underline{I} + \underline{\Lambda}_m(\theta) \qquad (3)$$

where

$$\underline{L}(\theta) = \begin{bmatrix} L_{11}(\theta) & L_{12}(\theta) & L_{13}(\theta) \\ L_{21}(\theta) & L_{22}(\theta) & L_{23}(\theta) \\ L_{31}(\theta) & L_{32}(\theta) & L_{33}(\theta) \end{bmatrix} \qquad (4)$$

L_{jj} is the self inductance of phase j, and L_{jk} when j≠k represents the mutual inductance between phase j and phase k. λ_{mj} represents the flux linkage associated with the permanent magnet and phase j. Equation (1) represents a system of differential equations with time varying (periodic) coefficients. It is known [10] that for sinusoidally distributed windings, a floquet transformation, frequently referred to as Park's transformation, can be used to transform the above equations to a system of differnetial equations with constant coefficients, represented in a coordinate frame attached to the rotor.

For a BLDCM with sinusoidally distributed stator windings, the elements of the inductance matrix and the permanet magnet flux linkage vector, $\underline{\Lambda}_m(\theta)$, are defined as follows

$$L_{kk} = L_a - L_g\cos(2n\theta + \frac{2(k-1)\pi}{3}) \quad \text{for } k=1,2,3 \qquad (5)$$

$$L_{12} = L_{21} = -\frac{L_a}{2} - L_g\cos(2n\theta - \frac{2\pi}{3}) \qquad (6)$$

$$L_{13} = L_{31} = -\frac{L_a}{2} - L_g\cos(2n\theta - \frac{4\pi}{3}) \qquad (7)$$

$$L_{23} = L_{32} = -\frac{L_a}{2} - L_g\cos(2n\theta) \qquad (8)$$

$$\lambda_{mk} = K_e\sin(n\theta - \frac{2(k-1)\pi}{3}) \quad k=1,2,3 \qquad (9)$$

where L_a and L_g are parameters defining the nominal inductance value and the amplitude of the inductance variation, respectively. K_e is the electromotivs force constant and n is is the number of permanent magnet pole pairs. After applying the transformation tc the rotating frame, the following governing equations are obtained

$$v_q = Ri_q + L_q\frac{di_q}{dt} + nL_d i_d\frac{d\theta}{dt} + nK_e\frac{d\theta}{dt} \qquad (10)$$

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$$v_d = Ri_d + L_d\frac{di_d}{dt} - nL_q i_q \frac{d\theta}{dt} \quad (11)$$

where $L_q = (\frac{3}{2})(L_a - L_g)$, and $L_d = (\frac{3}{2})(L_a + L_g)$. The torque expression in terms of the new variables is

$$T(i_q,i_d) = (\frac{3n}{2})\{K_e i_q + (L_d - L_q)\, i_q i_d\} \quad (12)$$

Equations (10)-(12) define a set of constant coefficient nonlinear differential equations.

III. Nonlinear Tracking Control of BLDCM

The control problem is first addressed by considering a mathematical model for BLDCM when magnetic saturation has been neglected. This step simplifies the derivation of the feedback control law. Having developed the contol law, we will then generalize it for the case when magnetic saturation is present. The control problem is attacked as a feedback linearization problem. The need for deriving explicit commutation strategies is eliminated by incorporating the transformation of the BLDCM representation to the rotating frame. This in turn eliminates the explicit dependence of the flux linkages and the torque equation on the rotor displacement.

IIIa. Control of BLDCM without Magnetic Saturation

Consider the dynamic system with the following state space representation

$$\frac{d\underline{x}}{dt} = \underline{f}(\underline{x}) + \underline{g}(\underline{x})\,\underline{u}(t) \quad (13)$$

where $\underline{x} \in \mathbf{R}^n$, $\underline{u} \in \mathbf{R}^m$, $\underline{f}$: $\mathbf{R}^n \rightarrow \mathbf{R}^n$, and $\underline{g}$: $\mathbf{R}^n \rightarrow \mathbf{R}^{n\times m}$. The BLDCM model, equations (10)-(12), constitutes a dynamic system of the form given by (13), where $x_1=\theta$, $x_2=\omega$, $x_3=i_q$, $x_4=i_d$. It can be shown[3] that there exists a transformation $\mathcal{T}(\underline{x})$ which transforms the BLDCM governing equations to a linear system of equations

$$\frac{d\underline{y}}{dt} = \underline{A}\,\underline{y} + \underline{B}\,\underline{\upsilon} \quad (14)$$

in the Brunovsky canonical form with the Kronecker indices $\kappa_1=3$ and $\kappa_2=1$. The nonlinear control which achieves this transformation is given by

$$\underline{u}(t) = \begin{bmatrix} v_q \\ v_d \end{bmatrix} = \underline{\beta}^{-1}(\underline{x})\,[\underline{\upsilon}(t) - \underline{\alpha}(\underline{x})] \quad (15)$$

where

$$\underline{\beta}(\underline{x}) = \begin{bmatrix} \frac{k_1+k_2x_4}{L_q} & \frac{k_2x_3}{L_d} \\ 0 & \frac{1}{L_d} \end{bmatrix} \quad (16)$$

$$\underline{\alpha}(\underline{x}) = \begin{bmatrix} (k_1+k_2x_4)(k_3x_3+k_4x_2+k_5x_2x_3)-k_2x_3(k_6x_4+k_7x_2x_3) \\ k_6x_4+k_7x_2x_3 \end{bmatrix} \quad (17)$$

where $k_1=3nK_e/2J$, $k_2=3n(L_d-L_q)/2J$, $k_3=-R/L_q$, $k_4=-K_e/L_q$, $k_5=-L_d/L_q$, $k_6=-R/L_d$, and $k_7=L_q/L_d$.

Linear control design techniques are used to compute the control inputs υ_1 and υ_2 of the linearized system which in turn are used to compute the control voltages v_q and v_d, using equation (15).

The non-singular transformation $\mathcal{T}(\underline{x})$ exists[3] if

$$K_e + (L_d-L_q)\, i_d \neq 0 \quad (18)$$

The coefficient of i_d in (18), i.e. (L_q-L_d), represents the degree of reluctance variation associated with the motor. This normally has a much smaller magnitude than the magnitude of K_e. As a result, this condition can only be significant if the magnitude of i_d becomes very large. The significance of this condition is further reduced by choosing a stabilizing control law for the state variable i_d.

IIIb. Control of BLDCM with Magnetic Saturation

For applications where large quantities of torque are required, (to achieve high acceleration and deceleration rates as in direct drive systems), the existence of magnetic saturation is inevitable. However, the task of modeling the saturation nonlinearity is quite complex. The fact that the flux linkages of the phase windings of BLDCM are mutually coupled makes the modeling task even more complex. Here, a mathematical model based on experimental results is used which accounts for the magnetic saturation effect.

The approach adopted[3] is to represent the variation of the inductance parameters L_a, L_g, and the back emf constant K_e, as functions of current. This has been done by collecting experimental data and computing the best fitting piecewise continuous polynomials to represent the dependence of these parameters on the phase current variable. Furthermore, to be able to exploit the properties associated with the representation of the dynamics of BLDCM in the rotating frame, the parameters L_a, L_g, and K_e are considered to be piecewise constant functions of the current variable. It is important to note that since explicit functions have been obtained, the intervals of current in which these parameters take on constant values can be made arbitrarily small.

Having defined the mathematical model of the BLDCM in the presence of magnetic saturation in terms of piecewise constant coefficients, we can now generalize the control law as given in (14) of section IIIa with minor modifications. The control law of section IIIa remains the same except that the parameters L_a, L_g, and K_e are now considered to be piecewise constant functions of the phase current variables.

IV. Case Study: Direct Drive Inverted Pendulum Driven by BLDCM

The effectiveness of the proposed control law is examined through computer simulations. A direct drive inverted pendulum actuated by the BLDCM whose model has been constucted and verified is considered. The dynamics of the arm and the payload are modeled by

$$T_L = Mgl\cos(\theta) + Ml^2\frac{d^2\theta}{dt^2} \quad (19)$$

where M=2 is the payload mass, l=1 is the arm length, g is the gravitational acceleration, and θ is the displacement of the arm relative to the horizontal plane. In all of the simulation results presented below, the BLDCM operates in the presence of magnetic saturation.

The task is defined as tracking a given trajectory $\{\theta_d(t), \omega_d(t), \alpha_d(t)\}$. Figure 7 depicts the block digram of the feedback control system . The control inputs υ_1 and υ_2 are defined as follows:

$$\upsilon_1(t) = -h_0\int(y_1-\theta_d)\,d\tau - h_1(y_1-\theta_d) - h_2(y_2-\omega_d) - h_3(y_3-\alpha_d) + \frac{d\alpha_d}{dt} \quad (20)$$

$$\upsilon_2(t) = -h_d y_4 \quad (21)$$

The control gains h_i, i=0,...,3, are computed based on a fourth order reference model with two pairs of complex conjugate poles with natural frequencies $\omega_{n1}=\omega_{n2}=40$, and damping ratios $\xi_1=\xi_1=1$. The current stabilizing control υ_2 is computed with $h_d=10^3$. A cubic trajectory is prescribed to examine the performance of the control system.

In an actual application, the mass and inertia properties associated with the payload are subject to significant variations and uncertainties. This is of particular importance in direct drive systems since the inertia variations are directly transmitted to the motor shaft. Figure 2 illustrates the time history of the position error when there exist errors in the inertial properties of the payload. Figure 3 illustrates the behavior of the control system when the BLDCM model is subject to uncertainties in its parameter values. The inaccuracies correspond to the model parameters K_e, L_a, L_g, and R. Since the model has been verified experimentally, the parameter uncertainties are not expected to be large. However, the parameter which could be subject to significant variations is the phase resistance R, due to the sensitivity to temperature variations. To see the effect of variations in R on the performance of the system, figure 4 illustrates the time histories of the postion error along the trajectory when the value of R is subject to different degrees of uncertainties. The position error in this case does not asymptotically approach zero, although it remains within reasonable bounds. It is apparent from the simulation results that the controller performance is most sensitive to variations in the resistance parameter.

Thus far we have assumed that accurate feedback information for position, velocity and acceleration are available to generate the appropriate control inputs. In the absence of acceleration measurements, approximate acceleration information can be computed based on the system dynamic model. Figure 5 illustrates the behavior of the control system when estimated acceleration is used and the system is subject to modeling uncertainties. The peak position error depicted is in the same range as in the case when accurate acceleration measurements were available (see figure 3). The behavior of the control system in the presence of uncertainties in model parameters, payload inertia uncertainties, and acceleration is depicted in figure 6.

V. Conclusions

The BLDCM control problem has been studied. A nonlinear control law has been presented which compensates for the nonlinearities of the system. The effects of magnetic saturation and reluctance variations have been included in the BLDCM mathematical model. An approach has been adopted which eliminates the need for derivation of explicit commutation strategies. The method is computationally simple and thus suitable for real time control applications. The effectiveness of the control algorithm is demonstrated by considering a direct drive inverted pendulum actuated by a BLDCM with magnetic saturation present. The control system performs well even when the system is subject to substantial parameter uncertainties, provided that accurate acceleration information is available. When an estimated acceleration information is used the performance of the sytstem subject to large parameter uncertainties may be degraded, this may be alleviated by including a robust control term in the overall controller[3].

VI. Acknowledgements

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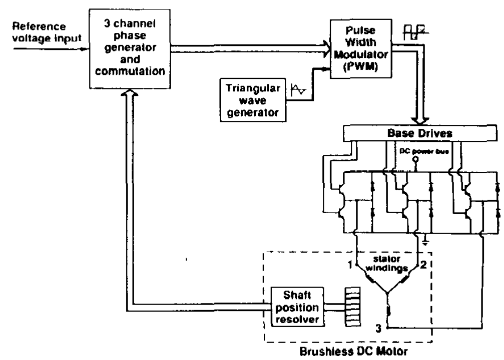

Figure 1: Typical BLDCM and its commutation.

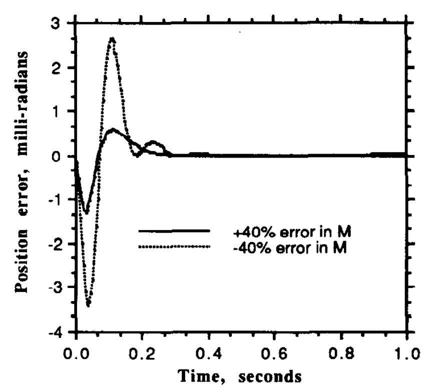

Figure 2: Time history of position error, in the presence of payload uncertainties.

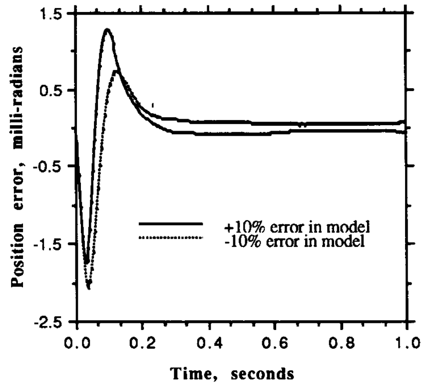

Figure 3: Time history of position error, in the presence of modeling errors.

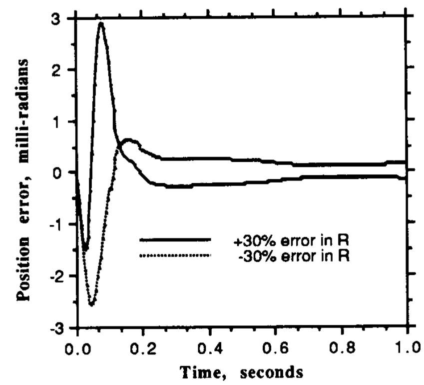

Figure 4: Time history of position error in the presence of resistance, R, errors.

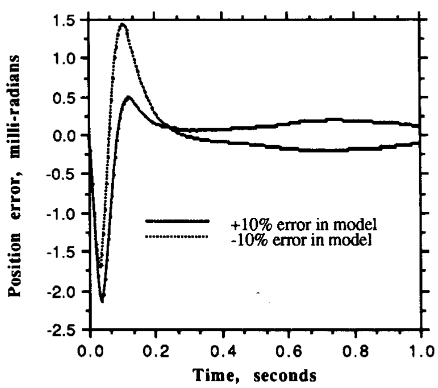

Figure 5: Time history of position error in the presence of modeling errors with estimated acceleration information.

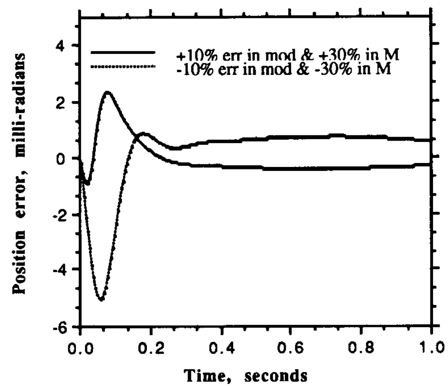

Figure 6: Time history of position error in the presence of modeling and payload uncertainties with estimated acceleration.

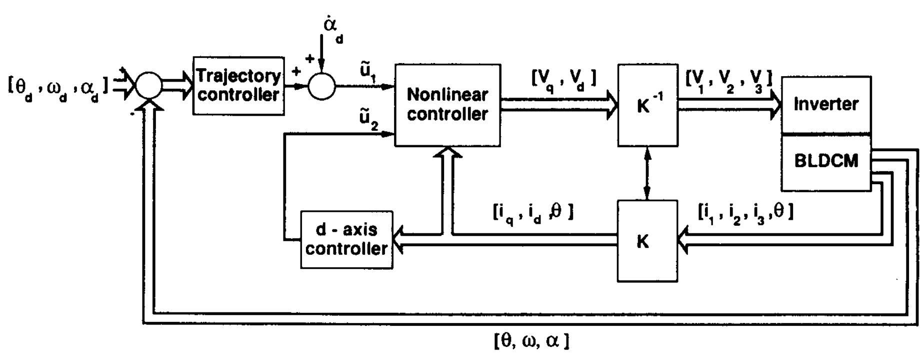

Figure 7: The block diagram of the nonlinear tracking control of BLDCM.
K and K^{-1} represent Park's transformation and its inverse.